# Internal Kink Stability with Trapped-Particle Kinetic Effects in ITER

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# Introduction

- In the standard operation scenario, ITER can be unstable to the $n = 1$, $m = 1$ internal kink.
- Trapped particles can significantly affect the stability threshold of internal kink.
- Previously analytic models were used to study the possibility of the suppression of internal kink by trapped-particle kinetic effects.
  - $\alpha$ particles: Coppi, Migliuolo, Pegoraro and Porcelli, Physics of Fluids B 2, 927 (1990).
  - Thermal+$\alpha$ particles: Porcelli, Boucher and Rosenbluth, Plasma Physics and Controlled Fusion 38, 2163 (1996).

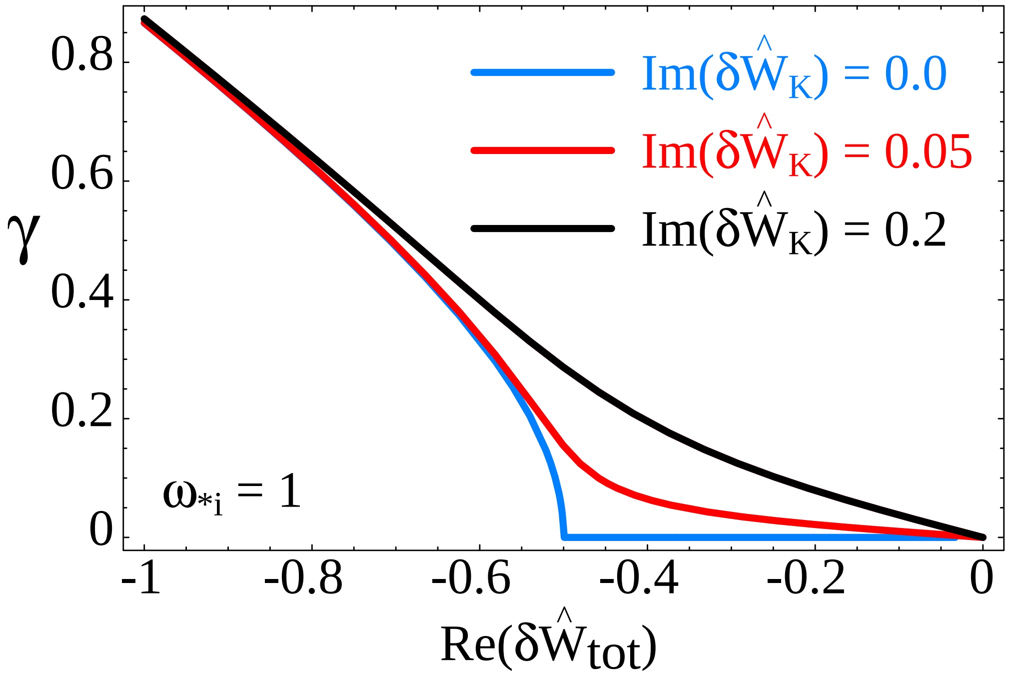
# Summary

**Trapped-particle ($\alpha$+thermal) kinetic effects significantly improve the internal kink stability in ITER equilibria. The internal kink is fully suppressed for $r_{q=1} \lesssim 0.5a$.**

- Since fluid stability of the internal kink is sensitive to the $q$ profile inside the $q = 1$ surface, we have generated ITER-like realistic equilibria with different $q$ profiles.
- The ideal MHD stability of those equilibria is computed with the ideal MHD code PEST-1.
- Using the ideal MHD eigenfunctions from PEST-1, the trapped-particle kinetic modification to the energy principle is computed.
- It is found that the internal kink is fully suppressed by trapped-particle kinetic effect as long as the radius of $q = 1$ surface is below a critical value($\simeq 0.5a$).

# Energy principle for internal kink

$$\sqrt{-\omega(\omega-\omega_{*i})} = -(\delta\hat{W}_F + \overbrace{\underbrace{\delta\hat{W}_K}_{\mathrm{Re}(\delta\hat{W}_K)+i\mathrm{Im}(\delta\hat{W}_K)}}^{\text{Kinetic}}) = -\delta\hat{W}_{\mathrm{tot}}$$



**Stability Condition:** $\delta W_F + \mathrm{Re}(\delta W_K) > 0$

# Calculation of $\boldsymbol{\delta W_K^{\mathrm{trapped}}}$

$$\begin{aligned}
\delta W_K^{\mathrm{trapped}} &= \sqrt{\frac{\pi}{2}} \sum_j \int d\Psi p_j \int d\Lambda \hat{\tau}_b \int_0^\infty d\left(\frac{\varepsilon}{T}\right)\left(\frac{\varepsilon}{T}\right)^{3/2} \exp\left(-\frac{\varepsilon}{T}\right) \\
&\quad \times \frac{\omega_{*j} + \omega_E - \omega}{\left\langle \omega_D^j \right\rangle + \omega_E - \omega - i\nu_{eff}} |\, \langle \boldsymbol{\kappa} \cdot \boldsymbol{\xi}_\perp \rangle \,|^2
\end{aligned}$$

- Resonant denominator: $\langle \omega_D \rangle + \omega_E - \omega - i\nu_{eff}$.
- $\omega_D$: trapped particle precession drift frequency.
- For toroidal rotation $\Omega < \omega_{*i}$, effect of $\boldsymbol{E} \times \boldsymbol{B}$ drift $\omega_E$ is relatively small ($\omega_E = \Omega - \omega_{*i}$), $\omega_r \gg \{\omega_E, \Omega, \omega_{*i}\}$.
- $j$: species, including thermal ions, electrons and $\alpha$ particles.

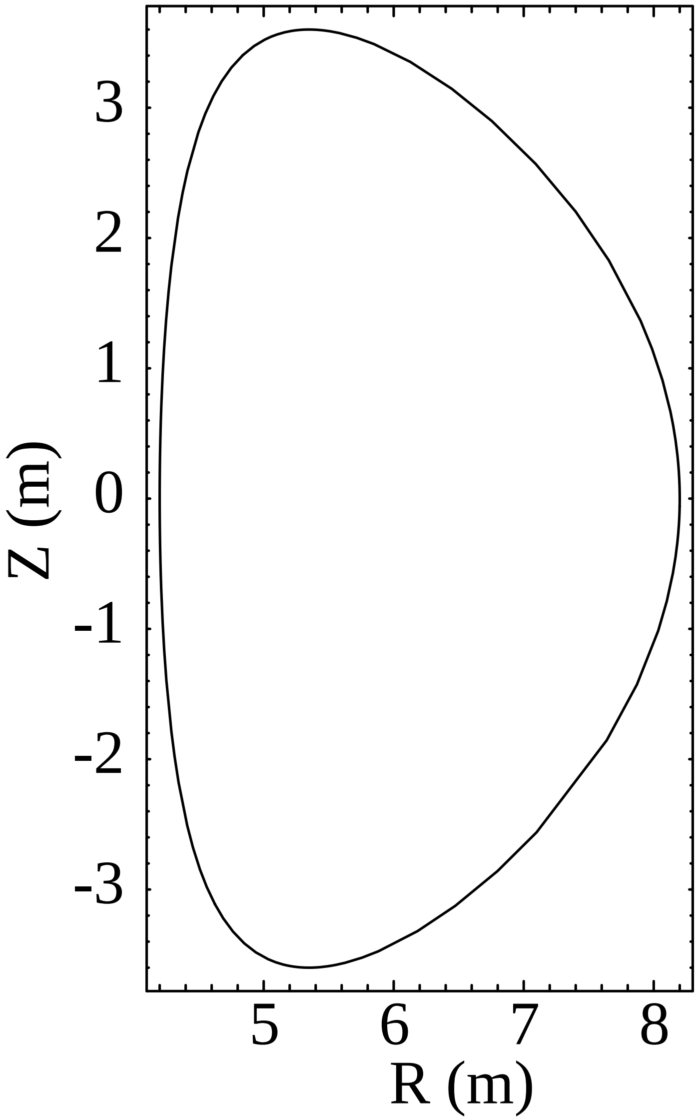
# Steps to calculate the internal kink growth rate

- For a given equilibrium, compute the ideal MHD eigenfunction $\boldsymbol{\xi}$ and $\delta W_F$ using the ideal MHD stability code PEST-1.

- Compute the kinetic correction $\delta W_K$ using the ideal MHD eigenfunction $\boldsymbol{\xi}$.

- Solve the internal kink dispersion relation $\sqrt{-\omega(\omega-\omega_{*i})} = -(\delta\hat{W}_F + \delta\hat{W}_K(\omega))$ for $\omega$, $\gamma = \mathrm{Im}(\omega)$.

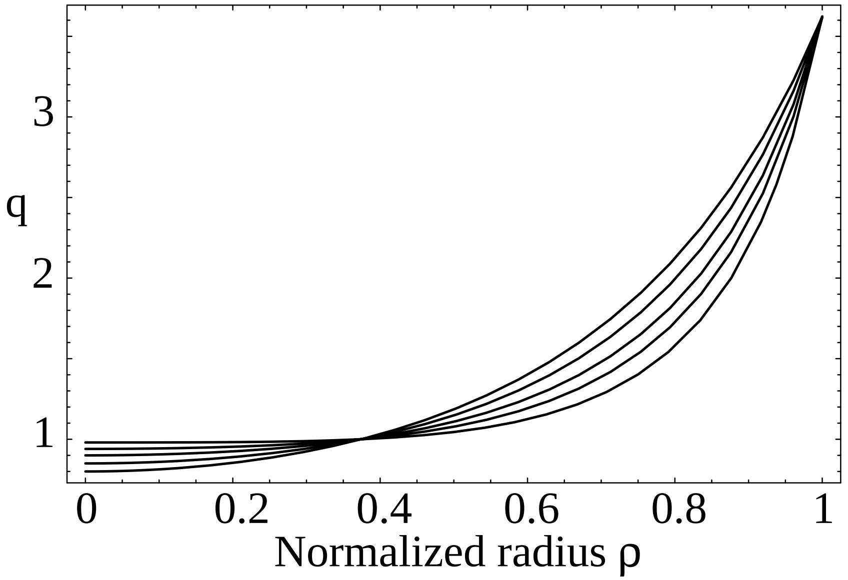
# Symmetrized ITER and its parameters

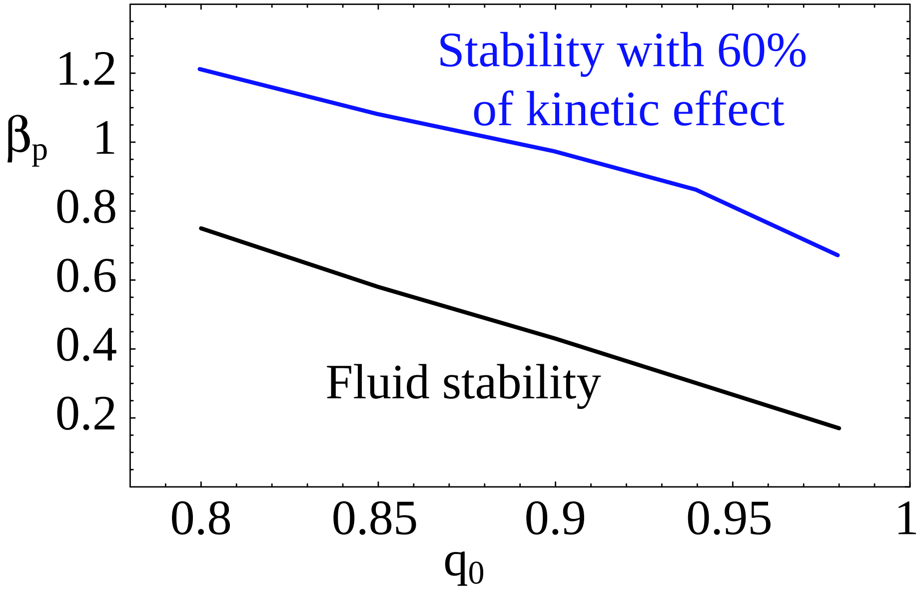


R = 6.2 m
a = 2.0 m
$\kappa$ = 1.8
$\delta$ = 0.44
B = 5.3 T
$n_e(0) = 10^{20} m^{-3}$
$T_i(0)$ = 19 keV
$T_e(0)$ = 23 keV

# Stability for $q$-profile series I $-$varying $\boldsymbol{q_0}$ with fixed $\boldsymbol{r_{q=1}}$ and $\boldsymbol{q_a}$

Full suppression of internal kink for $r_{q=1} \lesssim 0.5a$.



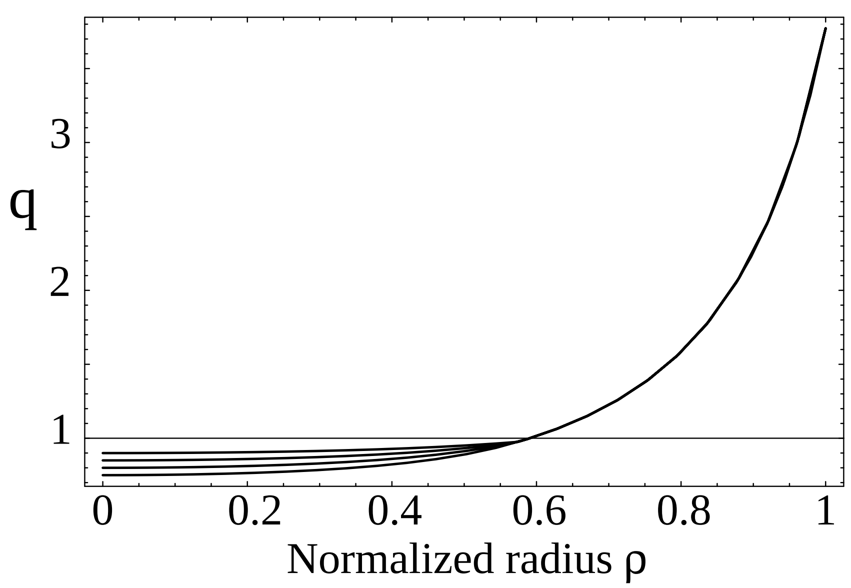


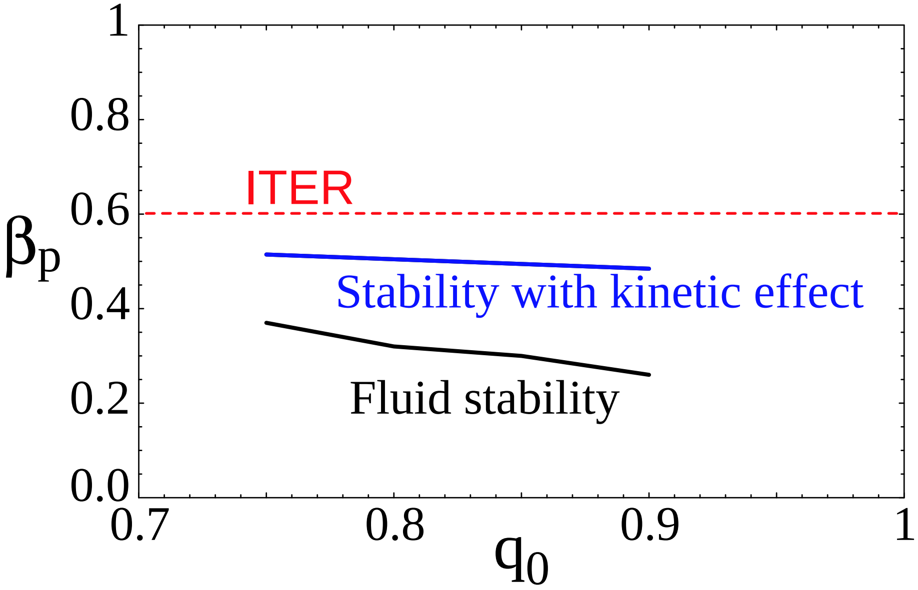
Mode is completely stabilized with all the kinetic effect up to $\beta_p = 1.0$.

$$\beta_p = \beta_p^{\mathrm{Bussac}} = \left(\frac{R}{r_1}\right)^2 \int_0^{r_1} \left(\frac{r}{r_1}\right)^2 \frac{-dp}{B^2/(2\mu_0)} \qquad r_1 = r_{q=1}$$

# Stability for $q$-profile series II – varying $q_0$ with fixed $r_{q=1}$ and $q_a$

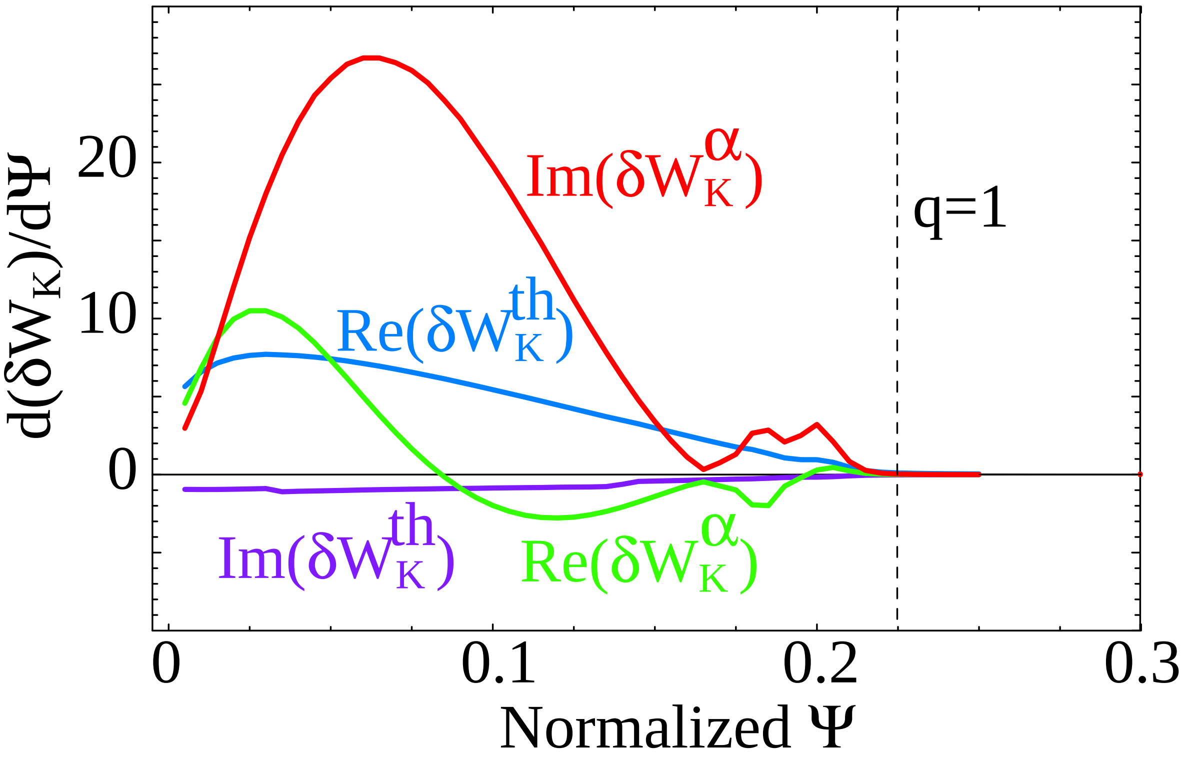
Unstable region for $r_{q=1} \gtrsim 0.5a$.





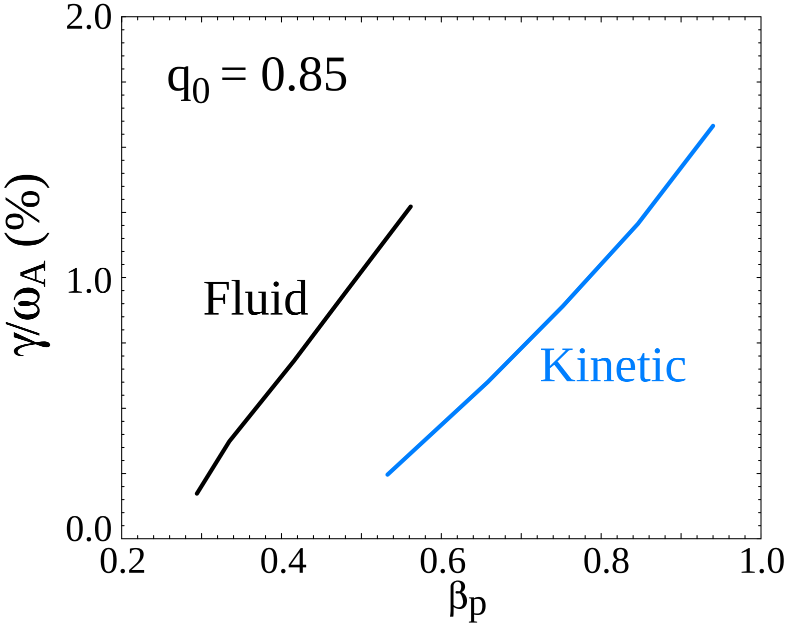
Kinetic effects significantly improve the stability threshold in $\beta_p$.

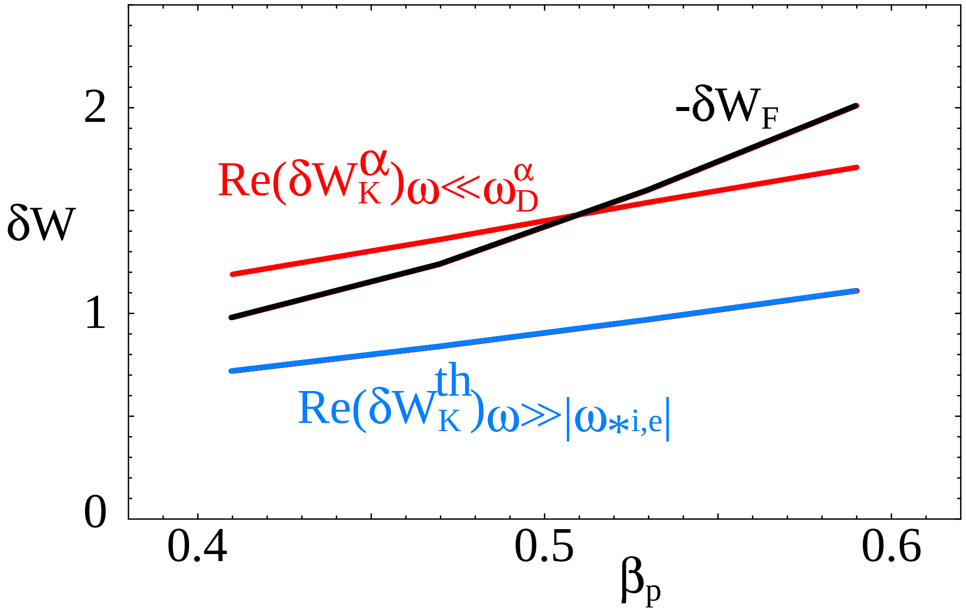
# Contributions to $\delta W_K$ along radial locations



- Real parts: $\mathrm{Re}(\delta W_K^{\mathrm{th}})$ is comparable to $\mathrm{Re}(\delta W_K^{\alpha})$,
- Imaginary parts: $\mathrm{Im}(\delta W_K^{\alpha})$ is much greater than $\mathrm{Im}(\delta W_K^{\mathrm{th}})$.
- Close to marginal stability, mode is fishbone-like($\omega_r \gg \gamma$), since resonance between $\omega$ and $\omega_D^{\alpha}$ gives a large $\mathrm{Im}(\delta W_K^{\alpha})$.

# Contributions to Re($\boldsymbol{\delta W}$) {$\boldsymbol{\delta W_F}$, Re($\boldsymbol{\delta W_K^\alpha}$), Re($\boldsymbol{\delta W_K^{th}}$)} have comparable amplitudes

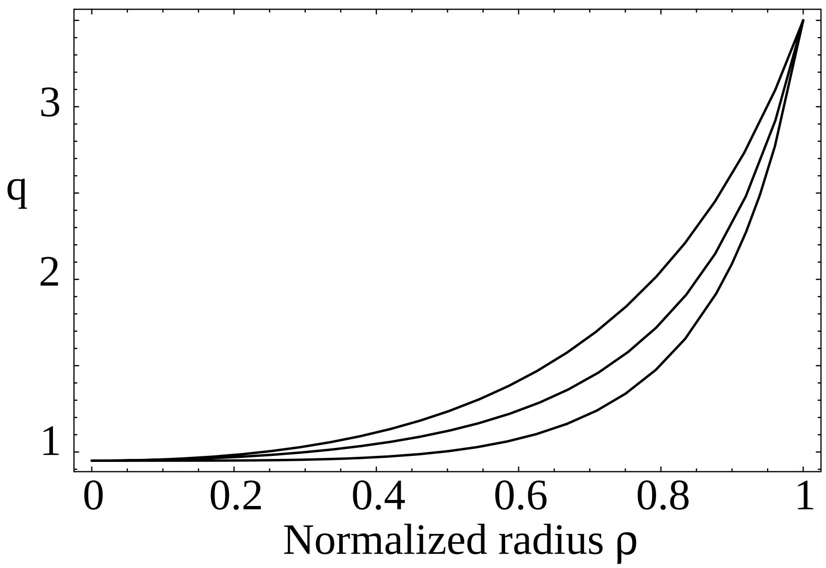


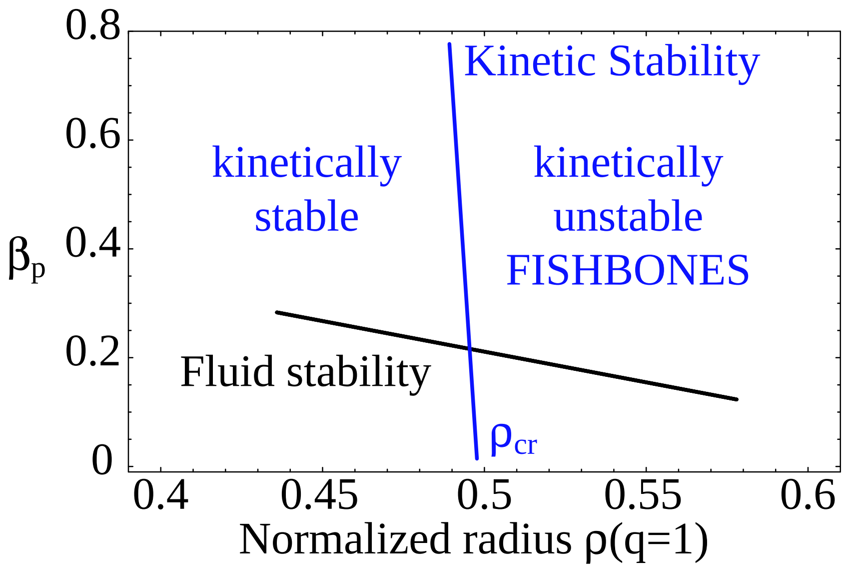


Though $\beta_\alpha/\beta \sim 6\%$, the $\alpha$ contribution Re($\delta W_K^\alpha$) is comparable to the fluid contribution $\delta W_F$ and the thermal contribution Re($\delta W_K^{th}$).

# Stability for $q$-profile series III $-$varying $r_{q=1}$ with fixed $q_0$ and $q_a$

The critical radius for kinetic stability is $r_{cr} \sim 0.5a$.





A critical radius $\rho_{\rm cr}$ determines the stability threshold.

If $\rho_{q=1} < \rho_{\rm cr}$, mode is fully stabilized with kinetic effect up to $\beta_p = 1.0$

# Explanation for the existence of a critical radius

- $m = 1$ mode has a large real frequency $\omega_r$.
- Alpha particle precession drift frequency $\omega_D^\alpha$ is a decreasing function of radius, and $\omega_r > \omega_D^\alpha$ beyond some radial location.
- For a large $r_{q=1}$, a large region develops within $r_{q=1}$ where $\omega_r > \omega_D^\alpha$, thus $\mathrm{Re}(\delta W_K^\alpha)$ can be negative, and reduce the stabilizing effect.
- If $\mathrm{Re}(\delta W_K^{\mathrm{tot}}) < 0$, kinetic effect is destabilizing.

# Conclusion

**Trapped-particle ($\alpha$+thermal) kinetic effects significantly improve the internal kink stability in ITER equilibria. The internal kink is fully suppressed for $r_{q=1} \lesssim 0.5a$.**

- Internal kink stability has a complicated dependence on $q_0$, $\beta_p$, $q'$, and $r_{q=1}$.
- According to fluid theory, the most unstable region has large $r_{q=1}$ with flat $q$ profile. Large $r_{q=1}$ also reduces the kinetic stabilization.
- Trapped-particle kinetic effects from thermal particles are comparable to those from $\alpha$ particles.
- Trapped-particle kinetic effects (thermal+$\alpha$) can fully suppress the internal kink mode, if $r_{q=1}$ does not exceed the critical radius($\simeq 0.5a$).