Error Field Assessment from Driven Rotation of External Kinks at Extrap-T2R

F.A. Volpe¹, L. Frassinetti², P.R. Brunsell², J.R. Drake², K.E.J. Olofsson²

¹ University of Wisconsin, Madison, U.S.A.
² Royal Institute of Technology (KTH), Stockholm, Sweden
Conventional EF Correction is a “destructive test” based on low-density LMs and needing 3-4 discharges.

[H. Reimerdes et al., Fusion Science Technol, 2011]
Locked Modes (LMs) lock to the TOTAL Error Field (EF)

i.e. to the resultant of **known** Magnetic Perturbations (MPs) and **unknown** EFs
Uniform MP rotation in presence of static EF causes *non-uniform* mode rotation

---

[F.A. Volpe et al., *Phys. Plasmas* 2009]
Discrepancy between applied I-coil phase and actual LM phase leads to set of equations for EF Amplitude and phase

- $2N$ equations in $2+N$ unknowns, including EF amplitude and phase.
  - Here $N=$ no. steps in toroidal scan.
- $N=2$ locked phases are sufficient.
- Toroidal number assumed $n=1$.

\[
\begin{align*}
A_{EF} \cos \phi_{EF} + A_{RMP,1} \cos \phi_{RMP,1} &= A_{tot,1} \cos \phi_{tot,1} \\
A_{EF} \sin \phi_{EF} + A_{RMP,1} \sin \phi_{RMP,1} &= A_{tot,1} \sin \phi_{tot,1} \\
\vdots \\
A_{EF} \cos \phi_{EF} + A_{RMP,N} \cos \phi_{RMP,N} &= A_{tot,N} \cos \phi_{tot,N} \\
A_{EF} \sin \phi_{EF} + A_{RMP,N} \sin \phi_{RMP,N} &= A_{tot,N} \sin \phi_{tot,N}
\end{align*}
\]
Animation shows how static $n=1$ EF and rotating $n=1$ MP nearly canceling out lead to non-uniform rotation.
Known (measured) LM locks to resultant of known (applied) MP and unknown EF.
Now uniform and non-uniform rotation during LM control are clear.

Compare uniform and non-uniform rotation in slide 5.

Other application: Sustained Rotation of LMs and NTMs.
Spiraling \( n=1 \) field scans in 2D. \( A_{\text{MP}} \) is the smallest \( A \) yielding a complete revolution. \( \phi_{\text{MP}} \) is phase opposite to phase of max accel./deceleration.

Other advantages:
- spiral can “cautiously” approach larger and larger LMs. No need for MP > EF.
- can be extended to multi-\( n \) EF by looking at multiple features and transitions.
Give me a compass and a magnet and I will tell you where the other magnet(s) is (are).

…and how strong they are and what multi-poles are they.
Which mode should be used, for proof of principle?

- **Mode does not** need to be preexistent and rotating  
  - At DIII-D, LM originated from rotating precursor (NTM) induced by $\beta$ ramp in low rotation plasma $\rightarrow$ ITER  
  - In few discharges, LM from EF penetration, w/o rotating precursor

- **Does not** need to be a LM or QSM (10-100Hz)  
  - Can be a TM or RWM (next slide)

- **Mode must:**  
  - …interact with EF $\rightarrow$ potential energy  
  - Mode and EF depend on $\phi$  
  - There is a $\phi$ minimizing the potential energy
Need to destabilize a mode (preferably saturated) and use it as a probe for EF. Which mode?

- Fast-rotating TM* \( n \leq -12 \)
- Growing RWM \(-11 \leq n \leq -3\)
- Marginal RWM \([-2 \leq n \leq +7\)\)
- Saturated, driven RWM \( n \geq +8 \)

*resonant, i.e. \( q = m/n \) in the plasma
Modes have **pros** and **cons**

- **Fast-rotating TM** ($\sim 10\text{kHz}$) [$n$$\leq$$-12$]
  - Shielding makes it impossible to couple rotating MP to fast mode
  - Interaction of fast TM with slow/static MP & EF $\rightarrow$ magnetic braking

- **Unstable RWM** [-11 $\leq n$ $\leq$ -3]
  - Mode growth is in competition with other effects on amplitude (rRMP, EF). Difficult to extract effect of EF.

- **Marginal RWM** [-2 $\leq n$ $\leq$ +7]
  - Decays or grows slowly $\rightarrow$ $\sim$constant $\rightarrow$ easier to extract effect of EF

- **Stable RWM** [$n$ $\geq$ +8]
  - Stable
  - Amplifies EF $\rightarrow$ easier to measure in sensor coils
  - Decays rapidly as soon as drive (rRMP+EF) is zeroed $\rightarrow$ also an indicator of good EFC

**Exp. with** $n$=$+1,2,3$ $\rightarrow$ $n$=$-1$

**Exp. with** $n$=$+10$
Multipole-multipole interaction is a generalization of dipole-dipole interaction.

Magnetic dipole of the rotating mode

Static Error Field

Array of measuring coils

Rotating magnetic perturbation
4x32 control coils apply: i) known **static EF** and ii) **rotating MP** to “drag” **mode** and “probe” **EF**.
Time-evolution of mode amplitude and phase can be predicted/interpreted from Newcomb equation for thin shell

\[ \tau \frac{db_r}{dt} - \tau \gamma b_r = b_{\text{ext}} \]

where \( b_{\text{ext}} = EF + \text{static RMPs} + \text{rotating RMPs} \):

\[ b_e(t) = b_0 + b_1 e^{i \omega t} \]

\[ b_r = \frac{\tau_w b_0}{\tau} \left( e^{\gamma \frac{t}{\tau_w}} - 1 \right) + \frac{b_1 \tau_w}{(i \omega \tau_w - \gamma)} \left( e^{i \omega t} - e^{\gamma \frac{t}{\tau_w}} \right) \]

\[ b_r = |b_r| e^{i \omega t + \Delta \phi} \quad (t > \tau_w/\gamma \approx 1 - 2 mS) \]

Amplitude

\[ |b_r| = \frac{\tau_w}{\tau} \frac{b_1}{\sqrt{\omega^2 \tau_w^2 + \gamma^2}} \]

Phase

\[ \Delta \phi = \text{atan} \left( \frac{\omega \tau_w}{\gamma} \right) \]
Predictions
Ad hoc \( n=+10 \) EF (~4G, \( A=0.2 \phi=0 \)) correctly characterized (\( A=0.22\pm0.02, \phi=1.1\text{rad} \)).
Smaller applied EF (~1G) also give expected response
Longer discharge confirms good EFC (and can be used to optimize the phase)
Better match of $\phi_{EF}$ makes mode rotation more uniform
Summary & Conclusions

• Applying a uniformly rotating RMP of constant amplitude results in a mode (LM, stable or unstable RWM, etc.) rotating non-uniformly and varying in amplitude.
  – Due to EF
  – Use non-uniform rotation and amplitude modulation to diagnose EF

• $n=+10$ stable external kink forced to rotate at 50Hz correctly measured known (applied) $n=+10$ EFs of 1-4G

• Evidence of good EFC from longer discharge and more uniform rotation

• Next test at Extrap-T2R: $n=-1$

• ITER: small $n=1$ LM?
Backup slides
Three approaches were explored

• Everything in vacuum
  – No mode $\rightarrow$ 2 entities (EF and rRMP) instead of 3
    (can’t test rotation & modulation of the 3rd in presence of other 2)
  – Direct measurement of how the two compensates each other.
    Only works if static EF is turned on during shot (otherwise not visible to saddle loops) and if MP rotates slowly relative to wall $\tau$

• Everything in open loop
  – Short discharges (~12-20ms)
  – Tried with machine-EF at n=+2
  – Clear oscillations $\rightarrow$ rotations transition observed

• Mode of interest in open loop, other in f/back
  – Longer discharges (40-70ms)
  – Tried with “phantom” EF at n=+3 (growing RWM)
In open-loop, constant-amplitude request doesn’t exactly yield constant amplitude

“Static” RMPs tend to decay

“Rotating” RMPs experience amplitude oscillations

---

**Graphs:**

1. **(c)** Plot showing $b_{r,1-12}$ (mT) over time (ms) for "Static" RMPs, with a peak around 25 ms and a decay afterwards.
2. **(d)** Plot showing $b_{r,1-12}$ (mT) over time (ms) for "Rotating" RMPs, with oscillations superimposed on the decay.
3. **(e)** Plot of phase $b_{r,1-12}$ over time (ms) for current simulation, showing a sharp change around 15 ms.
4. **(f)** Plot of phase $b_{r,1-12}$ over time (ms) for model simulation, with initial sharp changes followed by a smoother decay.

---
Mode locks to Error Field, not to Wall

Torque exerted by resistively delayed image currents in the wall:

\[
T_{\text{wall}} = - \frac{2\pi RB_R(b)r_{mn}}{\mu_0 b} \left( \frac{r_{mn}}{b} \right)^{2m-1} \frac{\omega_{mn} \tau_w}{1 + (\omega_{mn} \tau_w)^2} \rightarrow 0 \text{ for } \omega_{mn} \rightarrow 0
\]

Torque exerted by EF trying to align magnetic dipole to it:

\[
T_{\text{EF}} = - \pi^2 R^2 m \frac{a}{r_{mn}} I_{\text{EF}} B_R(a) \sin(n\omega_{mn} t - n\phi_{mn})
\]
Ultimately, technique was tested on intrinsic $n=10$ EF
Measurements at $n=-1$ need to be analyzed