Abstract

The uncertainty of electron density and temperature fluctuation measurements is determined by statistical uncertainty introduced by the sensor's noise and the difficulty in precisely, but simply, model the plasma. This project was the model of making the noise sources in the MST Thomson scattering experiment. The results of the principal components analysis routine to the most likely electron temperature and density, with confidence intervals. The results for the noise from scattered light and plasma background light is multiplied by the noise enhancement factor (P) of the avalanche photodiode (APD). Electronic noise from the amplifier and digitizer is added. The amplifier response function shapes the signal and induces correlation in the noise. The data analysis routine fits a characteristic pulse to the digitized signals from the amplifier, giving the integrated scattered signals. A finite digitization rate loses information and can cause numerical integration error. We find a formula for the variance of the scattered signals in terms of the background and pulse amplitudes, and three calibration constants. The constants are measured easily under operating conditions, resulting in accurate estimation of the scattered signals' uncertainty. We measure $F$ for our APDs, in agreement with other measurements for similar APDs. This value is wavelength-independent, simplifying analysis. The correlated noise we observe is reproduced well using a Gaussian transfer function. Numerical integration error can be made negligible by using an interpolated characteristic pulse, allowing digitization rates as low as the detector bandwidth. The effect of background noise is also determined.

Summary of Key Results

- Simple but comprehensive model of all uncertainty sources in integrated scattered signals
- Variance of photonic noise from pulse proportional to signal amplitude
- Constant background noise acts over effective integration time
- Numerical integration error scales quadratically with signal amplitude
- Correlation of noise (due to amplifier response function) treated

Simulation confirms and extends analytic results
- Integration error is inversely proportional to pulse resolution
- Background integration time depends on amplifier response function
- All other scalings confirmed

- Experimental data obeys model
- Pulled and constant light source results agree
- Provides easy way to calibrate uncertainty of scattered signals

Model of Scattered Signal Uncertainty Is Comprehensive

- $\text{Var}(S) = c \cdot S^2 + \frac{R^{-1}}{\text{tanh}(R^{-1})} GMF_S + \left(\text{GMFB}_{\text{ph}} + \text{GMFB}_{\text{pl}}\right) + \text{error}_{\text{num}}$

- Quadratic term: is the RMS relative numerical integration error

- Linear term: photonic noise inherent in signal $\text{Var}(S_{\text{ph}}) = \text{Var}(S_{\text{ph}})$

- G's amplifier gain in $V/s/electron$

- $M$ is APD avalanche multiplication factor

- $F$ is APD noise enhancement factor

- $R$ is the ratio of the digitization to amplifier bandwidth

- $R^{-1}/\text{tanh}(R^{-1})$ reflects loss of information if $R < \text{tanh}^{-1}(R^{-1})$

- $\text{error}_{\text{num}}$ is constant

- Background noise accumulated over integration time

- Background light signal $h_0$ causes photonic noise

- Electronic noise power spectral density is $\text{error}_{\text{num}}$

Monte Carlo Simulation Used To Verify Analytical Results

- Two 2J purple Nd:YAG lasers

- Inertial mass and fiber optics collect scattered light

- 21 GA Filter Polynomials decompose spectrum

- 128 Avalanche Photodiode / preamplifiers detect time-varying signal

- Simulation Details

- Photons drawn at 0.1 ns intervals, Poisson distribution

- Convolved with amplifier response function

- Output sampled at digitization period $\tau = 1$ ns

- Characteristic pulse fitted to output to get integrated signal

- Verified model for variance of output with constant input $\text{Var}(S) = \text{GMFB}_{\text{ph}}(\text{input})$

- Photonic Noise From Pulse Reproduced in Simulation

- Linear term: $\text{Var}(S)_{\text{lin}} = \frac{R^{-1}}{\text{tanh}(R^{-1})} GMF_S S$

- Quadratic term: $\text{Var}(S)_{\text{quad}} = c \cdot S^2$

- Integration error inversely proportional to interpolation rate

- Resolution of characteristic pulse, not digitizer resolution, important for integration

- Only require $R > \text{tanh}^{-1}(R^{-1})$ to get full integration on actual pulse

Simulation of Background Noise Yields New Behavior

- Constant term: $\text{Var}(S)_{\text{const}} = \left(\text{GMFB}_{\text{ph}} + \text{GMFB}_{\text{pl}}\right) + \text{error}_{\text{num}}$

- Measure of input pulse width is 2.4x too high

- Variance also depends on response function

- Not captured by analytical results

- Variance spikes up when SNR small

- Due to uncertainty in pulse location

Theory for Constant Input Fits Experimental Data

- Response functions modeled by fitting to correlation functions

- Amplifier response function

- Bandwidth $\text{BW}_{\text{APD}} = 5$ MHz

- Constant signal model fitted to experimental data

- $\text{BW}_{\text{APD}} = 0.32$ to $0.38$ MHz

- $M = 90$

- Photon noise is independent of wavelength

- Removes constant on light sources for noise calibration

- Simplifies experimental uncertainty estimation

Monte Carlo Simulation Used To Verify Analytical Results

- Best Fit: $c \cdot S^2 + \frac{R^{-1}}{\text{tanh}(R^{-1})} GMF_S$ for unfiltered mV ns

- Continuous: $0.29 \pm 1.08$ ns

- 10x: $1.41 \pm 1.00$ ns

- 30x: $0.40 \pm 0.24$ ns

- AC 1ns: $0.24 \pm 1.29$ ns

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