Internal Kink Stability with Trapped-Particle Kinetic Effects in ITER

Bo Hu and R. Betti

University of Rochester

J. Manickam

Princeton Plasma Physics Laboratory

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Introduction

- In the standard operation scenario, ITER can be unstable to the $n = 1, m = 1$ internal kink.
- Trapped particles can significantly affect the stability threshold of internal kink.
- Previously analytic models were used to study the possibility of the suppression of internal kink by trapped-particle kinetic effects.
Summary

Trapped-particle ($\alpha$-thermal) kinetic effects significantly improve the internal kink stability in ITER equilibria. The internal kink is fully suppressed for $r_{q=1} \lesssim 0.5a$.

- Since fluid stability of the internal kink is sensitive to the $q$ profile inside the $q = 1$ surface, we have generated ITER-like realistic equilibria with different $q$ profiles.
- The ideal MHD stability of those equilibria is computed with the ideal MHD code PEST-1.
- Using the ideal MHD eigenfunctions from PEST-1, the trapped-particle kinetic modification to the energy principle is computed.
- It is found that the internal kink is fully suppressed by trapped-particle kinetic effect as long as the radius of $q = 1$ surface is below a critical value ($\simeq 0.5a$).
Energy principle for internal kink

\[
\sqrt{-\omega(\omega - \omega_{\ast i})} = -(\delta \hat{W}_F + \delta \hat{W}_K) = -\delta \hat{W}_{\text{tot}}
\]

\[
\text{Kinetic: } \begin{cases} 
\delta \hat{W}_K \\
\text{Re}(\delta \hat{W}_K) + i \text{Im}(\delta \hat{W}_K)
\end{cases}
\]

Stability Condition: \( \delta W_F + \text{Re}(\delta W_K) > 0 \)
Calculation of $\delta W_K^{\text{trapped}}$

$$\delta W_K^{\text{trapped}} = \sqrt{\frac{\pi}{2}} \sum_j \int d\Psi_p \int d\Lambda \hat{\tau}_b \int_0^\infty \frac{d\left(\frac{\varepsilon}{T}\right)}{T^3} \exp \left(-\frac{\varepsilon}{T}\right) \times \frac{\omega_{*j} + \omega_E - \omega}{\langle \omega^j_D \rangle + \omega_E - \omega - i\nu_{\text{eff}}} |\langle \kappa \cdot \xi_\perp \rangle|^2$$

- Resonant denominator: $\langle \omega^j_D \rangle + \omega_E - \omega - i\nu_{\text{eff}}$.
- $\omega_D$: trapped particle precession drift frequency.
- For toroidal rotation $\Omega < \omega_{*i}$, effect of $\mathbf{E} \times \mathbf{B}$ drift $\omega_E$ is relatively small ($\omega_E = \Omega - \omega_{*i}$), $\omega_r \gg \{\omega_E, \Omega, \omega_{*i}\}$.
- $j$: species, including thermal ions, electrons and $\alpha$ particles.
Steps to calculate the internal kink growth rate

- For a given equilibrium, compute the ideal MHD eigenfunction $\xi$ and $\delta W_F$ using the ideal MHD stability code PEST-1.

- Compute the kinetic correction $\delta W_K$ using the ideal MHD eigenfunction $\xi$.

- Solve the internal kink dispersion relation
  \[ \sqrt{-\omega(\omega - \omega_{*i})} = - (\hat{\delta W}_F + \hat{\delta W}_K(\omega)) \text{ for } \omega, \gamma = \text{Im}(\omega). \]
Symmetrized ITER and its parameters

\begin{itemize}
  \item $R = 6.2 \text{ m}$
  \item $a = 2.0 \text{ m}$
  \item $\kappa = 1.8$
  \item $\delta = 0.44$
  \item $B = 5.3 \text{ T}$
  \item $n_e(0) = 10^{20} \text{ m}^{-3}$
  \item $T_i(0) = 19 \text{ keV}$
  \item $T_e(0) = 23 \text{ keV}$
\end{itemize}
Stability for $q$-profile series I
varying $q_0$ with fixed $r_{q=1}$ and $q_a$
Full suppression of internal kink for $r_{q=1} \lesssim 0.5a$.

Mode is completely stabilized with all the kinetic effect up to
$\beta_p = 1.0$.

$$\beta_p = \beta_p^{\text{Bussac}} = \left(\frac{R}{r_1}\right)^2 \int_0^{r_1} \left(\frac{r}{r_1}\right)^2 \frac{-dp}{B^2/(2\mu_0)} \quad r_1 = r_{q=1}$$
Stability for $q$-profile series II
- varying $q_0$ with fixed $r_{q=1}$ and $q_a$

Unstable region for $r_{q=1} \gtrsim 0.5a$.

Kinetic effects significantly improve the stability threshold in $\beta_p$. 
Contributions to $\delta W_K$ along radial locations

- Real parts: $\text{Re}(\delta W_{K}^{\text{th}})$ is comparable to $\text{Re}(\delta W_{K}^{\alpha})$,
- Imaginary parts: $\text{Im}(\delta W_{K}^{\alpha})$ is much greater than $\text{Im}(\delta W_{K}^{\text{th}})$.
- Close to marginal stability, mode is fishbone-like ($\omega_r \gg \gamma$), since resonance between $\omega$ and $\omega^\alpha_D$ gives a large $\text{Im}(\delta W_{K}^{\alpha})$.  

Contributions to $\text{Re}(\delta W) \{ \delta W_F, \text{Re}(\delta W_K^\alpha), \text{Re}(\delta W_K^{th}) \}$ have comparable amplitudes.

Though $\beta_\alpha/\beta \sim 6\%$, the $\alpha$ contribution $\text{Re}(\delta W_K^\alpha)$ is comparable to the fluid contribution $\delta W_F$ and the thermal contribution $\text{Re}(\delta W_K^{th})$. 
Stability for \( q \)-profile series III  
-varying \( r_{q=1} \) with fixed \( q_0 \) and \( q_a \)

The critical radius for kinetic stability is \( r_{cr} \sim 0.5a \).

A critical radius \( \rho_{cr} \) determines the stability threshold.  
If \( \rho_{q=1} < \rho_{cr} \), mode is fully stabilized with kinetic effect up to \( \beta_p = 1.0 \).
Explanation for the existence of a critical radius

- $m = 1$ mode has a large real frequency $\omega_r$.
- Alpha particle precession drift frequency $\omega_D^\alpha$ is a decreasing function of radius, and $\omega_r > \omega_D^\alpha$ beyond some radial location.
- For a large $r_{q=1}$, a large region develops within $r_{q=1}$ where $\omega_r > \omega_D^\alpha$, thus $\text{Re}(\delta W_K^\alpha)$ can be negative, and reduce the stabilizing effect.
- If $\text{Re}(\delta W_K^{\text{tot}}) < 0$, kinetic effect is destabilizing.
Conclusion

Trapped-particle ($\alpha$+thermal) kinetic effects significantly improve the internal kink stability in ITER equilibria. The internal kink is fully suppressed for $r_{q=1} \lesssim 0.5a$.

- Internal kink stability has a complicated dependence on $q_0$, $\beta_p$, $q'$, and $r_{q=1}$.
- According to fluid theory, the most unstable region has large $r_{q=1}$ with flat $q$ profile. Large $r_{q=1}$ also reduces the kinetic stabilization.
- Trapped-particle kinetic effects from thermal particles are comparable to those from $\alpha$ particles.
- Trapped-particle kinetic effects (thermal+$\alpha$) can fully suppress the internal kink mode, if $r_{q=1}$ does not exceed the critical radius($\approx 0.5a$).