Response of RWMs to pre-programmed, external perturbations in the EXTRAP T2R RFP

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What was done?

- Pre-programmed magnetic perturbations, in the spectral range of RWMs, have been applied using the RFX controller and the powered saddle coil array in EXTRAP T2R.
The measurements include

• Vacuum shots
• Plasma shots with and without applied perturbation.
The Results

- Wall penetration time ($\tau_{m,n}$) for vacuum shots.
- Growth and damping rates of the modes in the RWM spectral range.
- The ”Mutual” between active coil and sensor coil [Tesla/Amp] for the coil-wall-plasma system.
- Quantitative estimates of field errors and resonant field amplification.
- A multi-mode model has been developed.
Publication

• “Studies on the response of resistive wall modes to applied magnetic perturbations in the EXTRAP T2R reversed-field pinch”

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Actuators

- Active saddle coils outside the shell produce the external perturbation. \((m=1\) connected, 16 toroidal positions, 32 channels)
- L/R time constant 1 ms
- Coil current 20 A, magnetic field 1 mT (1% of equil. poloidal field)
- High-bandwidth audio amplifiers, output power 1 kW.
Pre-programmed controller

- RFX digital controller
- Pre-programme perturbation harmonic amplitude, phase and wave form.
- 32 control voltage outputs, one for each of the channels.
Sensors

- Array of one-turn flux loops measures radial magnetic flux through shell.
- $m=1$ connected saddle coils.
- 64 toroidal positions
- 128 signals
Configuration of sensor coil and active saddle coil toroidal arrays

**Sensor coil array**: 64x4 = 256 saddle coils. Each coil has 90° poloidal, 360/64 = 5.125° toroidal extent.

32x4 = 128 evenly spaced "m=1" pair-connected coils, 32x2 = 64 input sensor signals.

**Active coil array**: 16x4 = 64 saddle coils. Each coil has 90° poloidal, 360/32 = 11.25° toroidal extent.

Total surface coverage is 50%.

Coils are "m=1" pair connected into 16x2 = 32 independently driven coils.
The linear equations for the radial field, $b_{m,n}$, of an (m,n) harmonic measured by the sensor coils placed inside the wall but outside the vacuum vessel.
The linear equations for the radial field, $b_{m,n}$, of an $(m,n)$ harmonic measured by the sensor coils placed inside the wall but outside the vacuum vessel

Vacuum case (no plasma) for a mode with a wall penetration time $\tau_{m,n}^w$ and an applied external field $b_{m,n}^{ext}$

$$\frac{db_{m,n}}{dt} = \frac{1}{\tau_{m,n}^w} \left( b_{m,n}^{ext} - b_{m,n} \right)$$
The linear equations for the radial field, $b_{m,n}$, of an $(m,n)$ harmonic measured by the sensor coils placed inside the wall but outside the vacuum vessel.

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$$\frac{db_{m,n}}{dt} = \frac{1}{\tau_{m,n}^w} \left( b_{m,n}^{ext} - b_{m,n} \right)$$

Plasma case for a mode with a RWM growth (damping) rate $\gamma_{m,n}$ and the same applied external field.

$$\frac{db_{m,n}}{dt} = \frac{1}{\tau_{m,n}^w} \left( b_{m,n}^{ext} \gamma_{m,n} + \tau_{m,n}^w b_{m,n} \right)$$
Plasma case for a mode with a RWM growth (damping) rate $\gamma_{m,n}$, wall time $\tau_{m,n}^w$, and applied external field $b_{m,n}^{ext}$.

$$\frac{db_{m,n}}{dt} = \frac{1}{\tau_{m,n}^w} \left( b_{m,n}^{ext} + \gamma_{m,n} \tau_{m,n}^w b_{m,n} \right)$$

Saturation level determined by $$\frac{b_{m,n}^{ext}}{-\gamma_{m,n} \tau_{m,n}^w}$$

- $\gamma_{m,n} > 0$ (unstable)
- $\gamma_{m,n} = 0$ (marg. Stable)
- $-1 < \gamma_{m,n} \tau_{m,n}^w < 0$ (stable)
- $\gamma_{m,n} \tau_{m,n}^w = -1$ (simulates vac)
- $\gamma_{m,n} \tau_{m,n}^w < -1$ (robustly stable)
An important assumption is "mode rigidity".

- The stability index remains constant.
- The equilibria profiles in the plasma do not change.

\[ \frac{db_{m,n}}{dt} = \frac{1}{\tau_m^w} \left( b_{m,n}^{ext} + \gamma_{m,n} \tau_m^w b_{m,n} \right) \]

For example, if \( \gamma_{m,n} = 0 \), the mode grows linearly proportional to time until the assumptions are violated.
Single-mode model

• This equation is a single mode model.

\[
\frac{db_{m,n}}{dt} = \frac{1}{\tau_{m,n}^w} \left( b_{m,n}^{ext} + \gamma_{m,n} \tau_{m,n}^w b_{m,n} \right)
\]

• However, in T2R the active saddle coils are square wire loops and therefore produce both toroidal and poloidal sidebands.
Sidebands

- The toroidal sidebands have a spacing $\Delta n = 16$.
- Since there are 64 sensor coils, toroidal sidebands in the range $-32 < n < 31$ are resolved.
- The sensor coils ($m=1$ connected) are sensitive to the poloidal sidebands.
- The sensor signal picks up $m=1$ plus $m= -3, +5, -7$ etc.
Simulation using cylindrical model, with a smooth wall with T2R parameters and saddle coils with T2R parameters.

The saddle coil array produces poloidal sidebands, $m = -3, +5, -7$ etc.
To determine $\tau_{m,n}^w$, a short current pulse is applied.

- Pre-programme to produce $n = -2$ perturbation.
- Toroidal sidebands ($n = -18, +14, +30$) are resolved.
- The poloidal sidebands ($m = -3, +5, -7$) are in the sensor signal.
Multi-mode model.

\[
\begin{pmatrix}
\dot{b}
\end{pmatrix} = \left[ \gamma_w \right] \left( [M] I + [b_{err}] \right) + \left[ \gamma I b \right] + b^{ext}
\]

• The dynamics of the *measured* so-called \((m=1, n)\) mode is better described a multimode model (Set of ODEs).

• Where \([b], [M]\) are column arrays and \([\gamma], [\gamma_w]\) are diagonal matrices and \(I\) is the scalar current. Note \(\gamma_w = (1/\tau^w_{m,n})\).

• Note \([M]I\) is just \(b^{ext}\) due to the saddle coil current.

• \(b^{err}\) is the intrinsic error field.

• Each such set of ODEs has only one unstable pole.

• The sum of the complex terms of \([b]\) gives the amplitude and phase of the *measured* \((m=1, n)\) RWM.
Experimental estimates of $\tau_{m,n}^w$ and $M_{m,n}$. 

The vacuum pulse data is fitted to the multi mode model with two free parameters $\tau_{1,0}^w$ and a scaling factor $\beta$. 

![Graphs showing the fit of the vacuum pulse data to the multi mode model.](image)
Plasma shots

Plasma current and the RFP equilibrium parameters: $F$ and $\Theta$. 
The pulsed perturbation is applied to plasma shots

\[ \frac{d\left(b_{\text{with}} - b_{\text{without}}\right)}{dt} = \left(\gamma_n^w M_n I_{n0} + \gamma_{m,n} \left(b_{\text{with}} - b_{\text{without}}\right)\right) \]

• By subtracting w/o from w/, we have an equation for the plasma response to the applied perturbation.

• The mode spectrum and growth is very reproducible from shot-to-shot.

• Therefore we assume that the intrinsic error field is also reproducible.

• The "vacuum" parameters are known. We can then estimate the growth (damping rate) by fitting the data to the multi-mode model.
"n=-4" pulsed perturbation is applied to a plasma shots

Raw data for the three types of shots: Target $n=-4$

- vacuum
- without pert.
- with pert.

\[ b_n \left( 10^{-4} \text{ T} \right) \]

\[ n = -4 \]

\[ \text{phase (degrees)} \]

\[ \text{Time (ms)} \]
Subtracted raw data showing plasma response

\[ \frac{d\left(b_{n,\text{with}} - b_{n,\text{without}}\right)}{dt} = \left(\gamma_n^w M_n I_{n0} + \gamma_{m,n} \left(b_{n,\text{with}} - b_{n,\text{without}}\right)\right) \]

Vacuum (black) and Subtracted (red) signals
"\(n=+12\)" pulsed perturbation is applied to a plasma shot

Raw data for the three types of shots: Target \(n=+12\)

- vacuum
- without pert.
- with pert.

\[ n = 12 \]

Time (ms)
Subtracted raw data showing plasma response

$$\frac{d(b_{n,\text{with}} - b_{n,\text{without}})}{dt} = \left( \gamma_n^w M_n^0 I_{n0} + \gamma_{m,n} (b_{n,\text{with}} - b_{n,\text{without}}) \right)$$

Vacuum (black) and Subtracted (grey) signals

Time (ms)
Estimate growth and damping rates by fitting data to multi-mode model.

Then simulate data using the $\gamma$ and $M$ parameters.

Compare data and model
- green: vacuum
- blue: data
- red: simulation

Target $n=-6$
The growth (damping) rates for $m=1$ and $m=-3$ for the range of $-11 < n < 15$.

Compare experimental estimates (circles) with cylindrical model (dots).

Note that the multi-mode model with external pulsed perturbations gives a much better basis for comparison with theory than the mode amplitude spectrum.
The growth (damping) rates for $m=1$ and $m=-3$ for the range of $-11 < n < 15$.

Compare experimental estimates (circles) with cylindrical model (dots).
The resonant field amplification can be studied also for stable modes.

The \( n=22 \) mode is robustly stable.

For robustly stable modes, the amplitude of the response should saturate determined by an amplification factor.

\[
A_{RFA} = \frac{(b_{sat} - b_{ext})}{b_{ext}} = -\frac{(\gamma^w + \gamma)}{\gamma}
\]

The saturation data gives:

\( A_{RFA} = -0.31 \)
\( A_{RFA} = -0.15 \)

The previous pulsed data for the damping rate of the \( n=22 \) mode gives:

\( A_{RFA} = -\frac{(\gamma^w_{22} + \gamma_{22})}{\gamma_{22}} = -0.20 \)
## Summary

The multi-mode model gives a good simulation of the mode dynamics including the effects of poloidal sidebands.

The growth rates, damping rates and mutuals can be experimentally estimated and compared with the theory for a cylinder geometry.

Resonant field error effects are present in the system. They can be modeled. They can be suppressed.

## Plans

Run experiments using toroidal field pick-up coil array as sensors.

Look for mode coupling effects.