RFP helical equilibria reconstruction with V3FIT-VMEC

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Motivation

Helical states are routinely observed in the RFX-mod reversed field pinch and for these regimes different approaches were adopted aiming at plasma magnetic equilibrium reconstruction [1,2,3].

An important aspect is the possibility to interface the reconstruction procedure with experimental measurements and this was done both with the NCT/SHEq [4] and now with the V3FIT/VMEC code [5,6].

In this work we describe the **use of the V3FIT code to obtain fixed boundary RFX-mod helical equilibria compatible with the measurements** of the radial and toroidal components of the magnetic field as well as toroidal flux loops. Also the determination of the sensitivity of the $q$ profile to external constraints is assessed, in order to determine a reliable parametric representation of this profile.

Outline

- VMEC/V3FIT-NCT benchmark for an axi-symmetric case
- Helical equilibria (fixed boundary)
  - Vacuum fields determination
  - V3FIT convergence and comparison to experimental data
  - q profile parametrization
- Summary and future plans
Axi-symmetric benchmark
VMEC-NCT benchmark ($\beta=0$): direct problem

**NCT input**: $\alpha, \theta_0, \Delta_h \quad \Rightarrow \quad \left< \frac{J \cdot B}{B \cdot B} \right> = \mu(r) = \frac{2\theta_0}{a} \left[ 1 - \left( \frac{r}{a} \right)^{\alpha} \right]$

**VMEC input**: $q(s), \Psi_t, \text{LCFS (all from NCT output)}$

\[
\begin{align*}
F^* &= \frac{1}{\mu_0} \oint_{\Gamma_{\text{pol}}} B_{\text{tor}} \frac{1}{\Psi_t} \\
\Theta^* &= \frac{1}{\mu_0} \oint_{\Gamma_{\text{tor}}} B_p \frac{1}{\Psi_t}
\end{align*}
\]

$\Delta F^{*}_{(\text{VMEC-NCT})} < 0.5\% \quad \Delta \Theta^{*}_{(\text{VMEC-NCT})} < 0.9\%$

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**Parallel current**

\[
\begin{align*}
\mu & \quad \text{vs. } r (\text{m}) \\
\end{align*}
\]

**Safety factor**

\[
\begin{align*}
q & \quad \text{vs. } r (\text{m}) \\
|B| & \quad \text{vs. } \tau (\text{m})
\end{align*}
\]
V3FIT-VMEC and NCT equilibria: inverse problem

- V3FIT-VMEC (fixed boundary) and NCT (hybrid free-boundary) have different measurement matching algorithms
  - NCT assumes circular shape, assume $\Delta_h$, matches $F^*$ and $\Theta^*$ with $\alpha, \theta_0$
  - V3FIT-VMEC (fixed boundary), matches external measurements, but ignores $I_{tor}$, allows a flexible representation of $q(s)$ (8 points cubic spline in the case shown) but also allows to set constraints (i.e. $q(a)$). The shape of the plasma can also change
- Even if the inverse problem is degenerate for this axi-symmetric case there are indications (consistent with past observations) that the $\alpha, \theta_0$ modeling for the $\mu$ profile is too restrictive
Equilibria and measurements

In both cases equilibria provide a good match with experimental data.

NCT modelled data (computed a-posteriori) matches data pretty well.

V3FIT improves matching with Bp and Pol fluxes, by slightly changing The shape of the LCFS.
Helical equilibria reconstruction
The RFX-mod Reversed Field Pinch (RFP) [1] is equipped with a large set of magnetic measurements:

- 4 × 48 pick-up coils for Bt & Bp
- 4 × 8 pick-up coils for Bp
- 4 × 48 saddle coils for Br
- 10 toroidal flux loops
- 8 poloidal flux loops
The issue of external measurements

Up to now VMEC for the RFP has been used only in the fixed-boundary approach. The free-boundary is presently under test.

In fixed-boundary one can determine only the modelled data coming from plasma currents so that the vacuum part has to be estimated otherwise. This is not an easy task in non-stationary conditions: eddie currents in passive structures produce transient magnetic fields.

So far we have considered only stationary QSH (with static helical boundary): simple Biot-Savart law with a filamentary representation of active coils.
1. Magnetizing system  
2. Toroidal system  
3. Field shaping system  
4. Active coils (PR)

Some of these systems are axi-symmetric and others have a high spatial periodicity (e.g. 12 toroidal sectors and 192 active coils).

The plasma, through a self-organized process, reaches a state with a helical core embedded in an axisymmetric boundary.
Some approximations can be adopted in describing RFX-mod winding systems limiting the number of wires.

**Passive structures are neglected**, so that the result is reliable only for sufficiently stationary configurations.

The modelling was tested by applying a current reference to each system of coils in vacuum shots.
The determination of the variances ($\sigma^2$) of the measurements (used by V3FIT) is an important issue.

As the helical equilibrium determined with VMEC is obtained including only the dominant helicity and some of its harmonics (toroidal periodicity 7 for RFX-mod), one can determine $\sigma$ by considering the spread of data on this periodicity:

1. The measurements (+) are plotted on a single $n=7$ period.
2. One extracts the $n=7$ and some of its harmonics (●) from the data.
3. From the spread one obtains $\sigma^2$. 
Diagnostics used: $4 \times 48 \, b_r$, $4 \times 48 \, b_\phi$, 10 toroidal flux loops (poloidal flux loops and $I_{\text{tor}}$ are planned, $8 \times 4 \, b_\theta$ and $b_\phi$ array not used yet).

Evolution of modelled toroidal flux measurements through V3FIT convergence steps: already at step 2 the match with the measurements is good. However the matching to other magnetic diagnostics leads to further steps towards the final result at step 5.

VMEC runs in fixed boundary mode with the constraints:
- $N_{fp} = 7$ (due to the plasma, not to external coils)
- NPOL=9
- NTOR=6
- LCFS shape: $\Delta_{1,0}$ $\Delta_{1,-7}$ $\Delta_{0,7}$ of the LCFS (circular assumption)
- Toroidal flux at the edge
- Safety factor profile $q(s)$
- Pressure profile

$\chi^2_{\text{final}} = 137.29$
**B_r comparison**

--- Experimental data
--- Modelled data

NPOL=9
NTOR=6 (multiples of 7)

Phase-locking region: cannot be modelled with VMEC. It influences $\sigma^2$

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**B_t comparison**

--- Experimental data
--- Modelled data

NPOL=9  
NTOR=6 (multiples of 7)

**Phase-locking region:** cannot be modelled with VMEC.
In order to address the issue of finding the best parametrization of the q profile for the RFP, one can start from the typical profile in helical states: **a non-monotonic q** with field reversal at the edge.

The effect of changes in the basic parameters can give an indication on the sensitivity of the diagnostic system to these changes and the minimum requirement in the diagnostics to be considered in the equilibrium reconstruction.

For this analysis V3FIT is used to obtain modelled data neglecting any matching to experimental measurements. The aim is to understand how the equilibrium changes with respect to the free parameters.
$q_0$ scan: helical states are characterized by the reversed shear in the core. A monotonic profile is unable to reproduce the experimental evidence both in terms of magnetic axis position and flux surfaces shape.
**$q_{\text{max}}$ scan:** the increase of $q_{\text{max}}$ leads to a **larger warping of flux surfaces** (from a circular to an elliptic and then bean shape) and larger shift of the magnetic axis.
Δ₁,₇ scan

Δ₁,₇: dominant helicity deformation of the Last Closed Flux Surface (LCFS). The shape of the LCFS is a degree of freedom in the reconstruction. It is linked to the radial component of the magnetic field.
**Conclusion from scan studies:**

*Modelled data* are affected in a measurable way by changes in the main parameters characterizing the q profile.

Both global quantities (e.g. $I_p$, $E_{\text{mag}}$) and local measurements (e.g. $b_{\phi}$, $r_{\text{mag axis}}$) can provide good information for the reconstruction.

More precise indication will come from the use of internal diagnostics such as SXR emissivity and Thomson scattering data.
Summary and future plans

- V3FIT with VMEC-fixed boundary is being used for the reconstruction of helical equilibria for the RFP
  - Benchmarking for the direct problem in axi-symmetric case is ok
- We are refining the comparison to experimental data, implementing NCT algorithms to V3FIT diagnostics:
  - Including informations from external currents (hybrid fixed-free boundary)
  - Extracting from the huge amount of measurements only the relevant harmonics and treating the others as noise
- Estimate of vacuum measurements in non stationary conditions is in progress (i.e. a rotating SHAx): a transfer function for the br component is available, bt is in progress.
- We also plan to test different profile parametrizations, and to include internal diagnostics (TS and SXR) in V3FIT reconstructions.