Latest advances in system identification and synthesis of MHD controllers for EXTRAP T2R

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  – Improvements since last workshop.
• What is synthesis of a controller based on System Identification?
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  – T2R controller in action.
  – Generic study of minimum number of coils required to control MHD in T2R.
• What can this mean for future experiments?
Generic mode control strategy

MHD (automatic) control system (the Plant):
- Plasma, including the actual 3D boundary structure (wall, vessel, etc.)
- Arrays of magnetic sensor coils.
- Arrays of active saddle coils (actuators).
- Controller hardware plus software.

Advanced Control theory

*Feedback is implemented by a real-time controller which uses, eventually in a well-defined optimal way, the admissible actuators for MHD control in the form of currents in the active coils to constrain the MHD mode evolution at a specified reference state by responding to measured sensor voltages.*

A digital controller was originally developed by Consorzio RFX and implemented in a collaborative effort on both the EXTRAP T2R and RFX-mod experiments.

New software incorporating advanced control theory has been developed for EXTRAP T2R and then installed and tested.
Benefits

What is the benefit?
- Through system identification, one can measure and account for non-axisymmetric features in the wall $\tau_{mn}$.
- Use modern control theory. Optimise.
- Develop control to track an “arbitrary” reference state.
- Generic studies of minimum size requirements for coil array.

Use the controller for generic MHD studies.
- Measure growth rates for “dry” and “wet” plants (i.e. with and without plasma)
- RMP effect on a rotating tearing mode.
- Diagnose field errors
Discrete-time linear time-invariant system model

The system model to be estimated takes the form;
\[ x(k+1) = Ax(k) + Bu(k) + Ke(k) \]
\[ y(k) = Cx(k) + Du(k) + e(k) \]
(The model is known as an “innovations” model.)

where:
• \( k \) is an integer time-sample index.
• \( x \) is the unknown and nonunique system state vector.
• \( u \) is the output vector (coil currents) and \( y \) is the input vector (sensor signals).
• \( A, B, K, C \) and \( D \) are the unknown system matrices to be determined.
• \( e \) is the “innovations” vector which can be thought of as model residuals with no time-correlation and zero-mean.

The model is closely related to the Kalman filter with \( K \) being the KF gain matrix.
Discrete-time linear time-invariant system model

\[ x(k+1) = Ax(k) + Bu(k) + Ke(k) \]
\[ y(k) = Cx(k) + Du(k) + e(k) \]

Comments:

- The goal is to model the dynamics of the plant by processing data to create a model of this form.

- The state vector \( x \) is not comprised of harmonics.

- It will be seen that the state vector \( x \) must be comprised of hundreds of states in order to capture the dynamics of the system.

- The right-hand side includes the state vector plus (input and output) vectors (measured quantities) plus the innovations.

- The model models the change of the state from time \( k \) to \( k+1 \) for the given controller output vector (\( u \)) plus the given innovation vector (\( e \)).
Discrete-time linear time-invariant system model

\[
x(k+1) = Ax(k) + Bu(k) + Ke(k) \\
y(k) = Cx(k) + Du(k) + e(k)
\]

How do we create the model?

• Creation of the model includes determining the state vector of order \( n \) and the system matrices.
• Created from batches of data.
• The raw data is obtained by dithering.
• It will be seen that the state vector \( x \) must be comprised of hundreds of states in order to capture the dynamics of the system.
System Identification by Dithering

Vector dithering is to concurrently excite both spatial and temporal dynamics of the system.

During dithering, the Intelligent Shell controller is functioning so the baseline state is a smooth magnetic boundary.
The Swedish Fusion Association
EURATOM – VR

EXTRAP T2R experiment

- \( R/a = 1.24\,\text{m}/0.18\,\text{m} \)
- \( I_p \approx 150\,\text{kA}, n = 5 \times 10^{18}\,\text{m}^{-3}, T_i = 500\,\text{eV}, T_e = 300\,\text{eV} \)
- \( \tau_{\text{wall}} = 6\,\text{ms} \)
- \( \tau_{\text{pulse}} = \text{up to } 90\,\text{ms} \)
- 4 poloidal x 32 toroidal active and sensor saddle coils (m=1 connected)
Plucking the strings that are the plasma dynamics

Dithering shot #21716, visually

measured signals on the 64 different sensor coil pairs

short current pulses to the 64 different active coil pairs
Create the model

An advanced signal processing method is used to estimate the model from the data batches.

SSARX

(SS = Subspace system)

(ARX = Autoregressive exogeneous)

In time series modeling, an autoregressive exogenous model is an autoregressive model which has exogenous inputs. This means that the model relates the present value of the time series to both past values of the same series and present and past values of the driving (exogenous) series.

Reference:

M. Jansson, Subspace identification and ARX modeling in: IFAC Symposium on System Identification, 2003

(IFAC = International Federation of Automatic Control)
How many states?

Multi-fold cross-validation fit

\[ \log_{10} \eta \]

Number of states \( n \)

\begin{align*}
\text{Log}_{10} \eta & \quad 200 \quad 400 \quad 600 \quad 800 \\
\end{align*}
Visualize the model.

The model captures the dynamics of the system.
How do we visualise the order \( n=500 \) state space vector?
Decompose into a sum of eigenmodes.
Visualize the spectral density.

* Periodogram is an estimate of the spectral density of a signal.
We recognize.

Theoretical growth rates with plasma

Theoretical wall penetration time without plasma

Toroidal mode number $n$ ($m=1$)

$\exp(T_s Y_{mn})$

$G_1, \Gamma$

$G_0$
Visualisation of the model derived from the data

Vacuum response (wall penetration)

Toroidal mode number $n$ ($m=1$)

Toroidal mode number $n$ ($m=1$)
Visualisation of the model derived from the data

Plasma response

Toroidal mode number $n$ ($m=1$)
Generic study of minimum number of coils required to control MHD in T2R.

- Controllability versus coverage
Generic study of minimum number of coils required to control MHD in T2R.

Controllability versus coverage

- Based on the “model” (ca 500 states).
- Randomly remove an active coil and synthesize a controller.
- Evaluate “controllability” for the new controller.
- Etc.

The “in silico” result is that there is still controllability as coils are randomly removed.
What does this mean for the future?

What does this imply?
• Start to look at “minimum coverage” generically.
• Random arrays can have advantages.
• Process control of the 500 states and not targeted control of a mode.

Secondary implication.
• What happens if a coil is lost?