Understanding Spheromak Formation and Evolution by Ideal and Resistive MHD Modeling

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Examining some detailed MHD characteristics of SSPX:
— comparing experiment with NIMROD modeling

- **Mode amplitudes** previously examined by Sovinec (e.g. paper at ICC2004)
  - Drop when the gun is operated at $\lambda_{\text{gun}} \approx \lambda_{\text{flux conserver}}$ as in the experiment
  - Amplitudes about the same as the experiment
  - Mode structure (examined here) finds the $n=1$ mode is peaked on the current column — driven by the gun current

- **Mode frequencies**
  - Much lower than experiment
  - Non-MHD physics — e.g. rotation drive — required to explain the frequencies

- **Plasma startup and transition to quasi-steady state**
  - NIMROD — mode growth from “seed amplitude” determines when non-axisymmetric processes become important
  - SSPX — startup is more non-axisymmetric than can be initiated in the code, and non-axisymmetric processes occur earlier
  - Time evolution of SSPX double-pulse experiment may be explained by the growth of modes from an initially small amplitude
Mode frequency and rotation — NIMROD calculation for a high gun current \((\lambda_{\text{gun}} > \lambda_{\text{flux conserver}})\) discharge

The calculation found significant velocities in the spheromak

- Speeds equivalent to \(\sim 1\) eV
- Average velocity low — but significant shear
- Mode frequencies lower than experiment by an order of magnitude

Adding rotation brought the mode frequencies into agreement with experiment

- Only small changes observed in other properties
Velocity contours — NIMROD with no “imposed” rotation

High azimuthal velocities ($\pm 1.4 \times 10^4$ m/s) but considerable shear in profiles

- Average azimuthal velocity is small
  - Holcomb pointed out (from experiment) that there are only small (net) $j_x B$ forces to drive rotation — essentially resulting from current crossing the bias flux

- Large radial and axial velocities near surfaces, especially the cathode and bottom of the flux conserver (anode)
NIMROD “magnetic probe measurements” show modes at low frequency

- The calculated poloidal field (0.15 T) — consistent with experiment
- Amplitude of the oscillation (~ 7% rms) — in fair agreement for a strongly driven spheromak
- However, NIMROD frequency (1.6x10³ Hz) is an order of magnitude lower than experiment
Modes $n = 1, 2, 3$ (and perhaps higher) — present at low amplitude (2-4 %) and low coherence (Shot 7391), $f \sim 20$ kHz
The $n=1$ mode is coherent in Shot 7226, with $f \approx 14$ kHz
Rotating the plasma brings frequencies into approximate agreement with observations

Experiments typically have $f \sim 20$ kHz

- This corresponds to an artificial rotation with a velocity at the flux conserver ($R = 0.5$ m) of $v_\phi = 2\pi R f = 6.26 \times 10^4$ m/s (20 kHz)
- NIMROD results after $4 \times 10^{-4}$ sec:
Velocity after applied rotation still had considerable shear

\[ \text{Re VR} \]

\[ \text{Re VZ} \]

\[ \text{Re VR} \]

\[ \text{Re VZ} \]

w/o rotation

with rotation
The n=1 fields — much weaker at the wall than in the current column

- Consistent with the experimental observation that the n=1 mode is much stronger on probe-17 (near the geometric axis) than on probe-9 (on the outside wall)
- The mode energy \([E_m(1)]\) in NIMROD implies much higher fields than are present inside the spheromak separatrix
- Mode driven by column current
Torque/rotation — NIMROD and experiment

- The net torque due to $j \times B$ forces is
  \[ T_\varphi = \int dV r(j_z B_r - j_r B_z) = \frac{1}{\mu_0} \int dS r B \cdot \hat{n} \]
  \[ = \int d\ell_p r(\ell_p) I(\ell_p) B_n(\ell_p) \text{ with } \ell_p \text{ the poloidal length on the flux conserver} \]
  \[ \approx \frac{I_{\text{gun}} \Phi_{\text{gun}}}{2\pi R} f \text{ with } f \text{ the net cross-field current} \]
  In NIMROD this is presumably balanced by viscous forces in the wall boundary layer.

- The resulting torque has been shown to be in the wrong direction to drive the plasma rotation (C. T. Holcomb, et al., submitted to PRL; suggested due to ion wall losses.)
  - Differential ion losses can apply a torque which could spinup the plasma — however this has not been demonstrated experimentally.

An effective ion current to the wall, $I_+$, with average $v_\phi$ yields a net torque
\[ T_+ = 0.5 \times 10^{-8} v_\phi R I_+ \]

The ratio of torques is
\[ \frac{T_+}{T_\varphi} \sim \frac{1}{50} \frac{I_+}{f I_{\text{gun}}} \text{ at } 10 \text{ eV and } 30 \text{ mWb} \]
Torque (continued)

- The magnetic and kinetic torques are about equal if the ion loss (at 10 eV) is an order-of-magnitude greater than the gun current
- The effective particle confinement time is

\[ \tau_n = \frac{enV}{fI_{\text{gun}}} \sim 6 \times 10^{-6} \text{ s at } n = 10^{20} \text{ m}^{-3}, f = 0.1, \text{ and } I_{\text{gun}} = 200 \text{ kA} \]

This is rather short, although because of the many approximations the mechanism cannot be excluded
Spheromak startup — mode growth and formation of the spheromak configuration

SSPX — the initial current pulse shows breaking of axisymmetry very early in the pulse

- This is likely due to inherent asymmetries in the breakdown associated with initial formation of the arc

NIMROD — mode growth is slow enough that significant asymmetry is delayed until well into the pulse
Startup of a shot for modeling by NIMROD

The formation pulse and beginning of the sustainment pulse are shown

- Note the early start of the $n=1$ mode
  - Strong fluctuations on the central current column
  - The external power system is highly inductive — fluctuations ($n=0$) show up on the voltage

- Rapid appearance of magnetic fluctuations probably due to nonuniformities in the breakdown process

The gun current and flux are modeled accurately in a NINROD calculation
NIMROD — Mode growth from a low “seed” delays initial reconnection processes until after the current peaks

Energy in $n=0$ grows until a “dump” into higher $n$

Energy and flux drop during the initial reconnection event

Mode energy takes $100 \mu s$ to grow from $10^{-8} \text{ J}$

Before the burst of magnetic activity, the fieldlines lie on the open flux resulting from the “bubble-burst” from the gun

All fieldlines are open (except for a small island near the flux conserver)

— no indication of chaos

- Puncture plots are smooth
- $\lambda=\mu_0 j/B$ peaked on the geometric axis and small on the magnetic axis (not shown)
- Small poloidal field near magnetic axis — fieldlines make many toroidal transits
After the burst of magnetic modes, azimuthally-averaged flux surfaces are formed (a “good” mean-field spheromak) but fieldlines are chaotic

All fieldlines are open — chaos is apparent

- All fieldlines exit the flux conserver
- $\lambda=\mu_0j/B$ (azimuthally averaged) is patchy and non-monotonic (not shown)
NIMROD — Mode growth from a moderate “seed” — reconnection close to the current peak with continued energy growth

Modes reach large amplitude earlier due to their larger value at t=0

Total magnetic energy (at 0.25 ms) — about the same as for the low-seed case

The gun current has turned down — magnetic energy is still growing

“Magnetic probe” has a different time history than the experiment — $B_z$ (midplane) still growing
Good “mean-field” (azimuthally averaged) spheromak has formed by 0.33 ms

The current column is still quite peaked — see $\lambda$-profile near the geometric axis

The contour of $\lambda$ is flat within the mean-field spheromak separatrix — corresponds to chaotic fieldlines associated with high fluctuation amplitudes
Experiment — azimuthally averaged fit at 0.45 ms shows a pinched current column and deep $\lambda$-profile

Corsica fails to obtain a good fit at earlier times
Possible experimental example of slow mode growth with a “late” reconnection event

A current pulse was applied to an existing plasma in SSPX with low-amplitude fluctuations — the field at the flux conserver midplane was constant until after the peak of the pulse.

Reconstructed flux plots showed a pinching of the current column before the current peaked.

A voltage spike occurred after the current pulse, followed by a buildup of the field at the flux conserver midplane.

This was possibly a reconnection event similar to that seen in the NIMROD simulations.
Time history of Shot 11170

The poloidal field at probe p09 position (flux conserver midplane) does not start to increase in the second pulse until the current peak. Non-axisymmetric mode activity is also low until then.
Expanded time verifies that the poloidal field at p09 does not start to increase until the peak of the second current pulse.

I, V, 90° probes p09 and p17

- Note the break in dI/dt at ≈ 1.48 ms
- Note the spike in voltage after 1.4 ms
- Increasing B
- Probe p17 (bottom of FC) increases from 1.3 ms

Azimuthal array at p09

- Magnetic field at flux conserves midplane only builds after the peak of the current.
Probes p03 (in gun) and p17 (bottom of FC) are not axisymmetric before the current peak – contrary to simple model.
Contours of $|B|$ show the current column pinched until the voltage pulse

During current rise

$t = 1.38 \text{ ms}$

During current decay

$t = 1.52 \text{ ms}$
Hyper-resistivity is a diffusive approximation to resistive MHD

Hyper-resistivity yields a good fit to the evolution of the discharge

- The calculation is simple — fast comparison with experimental data
- In SSPX, the hyper-resistivity coefficient varies by a factor of 10, correlating with discharge characteristics — implying relationship with the magnetic turbulence

Predictive capability is limited — no good first-principles model

- Hyper-resistivity assumes differential transport — e.g. magnetic fluctuations are local — In the experiment, dominant modes are low order [e.g. n(toroidal) = 1 to 4]
- The model (as applied) assumes transport of helicity across open flux is very fast — MHD fits to magnetic probe data often indicate a peaking of current on the geometric axis

Resistive MHD is a much better physics model, but requires considerable computer time — limited in the number of comparisons
Shot 7391 — Fit at 0.9 ms starts the hyper-resistive calculation

- Confined current agrees with probe fit from Corsica for $\Lambda = 0.00315$
- $T_e(0) = 50$ eV, $T_e(\text{edge}) = 20$ eV
Hyper-resistivity gives a good fit to the current evolution

A hyper-resistive model for helicity and current transport fits the data quite well

$$\frac{\partial \Psi}{\partial t} = V_L = 2\pi \left[ \eta \langle \mathbf{j} \cdot \mathbf{B} \rangle - \frac{\partial}{\partial \Phi} \Lambda \left( \left\| \nabla \Phi \right\|^2 \right) \frac{\partial V}{\partial \Phi} \frac{\partial \lambda}{\partial \Phi} \right]$$

with \( \lambda = \frac{\mu_0 \mathbf{j} \cdot \mathbf{B}}{B^2} \), \( \langle \cdots \rangle = \int \frac{dl}{B} / \int \frac{dl}{B} \), and \( \hat{\Phi} \) the normalized toroidal flux

For shot 7391, a good fit to the time history was obtained by starting from an initial fit at 0.9 ms and evolving the current profile with \( \Lambda = 7 \times 10^3 \) volt-weber

Goal — relate this coefficient to NIMROD modeling
Summary

Comparisons between NIMROD and SSPX are generally good

• Calculated mode frequencies are low — physics of rotation probably outside of resistive MHD model
  – Adding rotation to match frequency with experiment has little effect on other spheromak properties

• Spheromak buildup is slower than experiment
  • Probably due to non-uniformities in experimental breakdown
  • However, injected energy is stored in the field and the modeled buildup continues longer – uncertain whether magnetic field at flux conserver reaches final experimental value

• Double-pulse experiment shows slow buildup similar to NIMROD

• Hyper-resistivity model shows good fit to experiment
  Issue — Can hyper-resistivity coefficient be deduced from NIMROD?