Simulations of sustained HIT-SI operation in the present experimental regime and beyond

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R.J. Smith  G.A. Andexler
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Also,
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I. HIT-SI DESCRIPTION: geometry, important parameters, Steady Inductive Helicity Injection

II. Decaying resistive MHD simulations: equations, Lundquist number definition, poloidal flux generation

III. Boundary conditions for driven simulations: normal magnetic fields, tangential electric fields (in two parts), resistive edge layer

IV. Preliminary driven simulation results: increased n=0 growth with S, spheromak formation after ramp down, dissipation in the edge layer

V. Future Work
The HIT-SI spheromak

λ of spheromak region ≈ 11 m⁻¹

Major radius: 0.33 m

Circumference of magnetic axis: ≈ 2 m
**Steady Inductive Helicity Injection (SIHI)**

- **Flux and voltage in each injector are oscillated in phase with each other and out of phase with the other injector.**

- **Helicity injection rate:**

  \[
  \dot{K} = 2V_{\text{inj}} \psi_{\text{inj}}
  \]

  Thus

  \[
  \dot{K}_{\text{Injector}_1} = 2V_0 \psi_0 \sin^2 \omega t
  \]

  \[
  \dot{K}_{\text{Injector}_2} = 2V_0 \psi_0 \cos^2 \omega t
  \]

  AND

  \[
  \dot{K}_{\text{total}} = 2V_0 \psi_0
  \]
Simulations are resistive MHD

Simulations are performed with the NIMROD code using a zero pressure, resistive MHD model, without the continuity equation. The two evolved equations are:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{1}{\rho} \mathbf{j} \times \mathbf{B} + \nabla \cdot (\mathbf{v} \nabla \mathbf{v})$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{j}$$

The three parameters $\rho$, $\eta$, and $\nu$ are uniform and constant in time.
In dimensionless form (the curl of) our Ohm's Law becomes

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{S} \nabla^2 \mathbf{B} \]

where the only parameter is the dimensionless number \( S \), which is the ratio of the resistive diffusion time to the Alfvén time

We will define \( \tau_{L/R} = \mu_0 / (\eta \lambda^2) \) and \( \tau_A = 2\pi R / v_A \)

If \( S = \mu_0 v_A L / \eta \), then \( L = 1 / (2\pi R \lambda^2) \approx R / 100 \) for HIT-SI
Decaying simulations show relaxation at sufficient $S$ [1]

- A transient period of $n=1$ magnetic activity occurs during the decay of initially unstable equilibrium.

- Relaxation flattens the current profile and generates poloidal flux at high $S$. At low $S$, $n=1$ is too damped for relaxation.

Boundary conditions replace the injectors in driven simulations

Injector $B_z$ fields are toroidal fields of an RFP in spheromak mode.

➜ Injector $B_z$ fields are toroidal fields of an RFP in spheromak mode.
Applied tangential $E$ fields have two terms

→ An electric field is applied to produce a voltage profile identical to the $B_z$ profile:

$$\vec{E} = -\vec{\nabla}V$$

$$\frac{1}{r} \frac{\partial}{\partial r} (rE_\phi) - \frac{1}{r} \frac{\partial E_r}{\partial \phi} = - \frac{\partial B_z}{\partial t}$$

→ A second, radial electric field is applied to be consistent with the time changing $B_z$ fields
A highly resistive boundary layer serves several purposes in the simulations.

- In a thin layer at the wall, the resistivity increases linearly from the plasma value to a much larger value.
- Layer prevents currents from flowing to the wall (HIT-SI copper flux conserver has ~0.3 mm insulating ceramic layer).
- High impedance at the injector “openings” produces current profile identical to applied voltage ⇒ force free RFP fields.
- Insulating layer at injector strike points prevents line-tying.
Injector Flux and current from a HIT-SI discharge during phase one of operation.
### Parameters for a simulation in the estimated experimental regime of phase one HIT-SI discharges

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak injector flux</td>
<td>0.47 mWb</td>
</tr>
<tr>
<td>Peak injector current</td>
<td>10.6 kA</td>
</tr>
<tr>
<td>Density</td>
<td>$3 \times 10^{19}$ m$^{-3}$</td>
</tr>
<tr>
<td>Resistivity</td>
<td>$1.26 \times 10^{-5}$ Ωm</td>
</tr>
<tr>
<td>Resistive diffusion time</td>
<td>0.83 ms</td>
</tr>
<tr>
<td>Alfvén time</td>
<td>36.8 µs</td>
</tr>
<tr>
<td>Lundquist number</td>
<td>22</td>
</tr>
</tbody>
</table>
S=22 case does not exhibit growth of n=0 fields until the drive is ramped off.

0.2 ms ramp up, 0.4 ms flat top, 0.2 ms ramp down.
After ramp down, islands appear followed by core of good flux surfaces.

→ Islands are m/n = 3/2

T = 0.87 ms (near the N=0 peak)

→ Closed flux is bounded by m/n = 2/3 rational surface

T = 1.23 ms (well into the decay)
# Higher S Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Injector Flux</td>
<td>0.47 mWb</td>
</tr>
<tr>
<td>Peak Injector Current</td>
<td>10.8 kA</td>
</tr>
<tr>
<td>Density</td>
<td>$3 \times 10^{19}$ M$^{-3}$</td>
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<tr>
<td>Resistivity</td>
<td>$6.28 \times 10^{-7}$ $\Omega$ m</td>
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<tr>
<td>Resistive Diffusion Time</td>
<td>16.5 ms</td>
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<tr>
<td>Alfvén Time</td>
<td>36.8 $\mu$s</td>
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<tr>
<td>Lundquist Number</td>
<td>450</td>
</tr>
</tbody>
</table>
Spontaneous growth in n=0 energy seen at S=450

At this S, n=0 magnetic energy builds up and n=1 kinetic energy becomes dominant over n=0 and n=2.
FIELDS SHOW CHARACTERISTICS OF ENVISIONED SIHI OPERATION

At an X injector peak, shorter field lines link one side of the core spheromak region.

Longer field lines loop many times through the core.

All field lines appear to be open at this time.
Conclusions

- Decaying HIT-SI simulations predict significant relaxation beginning in the S~300-600 regime

- At S=22 (estimated regime for initial operations) no significant n=0 fields are generated until the injectors are ramped down

- During decay at low S, islands appear, followed by a core of good flux surfaces

- At S=450 n=0 fields grow to many times the n=1 magnitude as helicity builds up in the spheromak

- Sustained fields take on the characteristics of envisioned SIHI operation
Future Work

- Complete $S=450$ simulation to steady state $n=0$ amplitude followed by injector ramp off
- Further explore parameter space to determine under what conditions closed flux will form during sustainment and scaling of $n=0$ amplitude with $S$
- Compare results with experimental data to check the validity of the physics model