Stabilization of the Resistive Wall Mode and Error Field Reduction by a Rotating Conducting Wall

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Rotating walls predicted to stabilize the RWM

- Wall stabilization first proposed by Gimblett, PPCF (1989)
- We compare our results to theory of Hegna, PoP (2004)
  - Linear Eigenvalue problem solved for ideal MHD plasma and resistive thin-walls

- Our Main Results:
  - Wall rotation can stabilize the RWM
  - Error fields are found to lock the RWM
  - Wall rotation profoundly affects error fields
$B_z \approx 0.1 \, \text{T}$
$I_p \approx 7 \, \text{kA}$
$L = 1.2 \, \text{m}$
$R = 8 \, \text{cm}$
$t \approx 20 \, \text{ms}$
$\tau_w = 7 \, \text{ms}$
$\tau_A \approx 2 \, \mu\text{s}$
$T_e \approx 3.5 \, \text{eV}$
$n_e \approx 10^{20} \, \text{m}^{-3}$

Paz-Soldan et al, RSI (2010)
System is differentially rotating

- The secondary wall rotates (at atmosphere)
- The vacuum vessel is fixed
- Fluxloops reside in between the walls
Cylindrical experiments allow rotating solid conductors

- A torus would require flowing liquid metal

\[ V_\theta(r) = r \Omega_w \]

\[ V_\theta(r) = f(r, R_e) \Omega_w \]
High-speed rotating wall built

\[ \tau_w = \sigma \mu_0 r_w \delta_w \quad R_m = \Omega_w \tau_w \]

- Want many \( \tau_w \)/pulse and large \( R_m \)
- Pulse is short, so \( \Omega_w \) must be fast
- Design: 7000 RPM = 300 km/h (!)
- \( r_w = 9 \) cm, \( \delta_w = 1 \) mm, \( L = 1 \) m
- \( \tau_w = 7 \) ms, \( R_m = 5 \)
• But, other physics are present

• Wall rotation matters to these phenomena too!
  – We will explore each independently
First consider error field penetration:

- Insert $m = 1$ field, what happens as wall rotates?
Error fields are excluded (shielded), rotated

Shielding:

\[ \frac{1}{\sqrt{1 + \frac{R_m^2}{4}}} \]

\[ B/B_0 \]

\[ R_m \]

Rotation:

\[ \arctan \left( \frac{R_m}{2} \right) \]

\[ \phi \text{ (rad)} \]

\[ R_m \]
Error field thus depends on applied field and $R_m$

(This is experimental data)
Field penetration time decreases as wall rotates

- Shielding still seen at $t = \infty$
- However, error field reaches a smaller value faster
- Consider $m=1$ vacuum field Eigenmodes
  - We will see this is a consequence of differential rotation
• Single wall Eigenmode is dipole-like, and \( \gamma = \frac{2}{\tau_w} \)

• Define a coupling (‘gap’) parameter, \( \alpha \equiv \frac{r_a^2 - r_b^2}{r_b^2} \)

• As coupling increases, Eigenmodes branch into a slow and fast root
Rotation decouples Eigenmodes

- Rotation brings Eigenvalues towards single-wall values
  - Fast mode slows down
  - Slow mode speeds up
Speeding up of slow root is the observation

- The slowest mode dominates the observation after few ms
- Rotation increases the decay constant (eigenvalue)
- Results of calculation consistent with data
  - Dotted lines are +/- 2% on wall time
  - No free parameters
- This would not be observed on single-wall system
Potential measurements indicate plasma has intrinsic ExB

- Cold plasma (big $\eta$), large $J_z$, yields big $E_z$ (large $V_{bias}$ needed)

- MHD instabilities thus naturally have a real frequency in lab frame
**Micro-tutorial: equilibrium and mode fields**

- $B_{r,m=1}$: what the fluxloop measures, it is like the current centroid.
- $B_{\text{ext}}$: externally applied $m=1$ error field.
  - It is never measured as it is applied prior to fluxloop integration.
- $B_{\text{eq}}$: the $m=1$ equilibrium field from a misalignment between magnetic and geometric axis (a field error, arising from $B_{\text{ext}}$).
- $B_{\text{mode}}$: the field signature of the MHD instability.
  - It usually has a real frequency $\omega$ in the lab frame.
Micro-tutorial: equilibrium and mode fields

- Experimental data will be presented in this form
- \( B_{\text{mode}} \) here is a rotating RWM saturated for entire shot
- As \( \omega \gg \Omega_w \), **we must slow** \( \omega \) to see \( \Omega_w \) effect mode
Guide field ripple brakes plasma

- Axially localized $m = 0$ used
- Equilibrium and thus ExB flow is modified
  - Neoclassical effects small due to large collisionality
Mode locking observed at slow rotation

- As $m = 1$ error field increased, three regimes seen:
  - Rotating RWM: mode never locks during the discharge
  - Locking RWM: mode locks during the discharge
  - Born-locked RWM: oscillations never observed

\[ \frac{B_{\text{ext}}}{B_z} < \mathcal{O}(10^{-2}) \]
Error field defines RWM regime

- Convolution present: $R_m$ affects error field, which also affects RWM
- Error fields are always small: $\frac{B_{ext}}{B_z} < \mathcal{O}(10^{-2})$
- Effect of wall rotation in each regime must be studied independently
Wall rotation stabilizes locking RWM

- Increasing $R_m$ lowers amplitude, growth rate of the locked RWM
- Wall is seen to couple $\omega$ to locked mode
- Picture qualitatively consistent with theory
- Post-lock growth is non-linear:
  - cannot be quantitatively compared to theory
Asymmetry in locked RWM rotation

• Notice locked RWMs rotate slowly even if $\Omega_w=0$
  – This rotation is counter-initial ExB
  – Thus, an anomalous torque present

• Locked RWM response is resultantly asymmetric in $\Omega_w$

• Reversing $B_z$ reverses the asymmetry
Stability window of RWM extended

- Born-locked modes studied
  - remove ambiguity between locking threshold and RWM onset
- RWM onset defined as where $B_{r,m=1}$ deviates from $B_{eq}$
- $B_{mode}$ is found by vector subtraction of $B_{eq}$
- Onset occurs at considerably higher $I_p$ for fast rotation
Stability window of RWM extended

- $B_{\text{mode}}$ onset occurs at lower $q (=q_{\text{crit}})$ as $R_m$ increased
- $q_{\text{crit}}$ is everywhere somewhat above theoretical prediction
- Offsetting $q_{\text{crit}}$ such that $q_{\text{crit}}=1$ @ $R_m=0$ yields better agreement
- Theory assumes a top-hat current profile with radius $r_p$,
  - thus $q=q(r_p) =$constant everywhere inside plasma.
  - $q$ (5 cm) is as measured by the mid-radius anode ring, $q(r)$ is not constant
RWM Eigenfunctions are anode-localized

- Helicity seen in RWM
- Symmetry of theoretical model not observed in experiment
- Flows are suspected to be the cause
  - Axial flow of $M \sim 0.3$ measured by Mach probe
  - Predicted by Ryutov to advect Eigenfunction
  - Sheared azimuthal flow at cathode may be stabilizing
  - Neither flow is treated in Hegna PoP (2004) model

Experiment

Theoretical

$V_z = 0$
$V_z = \sim 0.1 M_a$

Ryutov, PoP (2006)
Conclusions

• Rotating wall has been shown to stabilize RWM, increase stable operation window
• Error fields are required to brake intrinsic plasma rotation and thus define the observed RWM regime
• Wall rotation affects error fields strongly
• Qualitative agreement with theory observed
  – Inclusion of more physics (flows, magnetic shear, resistivity) likely needed to make quantitative comparisons
How do we measure $B_{eq}$

- $B_{eq}$ is a baseline shot with low field ripple
  - Plasma is unbraked and never locks, don’t see big $B_{mode}$
- $B_{eq} + B_{mode}$ is a shot with larger ripple
  - Ripple doesn’t affect $B_{eq}$
Microtutorial: Mode Locking

- Consider a simple ad-hoc torque model:

\[ I_{zz} \dot{\omega} = \Gamma = \frac{A_{vis}(\Omega_0 - \omega)}{\Gamma_{vis}} - \frac{A_{EM} \omega}{1 + (\omega \tau_w)^2} \]

- ‘viscosity’ to a natural frequency \((\Omega_0)\)
- Electromagnetic torque is non-monotonic
- We look at zero-inertia limit (zero net torque)
- Find that allow zero torque (roots of the equation)
Several regimes of mode locking model

- Large natural frequency ($\Omega_0$):
  - only one $\omega$ is a root
  - It is stable
  - Called ‘fast branch’

\[
\Gamma = A_{vis}(\Omega_0 - \omega) - A_{EM} \frac{\omega}{1 + (\omega \tau_w)^2} \]

\[
\Gamma_{vis} \quad \Gamma_{EM}
\]
Several regimes of mode locking model

- Mid-range natural frequency ($\Omega_0$):
  - Three $\omega$ roots
  - 2 stable, 1 unstable
  - All branches seen

\[
\Gamma = \frac{A_{vis}(\Omega_0 - \omega)}{\Gamma_{vis}} - \frac{A_{EM}}{1 + (\omega \tau_w)^2} \frac{\omega}{\Gamma_{EM}}
\]
Several regimes of mode locking model

- **Bifurcation frequency** ($\Omega_0$):
  - $2\omega$ roots
  - Fast branch unstable
  - Slow branch stable

\[
\Gamma = A_{vis} \left( \Omega_0 - \omega \right) \frac{1}{\Gamma_{vis}} - A_{EM} \frac{\omega}{1 + (\omega \tau_w)^2} \frac{1}{\Gamma_{EM}}
\]
Several regimes of mode locking model

- Small natural frequency ($\Omega_0$):
  - Only slow branch remains

\[ \Gamma = \frac{A_{vis}(\Omega_0 - \omega)}{\Gamma_{vis}} - \frac{A_{EM}}{1 + (\omega \tau_w)^2} \frac{\omega}{\Gamma_{EM}} \]
Mode locking: bifurcations

- Decreasing $\Omega_0$ can cause $\omega$ to ‘jump’ from fast root to slow root (bifurcate)
  - This is the mode lock
- Increasing $\Omega_0$ again would not unlock mode until slow branch bifurcated
  - Hysteresis is present
Wall rotation and EM torque

- Electromagnetic torque in presence of differentially rotating walls can be calculated using thin-wall, long-cylinder approximations.
- For fast wall rotation, the bifurcation is lost:
  - No more mode locking
  - Mode unlocking is even more strongly affected
  - It is much easier to unlock a mode when the wall rotates

\[ \Omega_{0,\text{lock}} \quad \Omega_{0,\text{unlock}} \]

\[ \omega/2\pi \text{ (kHz)} \]

\[ \Omega_0/2\pi \text{ (kHz)} \]

\[ R_m = 0 \]

\[ R_m = 1 \]

\[ R_m = 3 \]

\[ R_m = 5 \]

\[ R_m = 7 \]

\[ R_m = 9 \]

\[ \omega_{\text{lock}} \]

\[ \omega_{\text{unlock}} \]

\[ f \text{ (kHz)} \]

\[ R_m \]

\[ \Omega_{0,\text{lock}} \]

\[ \Omega_{0,\text{unlock}} \]
Experiment overview: plasma guns

- Plasma is generated by an array of 7 washer-stabilized hollow-cathode plasma guns
- Dense plasma in gun allows large space-charge limited currents
- Allows current-driven MHD modes at modest $B_z$

Experiment overview: anode

- Segmented (bullseye) anode allows a coarse measure of the $q$-profile
- This is the parameter of importance to current-driven RWM stability
- Anode also provides a highly conducting boundary condition
Resistive wall modes are a performance limiting instability

- Plasma pressure (\(\sim \beta\)) limited by long wavelength ideal MHD modes
- Plasma current (\(I_p\)) is also limited by ideal MHD stability
- The resistive wall mode (RWM) can be excited in both these limits

\[
P_{\text{fus}} \propto \beta^2
\]

\(\beta_N = 3.5\)

Strait et al, PoP 1994
## Typical Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial field</td>
<td>$B_z$</td>
<td></td>
<td>500 G</td>
</tr>
<tr>
<td>Plasma current</td>
<td>$I_p$</td>
<td></td>
<td>2.1 kA</td>
</tr>
<tr>
<td>Electron density</td>
<td>$n_e$</td>
<td></td>
<td>$5 \times 10^{14}$ cm$^{-3}$</td>
</tr>
<tr>
<td>Electron temperature</td>
<td>$T_e$</td>
<td></td>
<td>3.5 eV</td>
</tr>
<tr>
<td>Spitzer resistivity</td>
<td>$10^{-4} ln(\Lambda)T_e^{-\frac{3}{2}}$</td>
<td>$\eta$</td>
<td>230 $\mu\Omega$m</td>
</tr>
<tr>
<td>Electron thermal speed</td>
<td>$(2k_b T_e/m_e)^{\frac{1}{2}}$</td>
<td>$v_{Te}$</td>
<td>110 km/s</td>
</tr>
<tr>
<td>Sound speed</td>
<td>$(\gamma k_b T_e/m_i)^{\frac{1}{2}}$</td>
<td>$C_s$</td>
<td>24 km/s</td>
</tr>
<tr>
<td>Alfven speed</td>
<td>$B_z/(n_e m_i \mu_0)^{\frac{1}{2}}$</td>
<td>$v_A$</td>
<td>49 km/s</td>
</tr>
<tr>
<td>Mach number</td>
<td>$v_z/C_s$</td>
<td>$M$</td>
<td>0.3</td>
</tr>
<tr>
<td>Alfven time</td>
<td>$r/v_A$</td>
<td>$\tau_A$</td>
<td>2 $\mu$s</td>
</tr>
<tr>
<td>Resistive diffusion time</td>
<td>$r^2 \mu_0/\eta$</td>
<td>$\tau_{res}$</td>
<td>52 $\mu$s</td>
</tr>
<tr>
<td>Energy confinement time</td>
<td>$P_{ohm}/W$</td>
<td>$\tau_E$</td>
<td>10 $\mu$s</td>
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<tr>
<td>Lundquist number</td>
<td>$\tau_{res}/\tau_A$</td>
<td>$S$</td>
<td>26</td>
</tr>
<tr>
<td>Plasma Beta</td>
<td>$2\mu_0\langle p\rangle/B^2$</td>
<td>$\beta$</td>
<td>10 %</td>
</tr>
<tr>
<td>Volumetric ohmic heating</td>
<td>$\int \eta J^2 dV$</td>
<td>$P_{ohm}$</td>
<td>200 kW</td>
</tr>
<tr>
<td>Ion mean free path</td>
<td>$(n_i \sigma_i)^{-1}$</td>
<td>$\lambda_i$</td>
<td>3 $\mu$m</td>
</tr>
<tr>
<td>Electron mean free path</td>
<td>$(n_e \sigma_e)^{-1}$</td>
<td>$\lambda_e$</td>
<td>500 $\mu$m</td>
</tr>
<tr>
<td>Ion skin depth</td>
<td>$c/\omega_{ci}$</td>
<td>$\delta_i$</td>
<td>13 mm</td>
</tr>
<tr>
<td>Electron skin depth</td>
<td>$c/\omega_{ce}$</td>
<td>$\delta_e$</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Electron larmor radius</td>
<td>$v_{Te}/\omega_{ce}$</td>
<td>$\rho_e$</td>
<td>0.1 mm</td>
</tr>
</tbody>
</table>

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* $L = 1.22$ m is the plasma length, $r \approx 10$ cm is the plasma diameter. $k_b$ is Boltzmann’s constant, $\gamma$ is the adiabatic index, and $\Lambda$ is the coulomb logarithm.
Decoupling is RWM stabilization

- Including plasma effects, the two-mode picture is preserved
  - Now fast mode is stable
    - Rotation destabilizes
  - Slow mode is unstable
    - Rotation stabilizes
  - Transition to stability happens at critical $R_m$

\[
\vec{B}(r, \theta, t) = \vec{B}(r, \theta) e^{-\gamma t}
\]
Micro-tutorial: error field penetration

- To gain intuition on the effect of the rotating wall, consider:
  - A purely $m = 1$ DC field is applied ($B_{\text{ext}}$)
  - Wall is allowed to rotate at a given $R_m$
  - All transients are allowed to decay
  - What happens to the field inside, outside the wall?
Potential measurements indicate plasma has intrinsic ExB

- Resulting rotation profile is complex
  - Strongest at cathode mid-radius
  - Vanishing at anode
  - Rigid body rotation in core, viscosity appears large
  - Sheared both radially and axially
Potential measurements indicate plasma has intrinsic ExB

- Cold plasma (big \( \eta \)), large \( J_z \), yields big \( E_z \) (large \( V_{bias} \) needed)
- Radial gradients in \( J_z(R) \) yield \( E_r \), as :

\[
V_p(R, Z) = \int_{L}^{Z} \eta J_z(R) dZ
\]

- Anode must be an equipotential, \( \eta \) is constant (isothermal plasma)