Modeling the Sawtooth Cycle and Stochastic Transport in MST

Joshua A. Reusch


2011 IEA RFP Workshop • Madison, WI • October 2011
Single fluid zero-β MHD simulations at $S \sim 4 \times 10^6$ reproduce standard MST discharges well.

- Equilibrium evolution is reproduced well including sawtooth period and duration

- Much of the measured heat transport in MST throughout the sawtooth cycle is due diffusing magnetic field lines
Outline

• Modeling the sawtooth cycle in MST
  • Comparison of the equilibrium evolution
  • MHD dynamo
  • Magnetic fluctuations

• Determining the magnetic diffusion induced transport
  • Field line tracing
  • Trapped particle effects
  • Comparison to measurement
Single fluid, 3D, nonlinear, resistive MHD simulations reproduce the MST sawtooth cycle well.

- Cylindrical, force-free MHD model was used (DEBS* code run with zero $\beta$)

- Theta, resistivity profile, and Lundquist number from MST ($S=3.8\times10^6$)

- High Prandtl number used to damp sub grid scale fluctuations ($Pr_m\sim200$)

- Simulations reproduce sawtooth period, duration, and equilibrium evolution of MST well

- Magnetic mode amplitudes are higher than in MST

Viscosity in DEBS is dynamically adjusted to damp subgrid scale fluctuations.

- Viscosity in DEBS is purely numerical used for numerical stability
- Much larger than classical viscosity for MST
- Increases by more than an order of magnitude at sawtooth crash
These simulations produced large, well defined sawteeth similar in period and duration to MST.

- Code produces sharp, quasi-periodic sawteeth
  - Quantities of interest were sawtooth ensembled

![Simulated $\tilde{b}$](image)

![Measured $\tilde{b}$](image)
The simulated evolution of the magnetic equilibrium through the sawtooth cycle also matches MST.
The simulation agrees well with equilibrium reconstructions across the radius.
Simulated lambda profile is also in good agreement with the inferred lambda profile from MSTFit.

\[ \lambda = \mu_0 a J_\parallel / |B| \]

Experiment
Simulation

\[ \lambda(0) \]
Simulated dynamo is smaller than that measured in MST away from sawteeth

- $E_\parallel$ from MSTFit shows a dynamo electric field away from sawteeth
- This electric field has been measured with CHERS

- Simulations show almost no MHD dynamo between sawteeth
- This may be due to the high viscosity damping tearing mode flows
At the sawtooth, the simulated dynamo is somewhat larger than that measured in MST

- $E_{\parallel}$ from MSTFit shows a large dynamo electric field at sawteeth

- Simulations show a larger MHD dynamo at crash than seen in MST
Measured magnetic mode spectrum is reasonably well reproduced by the simulation.  

- Edge magnetic pick up coil data compared to synthetic diagnostic  
- Simulated mode amplitudes ~2x larger than measured
The ratio of the edge poloidal fluctuation to the core radial fluctuation matches that seen in experiment.

$m=1, n=6$: Experiment*

In MST the core $B_r$ is about 2.5 times the edge $B_\theta$

This ratio is also seen in the simulated fluctuations

*W.X. Ding, et al., PRL, 103, 025001, 2009
Outline

• Modeling the sawtooth cycle in MST
  • Comparison of the equilibrium evolution
  • MHD dynamo
  • Magnetic fluctuations

• Determining the magnetic diffusion induced transport
  • Field line tracing
  • Trapped particle effects
  • Comparison to measurement
Field line tracing of simulated magnetic field shows varying stochasticity through the sawtooth cycle.
Tracing the magnetic field lines allows direct computation of $D_{mag}$ and from this, $\chi_e$.

- The magnetic diffusion is defined as:

  $$D_{mag} = \frac{\left\langle (r - r_0)^2 \right\rangle}{2L}$$

- The electron thermal conductivity is:

  $$\chi_{MD} = V_\parallel D_{mag} \approx V_{th,e} D_{mag}$$
\[ \chi_{\text{MD}} \text{ is compared to characteristic values of the } \chi_e \text{ in different regions of MST.} \]

- Best fit \( \chi_e \) for the experiment found by solving:

\[
1.5 \frac{\partial (n_e T_e)}{\partial t} = \nabla (n_e \chi_e \nabla T_e) + \eta J^2 - \text{Sink}
\]

for \( T_e \) and comparing to measurements

The trapped particles in MST can be more than half the electron density making the circulating fraction less than 0.5**

The effective electron thermal conductivity can be defined as:

\[ \chi_{\text{eff}} \approx f_c V_{th,e} D_{\text{mag}} \]

Agreement is found through much of the sawtooth cycle when trapped particles are taken into account.

- A significant amount of the core $\chi_e$ is due to magnetic stochasticity
- Other effects (islands, electro-static transport) are still important

$\chi_e$ in the mid-radius is due to magnetic stochasticity throughout sawtooth cycle
Summary and Conclusions

• High S nonlinear single fluid resistive MHD simulates MST sawtooth well, reproducing period and crash duration

• Simulations reproduce the sawtooth evolution of the magnetic equilibrium well

• Direct field line tracing using scaled mode amplitude profiles yields the level of magnetic diffusion

• Trapped particles do not carry heat along diffusing magnetic field lines and must be taken into account for the predicted $\chi_e$ to match the measured $\chi_e$.

This work was supported by the U. S. Department of Energy and the National Science Foundation