MARS analysis of n>1 RWM stability
(+ update on role of q=2)

J. Menard, PPPL
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Motivation

• Understand role of q=2 surface in n=1 RWM stabilization via rotation + dissipation
  – Relevant to AT discharges with $q_{\text{MIN}} > 2$

• Compute stability of $n > 1$ RWMs stabilized with rotation + dissipation
  – Multiple unstable RWMs complicate feedback

• Look for theory predictions that can be compared to experiment
  – Test dissipation models for RWM damping
Present work is extension of systematic stability study of DIII–D AT scenario started last year

- Profiles from high $\beta_N = 4.1$ shots of AT shape expt.
  - $\beta_N > 4$ achieved transiently
  - High-$\kappa$ DND (like modification)

- Vary J profile to scan $q_{\text{min}}$
  - Weakly reversed, $q_0-q_{\text{min}} < 1$
  - $\rho_{q_{\text{min}}} = 0.4 - 0.5$
  - $q_{95} = 5–5.5$
  - $q_{99.7}$ fixed at 7.2

- AT scenario above extended to somewhat higher $\beta_N$ with larger $\rho_{q_{\text{MIN}}}$ using $I_p$, $B_T$ ramps by Garofalo, et. al this year – see his APS talk/paper
DIII-D AT scenarios exhibit systematic decrease in ideal-wall (IW) $\beta_N$ limit with increasing $q_{MIN}$ and $n$

- Computed $n=1,2,3$ kink limits with and w/o DIII–D vessel (CHEASE + DCON)

Focus on this region of stability space for this talk

- $n=3$ most unstable mode at the ideal wall limit
- Small gap between NW and IW limits for $n=3$
- Treat $n=1$ and $n=2$ where $\Delta \beta_N$ is $\geq 0.8$
Sound wave damping model predicts n=1 critical rotation increases strongly with SW damping strength \( \kappa_{||} \) when \( q_{\text{MIN}} > 2 \)

\[ \kappa_{||} = 0.05 \] \( n=1 \) \( \Omega_{\text{crit}} \tau_A (q=3) = 2.3\% \)

\[ \kappa_{||} = 0.5 \] \( n=1 \) \( \Omega_{\text{crit}} \tau_A (q=3) > 7\% \)

NOTE: \( n=1 \) RWM never fully stabilized at high rotation with larger \( \kappa_{||} \).

Consider lower dissipation limit to assess role of \( q_{\text{MIN}} \).
Removal of $q=2$ surface increases $n=1$ critical rotation by 35% at $q=3$ surface using low-$\kappa_{||}$ SW damping model

\[ q_{\text{MIN}} = 1.8 \quad \Omega_{\text{crit}} \tau_A (q=3) = 1.7\% \]
\[ q_{\text{MIN}} = 2.2 \quad \Omega_{\text{crit}} \tau_A (q=3) = 2.3\% \]

NOTE: These results only hold in the weak damping limit for SW damping
Using semi-kinetic damping model, removal of $q=2$ surface also increases $n=1$ critical rotation by 35% at $q=3$ surface.

$q_{\text{MIN}} = 1.8 \quad \Omega_{\text{crit}} \tau_A (q=3) = 0.46\%$

$q_{\text{MIN}} = 2.2 \quad \Omega_{\text{crit}} \tau_A (q=3) = 0.62\%$

NOTE: Abs. rotation value changes by only 5% for SK and low $\kappa_{\|}$ SW damping.
As in DIII-D, AT reactor scenarios exhibit decreasing ideal-wall limits with increasing toroidal mode number.

![Graph showing plasma and marginal wall comparison for ARIES-RS and ARIES-ST.](image)

**ARIES-RS**

\[ \gamma^2 / \gamma_A^2 \]

- \( n = 4 \) (x12.0)
- \( n = 3 \) (x4.5)
- \( n = 2 \) (x2.0)
- \( n = 1 \) (x1.0)

- Ideal \( n = 1-4 \) modes computed using GATO
- \( n = 2-5 \) modes are stable with a conducting wall at 1.2 \( a \)

\[ A = 4.0, \ \beta_N = 5.3, \ q(0) = 2.8 \]

**ARIES-ST**

- \( \delta = 0.57, \ \beta_N = 7.0, \ \beta = 39.7\% \)
- \( \delta = 0.35, \ \beta_N = 6.0, \ \beta = 32.0\% \)

**PHYSICS OPTIMIZATION OF THE ARIES-RS FUSION POWER PLANT - V.S. CHAN, APS 1999**

**Fusion Engineering and Design 65 (2003) 165/197**

**Fig. 4.** Conformal wall separation which marginally stabilizes \( n = 1-6 \) kink modes as a function of toroidal mode number for ballooning stability optimized equilibria with \( A = 1.6, \ \kappa = 3.2, \) and triangularities 0.57 and 0.35.
Rotation required for stabilizing n > 1 RWM largely unknown experimentally – very few calculations for n > 1

• ARIES-AT at limiting $\beta_N=5.9$

Physics Basis for the Advanced Tokamak Fusion Power Plant ARIES-AT - PPPL-3878

Marginal wall position ($r_{wall}/a$)

$n = 1$  $n = 2$  $n = 3$

1.575  1.450  1.400

• Sound-wave (SW) damping model predicts $\times 2$ decrease in $\Omega_{crit}$ for $n=1 \rightarrow n=2$

• NOTE: MARS ideal mode $n=1$ marginal wall position < $n=2$ marginal wall position

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Sanity check: MARS-F n=2 NW and IW limits match DCON values in the appropriate limits of dissipation and rotation

Weak damping, low rotation reproduces DCON n=2 NW limit

Weak damping, higher rotation reproduces DCON n=2 IW limit

**MARS n=2 mode $\gamma$ vs. $\beta_N$ and $\Omega_\phi$**

![Graph showing MARS n=2 mode $\gamma$ vs. $\beta_N$ and $\Omega_\phi$](image)

**SW damping**

$\kappa_{||} = 0.05$

$\Omega_0/\Omega_\phi$ (Expt)

0.200 (+)

0.800

$\beta_N$

3.2 3.4 3.6 3.8 4.0

$\gamma_{\text{wall}}$

0 50 100 150 200

Left: No-wall limit for $\Omega_\phi = 0$

Ideal-wall limit →

Right: No-wall limit for $\Omega_\phi = 0$

Ideal-wall limit →
Sound wave damping model predicts $n=2$ critical rotation $< \text{half of value for } n=1$ in weak damping limit

$n=1$ $\Omega_{\text{crit}} \tau_A (q=3) = 2.3\%$

$n=2$ $\Omega_{\text{crit}} \tau_A (q=3) = 1.0\%$

**NOTE:** $n=2$ $\Omega_{\text{crit}}$ increases with rotation at high rotation…

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Sound wave damping model again predicts n=2 critical rotation < half of value for n=1 in stronger damping limit

\[
\begin{align*}
n=1 & \quad \Omega_{\text{crit}}^\tau_a (q=3) > 7\% \\
n=2 & \quad \Omega_{\text{crit}}^\tau_a (q=3) = 3\%
\end{align*}
\]

NOTE: n=2 \( \Omega_{\text{crit}} \) increases with rotation at high rotation as did n=1 in the strong damping limit

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Unlike SW model, semi-kinetic damping model finds similar critical rotation values for n=1 and n=2 RWMs

\[ n=1 \quad \Omega_{\text{crit}} \tau_A (q=3) = 0.61\% \]

\[ \text{MARS n=1 mode } \gamma \text{ vs. } \beta_N \text{ and } \Omega_{\phi} \]

\[ n=2 \quad \Omega_{\text{crit}} \tau_A (q=3) = 0.68\% \]

\[ \text{MARS n=2 mode } \gamma \text{ vs. } \beta_N \text{ and } \Omega_{\phi} \]

\[ \eta=0.00 \times 10^{-9}, \text{ Perpend. SK damping} \]

NOTE: $\Omega_{\text{CRIT}}$ values from kinetic damping model systematically 3-4 x LOWER than those obtained from sound wave damping model
Summary of Results (1)

• n=1 RWM
  – SW in strong damping limit ➔ no access to IW limit w/ rotation alone
    • Recent DIII-D experiments marginally stable here…
      – Does this contradict SW damping in strong damping limit?
    • Lower $\kappa_\parallel$ SW & kinetic models allow access to IW limit
  – Lower $\kappa_\parallel$ SW and kinetic models predict 35% increase in $\Omega_{\text{CRIT}}$ when $q = 2$ surface is removed from the plasma
    • Increase consistent with DIII-D results
  – Kinetic damping model 3-4× lower critical rotation than SW
    • $\Omega_{\text{CRIT}}(n=1)$ ordering: Kinetic < experiment < SW
Summary of Results (2)

• n=2 RWM for $q_{\text{MIN}} > 2$
  – SW damping model predicts $\Omega_{\text{CRIT}}(n=2)/\Omega_{\text{CRIT}}(n=1) \leq \frac{1}{2}$
    • Crudely expect $1/n$ scaling from # of sideband resonances
  – Kinetic damping model finds $\Omega_{\text{CRIT}}(n=2)/\Omega_{\text{CRIT}}(n=1) \approx 1$
    • Dissipation $\propto (\Omega/\omega_S)^6 \Rightarrow$ weaker scaling w/ # of resonances
  – Can we measure $n=2 \Omega_{\text{CRIT}}$ or RFA to distinguish between RWM damping models?
  – What about $n=2$ in ITER AT scenario?
    • What is $n=2$ ideal-wall limit?
    • Is $n=2$ RWM unstable for ITER AT? (Impact on feedback?)
RWM damping physics remains unclear

• Neither model in “agreement” with experimental $\Omega_{\text{CRIT}}$
  – Presently within factor of 2 on DIII-D (SK works for JET)
  – SW damping model does not include particle trapping
  – Expect particle trapping to be strong effect
  – Trapping taken into account with kinetic model
    • Low energy particles near TP boundary dominate dissipation
    • MARS-F model derived in high-A circular cross-section limit
    • This limit over-predicts particle bounce-time by as much as factor of 2 near the edge of DIII-D plasmas
    • Dissipation $\propto (\Omega \tau_B)^6$ for trapped particles $\Rightarrow$ correct bounce time tends to improve agreement between kinetic model and experiment
    • Using experimental $T_i/T_e > 1$ also improves agreement
      – $\omega^*$ effects also important in low $E \times B$ rotation limit
    • I’m not a theorist – please help!