Simulation of Nonlinear Interactions between $n=1$ Internal Kink and High-$n$ Pressure-driven MHD Instabilities

Y. Kagei$^1$, Y. Kishimoto$^{2,1}$, and T. Miyoshi$^3$

$^1$ Naka Fusion Institute, Japan Atomic Energy Agency, Japan
$^2$ Graduate School of Energy Science, Kyoto University, Japan
$^3$ Graduate School of Science, Hiroshima University, Japan

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Outline

1. Introduction
   Development of fully compressible nonlinear MHD simulation code under the NEXT (Numerical EXperiment of Tokamak) Project in JAEA

2. Objective

3. Modeling

4. Results
   Our first simulation results showing that m/n=1/1 internal kink mode have an influence on pressure-driven high-m/n modes

5. Summary
Introduction

- Fully compressible nonlinear MHD Code development under the NEXT Project in JAEA

Basic equations:
  Fully-compressible, single-fluid MHD

Grid:
  (I)Structured rectangular grids,
  (II)Orthogonal flux coordinate grids,
  and (III)Unstructured grids
  * The development of the unstructured code is still underway.

Discretization:
  Finite-volume(Poloidal)/Pseudo-spectral(toroidal) method
  ••• Satisfy divergence free condition

Time integration:
  (I)Explicit (Runge-Kutta) or
  (II)Semi-implicit scheme
Objective

- Understanding nonlinear interactions between various unstable modes which differ in their scales and their sources of instabilities
  
  e.g. Current-driven 1/1 kink mode and Pressure-driven high-m/n modes

- Importance of understanding saturation mechanisms of their modes in such a plasma
Simulation Model (1)

Rectangular plane:
Quadrilaterals, \((N_R, N_Z) = (256, 256)\)

Toroidal direction:
Pseudo-spectral representation \((n)\) from -32 to +31

Boundary:
No-slipping and perfect conducting wall

Initial Configuration:
An axisymmetric relaxed state obtained by a 2-D simulation with the resistivity \(\eta = 0\), but with a nonzero viscosity \(\nu\)

Basic equations:
Finite-volume(finite-differential)/spectral discretization of 3-D fully compressible nonlinear MHD equations

*) The finite volume method on the structured grid, such as this work, corresponds to the finite difference method in the generalized coordinate system.
Simulation Model (2)

- Mass conservation
  \[ \frac{\partial}{\partial t} (\sqrt{g} \rho) = - \frac{\partial}{\partial x^i} (\sqrt{g} \rho V^i) \]

- Momentum conservation
  \[ \frac{\partial}{\partial t} (\sqrt{g} \rho V) = - \frac{\partial}{\partial x^i} (\sqrt{g} \rho V V^i) - \frac{\partial}{\partial x^i} (\sqrt{g} P g^i) + g (J^j B^k - J^k B^j) g^i + \nu \frac{\partial}{\partial x^i} \left( \sqrt{g} g^{ij} \frac{\partial}{\partial x^j} V \right) \]

- Energy conservation
  \[ \frac{\partial}{\partial t} (\sqrt{g} P) = - \frac{\partial}{\partial x^i} (\sqrt{g} P V^i) - (\gamma - 1) P \frac{\partial}{\partial x^i} (\sqrt{g} V^i) + (\gamma - 1) \eta \sqrt{g} g_{ij} J^i J^j \]

- Faraday's law of induction
  \[ \frac{\partial}{\partial t} (\sqrt{g} B^i) = - \left( \frac{\partial}{\partial x^j} E_k - \frac{\partial}{\partial x^k} E_j \right) \]

- Ampere's law
  \[ \sqrt{g} J^i = \frac{\partial}{\partial x^j} B_k - \frac{\partial}{\partial x^k} B_j \]

- Ohm's law
  \[ E_i = - \sqrt{g} (V^j B^k - V^k B^j) + \eta J_i \]

- Control volume
  \[ \sqrt{g} \equiv (g_R \times g_Z) \cdot g_\varphi \]
Initial state dependence of the linear growth rate

- A high beta tokamak plasma with $q_0 < 1$ linearly unstable to the resistive internal kink mode ($n=1$) and to the resistive pressure-driven modes (Peaked at $n=12$)
- Growth rates about comparable to each other at $\beta_p \approx 0.7$
  $\Rightarrow$ Nonlinear simulation for the parameter value are conducted
Nonlinear evolution of instabilities

$\beta_p = 0.01$

$\beta_p = 0.7$

$\beta_p = 0.01$: The growth of $n > 1$ modes are nonlinearly excited by the $n=1$ internal kink

$\beta_p = 0.7$: $n=1$ and $n > 1$ modes are linearly unstable

Some of the mode with $n > 1$ are accelerated in the early nonlinear regime

$n > 1$ modes appears to saturate at last
Nonlinear evolution of instabilities

$n=1$ mode in the early nonlinear regime ($t/\tau_A \sim 2000-2500$)

- the same growth rate
- On a $q=1$ rational surface with the same helicity
Nonlinear evolution of instabilities

A Similar behavior to that observed for $2 \leq n \leq 5$ (as shown in the previous viewgraph) can be observed for other $n$, excepted around the fastest growing mode $n=12$. 
Nonlinear saturation of high-n instabilities

Fully nonlinear regime (t/τ_A > 2500)

Several phase reversal events & Saturation of high-n modes

The first phase reversal event occurs on the q=1 rational surface (t/τ_A ~ 2640)

The amplitude of reversed phase increases with time, and after that, the next event occurs

At last, a coupling structure between m and m±1 becomes different from that in early nonlinear regime (t/τ_A ~ 2700)
Lattice model of nonlinear mode coupling

The growth of the internal kink mode (1/1) ⇒
⇒ Early nonlinear regime:
   An excitation of the high-m/n helical modes by the nonlinear coupling effect (yellow) ⇒
⇒ Fully nonlinear regime:
   A change of an original coupling structure of pressure-driven high-m/n modes (blue)

The change of the coupling structure may be related to the saturation of high-m/n modes
Time evolution of pressure profiles

$\varphi = 0$ (\(\varphi\) is toroidal angle)

$\varphi = \pi$

- The phase reversal events prevent fingers from growing in the bad curvature region
Helical finger structure in saturated state

- Helically distorted fingers are formed accompanied by $m/n=1/1$ kink motion
- The phenomena is as if the plasma transfers fingers from bad- to good- curvature region
- More detailed is under investigation. Consistency of the results will be checked in the future.
Summary

- Development of a 3-D fully-compressible nonliner MHD simulation code based on the finite-volume/spectral scheme

- A nonlinear simulation of MHD instabilities of a high beta plasma unstable to pressure driven modes (high-n) whose growth rates are about comparable to resistive internal kink mode (low-n)

- Findings
  1) Appearance of a coupled modes with the same helicity around a q=1 rational surface in the early nonlinear regime
  2) A change of the poloidal mode coupling structure and a saturation of high-m/n modes in the fully nonlinear regime
  - Prevention of fingers from growing at bad curvature region
  - Formation of a helical finger structure

- Consideration using a lattice model
  Consistency will be checked and then more detailed will be investigated in the future.