Control of ideal and resistive magnetohydrodynamic modes in reversed field pinches with a resistive wall

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Outline

- Introduction
- Plasma model and numerical methods
- Results w/o feedback control: MHD mode spectrum, stability thresholds
- Results with feedback control: feedback stabilization and limits of stabilization, analysis of marginally stable modes, dependence on system parameters
- Summary and future work
Background

- Stabilization of resistive wall modes (RWMs) is important for devices with thin walls, and some Reversed Field Pinches (RFPs) (e.g. RFX, EXTRAP T2R) use feedback to stabilize the RWMs.

- The idea is to cancel out the radial component of the magnetic perturbation, thus creating a “virtual shell” which restores the stabilizing features of a thick conducting wall.

- This feedback is “1D”, i.e., proportional to only one quantity; the radial or one tangential component of the fluctuation.
Introduction

- In [Finn, Phys. Plasmas 13, 082504 (2006)], it was proposed to use feedback proportional to two components of the perturbed field, the radial and one tangential component.
- “Virtual wall inside resistive wall”
- Since modes which are unstable with an ideal wall have zero perturbed $B_r$, this opens the possibility of stabilizing modes above the ideal wall limit.
- Stabilization up to the ideal-wall ideal-plasma limit was found in [Finn 2006], which used a very simplified plasma model.
- We have applied this feedback to a more realistic model: viscoresistive MHD, realistic RFP equilibria, cylindrical geometry.
Model & methods:

Plasma model

- **Linearized MHD equations in a cylindrical plasma with zero pressure:**
  \[ \partial_t \tilde{v} = \left[ \nabla \times \tilde{B} - \lambda(r) \tilde{B} \right] \times B_0 + \nu \nabla^2 \tilde{v} \]
  \[ \partial_t \tilde{B} = \nabla \times \left( \tilde{v} \times B_0 - \eta \nabla \times \tilde{B} \right) \]
  \[ \tilde{B} = \tilde{B}(r) \exp(i(m \theta + k z)) \]
  \[ n = -k R \]

- **RFP equilibria:**
  \[ j_0 = \lambda(r) B_0 \]
  \[ \lambda(r) = \frac{\lambda_0}{1 + (r/a)^2} \]
Model & methods:

Boundary conditions

- **At** \( r = 0 \):
  \[
  \partial_r (\vec{v}_r - i\vec{v}_\theta) = (\vec{v}_r + i\vec{v}_\theta) = \vec{v}_z = 0 \\
  \partial_r (\vec{B}_r - i\vec{B}_\theta) = (\vec{B}_r + i\vec{B}_\theta) = \vec{B}_z = 0
  \]

- **At** \( r = r_w \):
  - **Ideal Ohm’s law:** \( i\mathbf{k} \cdot \mathbf{B}_0 \vec{v}_r = \gamma \vec{B}_r (r_w) \)
  - **No stress boundary condition:** \[
  \begin{align*}
    \text{i}m\vec{v}_r / r + r \partial_r (\vec{v}_\theta / r) &= 0 \\
    \text{i}k\vec{v}_r + \partial_r \vec{v}_z &= 0
  \end{align*}
  \]
  - **Thin wall approximation:** \( \gamma \tau_w \vec{B}_r = [\vec{B}_r']_{r_w} \)
  - **Zero tangential current:** \[
  \begin{align*}
    \partial_r (r \vec{B}_\theta) - \text{i}m \vec{B}_r &= 0 \\
    \partial_r \vec{B}_z - \text{i}k \vec{B}_r &= 0
  \end{align*}
  \]
Feedback control

- Feedback applied by setting boundary condition at $r_c$ to be proportional to a linear combination of components of the fluctuation measured at the wall:

$$\tilde{B}_r(r_c) = G\tilde{B}_r(r_w) + iK[\tilde{k} \cdot \tilde{B}(r_w^-)]$$

- This “2D” scheme can affect modes, even if $\tilde{B}_r(r_w) = 0$

$k \equiv (m/r)\hat{\sigma} + k\hat{z}$

$\sigma \equiv \hat{r} \times k = (m/r)\hat{z} - k\hat{\theta}$
Results without feedback:

Spectrum and mode structure

- Partial spectrum of $m = 1, n = 8$ MHD fluctuations ($a = 0.9$, $\lambda_0 = 3.5$, $\tau_w = 5 \times 10^4$, $\eta = 10^{-7}$, $\nu = 2 \times 10^{-5}$, $R = 4$, $r_c = 1.2$)

$$\tau_w = 5 \times 10^4, \eta = 10^{-7}, \nu = 2 \times 10^{-5}$$
Results without feedback:

Plasma stability

- Growth rate of the fastest growing mode depends on the value of $\lambda_0$
- Four stability limits can be identified, and are associated with resistive/ideal wall, and resistive/ideal plasma

Conceptual sketch of stability limits

Computed stability limits

$\gamma$

$\lambda_0$

$\eta \neq 0$

$\eta = 0$

$\gamma$

$\lambda_0$

Resistive wall

Ideal wall
Results with feedback:

Outline

- Regions of stability in \((G,K)\) parameter space
- Variation of stable region with \(\lambda_0\), and limits of stabilization
- Analysis of marginally stable modes
- Dependence on system parameters
Results with feedback:

**Feedback stabilization**

\[ \tau_w = 5 \times 10^4, \eta = 10^{-7}, \nu = 2 \times 10^{-5} \]
Results with feedback:

Feedback stabilization

\[ \tau_w = 5 \times 10^4, \eta = 10^{-7}, \nu = 2 \times 10^{-5} \]

Stabilized above the resistive-wall ideal-plasma threshold at 3.76!
Results with feedback:

Marginally stable modes

Simple model for MS mode stability line: $K_c = K_0 + K_1 \tau_w (\nu + \eta)$
Results with feedback:

Magnetosonic mode

- Simple model of MS mode with global viscosity and resistivity:
  \[ \psi''(x) + \omega^2 \psi(x) - i\omega(\nu + \eta)\psi'' = 0 \quad \text{for } 0 < x < a \]
  \[ \psi''(x) = 0 \quad \text{for } a < x < b \]

- Boundary conditions:
  \[-i\omega\tau_\omega \psi(a) = [\psi']_a \quad \psi(b) = -G\psi(a) + K\psi'(a^-)\]

- Piecewise solutions:
  \[ \phi_1(x) = \begin{cases} 
  \sin(\kappa x), & (0 < x < a) \\
  \sin(\kappa a)[(b - x)/(b - a)], & (a < x < b) 
\end{cases} \]
  \[ \phi_2(x) = \begin{cases} 
  0, & (0 < x < a) \\
  (x - a)/(b - a), & (a < x < b) 
\end{cases} \]
Results with feedback:

Magnetosonic mode

- Let $1/\tau_w \sim \eta \sim \nu$ be a small parameter
- Expand by orders in the small parameter:
  \[
  \omega = \omega_0 + i\gamma_1 + \ldots \\
  \kappa = \kappa_0 + i\kappa_1 + \ldots
  \]
- Solve order by order:
  \[
  \omega_0 = \kappa_0 = n\pi/a \\
  \gamma_1 = \kappa_1 \left(1 - \frac{(\nu + \eta)\omega_0^2}{2}\right) = \frac{1}{\tau_w a} \left[\frac{K}{b - a} - 1\right] - \frac{(\nu + \eta)\omega_0^2}{2}
  \]
- Could limited amplifier bandwidth (filtering) help with this high frequency mode?
Results with feedback:

Dependence on viscosity

As predicted, $K_c$ depends approximately linearly on viscosity. Also, when the MS boundary moves left, stabilization is lost at a smaller value of $\lambda_0$. 
Results with feedback:

Dependence on resistivity

$K_c$ depends only weakly on resistivity, since $\eta \ll \nu$. The lower boundary moves up with increasing resistivity, and this also causes loss of stabilization at a smaller value of $\lambda_0$. 

\( \lambda_0 = 3.5 \) 
\( \eta = 2 \times 10^{-7} \) 
\( \eta = 5 \times 10^{-8} \) 

\( \lambda_0 = 3.72 \) 
\( \lambda_0 = 3.71 \) 
\( \lambda_0 = 3.70 \)
Results with feedback:

Dependence on wall time

K_c depends strongly on wall time, moving to the right for larger wall time. While the stabilized region is smaller for smaller wall time, the limit of stabilization is about the same.
Tangential measurement inside:
3rd component stabilises above ideal wall limit

- Consider “3D” feedback scheme G-K & H?

\[ \tilde{B}_r(r_c) = G\tilde{B}_r(r_w) + iK[\tilde{k} \cdot \tilde{B}(r_w)] + iH[\tilde{\sigma} \cdot \tilde{B}(r_w)] \]

- \( m =1 \)
- \( n = 8 \)
- \( \eta = 1e-7 \)
- \( v = 5e-5 \)
- \( \Lambda_0 = 3.5 \)
- \( \tau_w = 5e4 \)
- \( r_w = 1.0 \)
- \( R = 4.0 \)

\( \tilde{\sigma} = \hat{r} \times \tilde{k} \)

3rd component used

Use of 3rd component effects stability threshold of magnetosonic mode very strongly

![Diagram showing coloured regions as stable regions.](image)
Tangential measurement outside: 3rd component destabilises magnetosonic mode reducing stability

Below ideal wall tearing limit
(Λ₀ = 3.1, ν = 2e-5)

H = 0 Case same as (Finn 2006)

1) Above ideal wall tearing limit (Λ₀ = 3.2) feedback no longer able to stabilise
2) 3rd component drives magnetosonic mode unstable partially masking Alfven mode’s stability region
Tangential measurement outside:
Magnetosonic and Alfven mode contributions to stability

Magnetosonic

Alfven

magnetosonic mode unstable for rightmost Alfven stable region

magnetosonic mode unstable for leftmost Alfven stable region
Summary

- Modeled RFP plasmas using viscoresistive MHD in cylindrical geometry and realistic equilibrium profiles.
- Applied a new “2D” feedback scheme (G,K), and analyzed the dependence on system parameters.
- Boundaries of stable region in (G,K): original mode; original mode made complex and driven unstable by feedback; magnetosonic mode driven unstable by feedback.
- Demonstrated stabilization of tearing modes and ideal plasma modes, above the ideal-wall tearing limit, and up to the ideal-wall ideal plasma limit.
- Further improvement in feedback performance only possible through use of internal measurements.
Future Work

- Use this feedback scheme in experiments
  - At APS-DPP we talked with researchers from the RFX experiment in Italy, and they have now submitted a proposal to try this feedback method on their experiment

- Add effects of plasma rotation, control system delays

- Signal processing techniques can be used to prevent driving the magnetosonic mode