Electron Heat Transport Measured in a Stochastic Magnetic Field

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New profile measurements have allowed the electron thermal diffusivity profile to be estimated from power balance in the Madison Symmetric Torus where magnetic islands overlap and field lines are stochastic. The measurements show that (1) the electron energy transport is conductive not convective, (2) the measured thermal diffusivities are in good agreement with numerical simulations of stochastic transport, and (3) transport is greatly reduced near the reversal surface where magnetic diffusion is small.

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Toroidal plasma confinement systems rely upon symmetry for formation of nested magnetic flux surfaces. When flux surfaces exist, magnetic field lines and hence particle orbits are constrained to lie on a toroidal surface. Resonant, symmetry-breaking magnetic perturbations such as those generated from magnetohydrodynamic (MHD) instabilities can change the magnetic topology. If these fields are small, the magnetic field lines break into chains of magnetic islands at mode-rational surfaces where the perturbations are resonant [1,2]. When two or more nearby magnetic islands grow to the point where they overlap, the flux surfaces in between can be completely destroyed. In this case field lines are no longer constrained to a toroidal surface, but can wander stochastically [3,4].

The breakup of magnetic surfaces is predicted to have a dramatic effect on energy confinement. The free streaming orbits of particles parallel to the magnetic field result in a radial excursion of the particle's trajectory, leading to radial energy diffusion. The thermal conductivity is often assumed to be given by

$$\chi = v_{ll} D_M, \tag{1}$$

where χ is the radial thermal conductivity, v_{ll} is the velocity along the magnetic field, and D_M is the magnetic field diffusivity. Rechester and Rosenbluth (RR) [5] estimated the energy transport in the limit of strongly stochastic magnetic fields, quantified by large values of the Chirikov overlap criteria, or "stochasticity parameter,"

$$s = \frac{1}{2}\frac{w_{mn} + w_{m'n'}}{r_{mn} - r_{m'n'}}. \tag{2}$$

Here the magnetic island width w_{mn}, for toroidal mode number n and poloidal mode number m, can be determined from the strength of the magnetic perturbation at the radii r_{mn} of rational surfaces, $w_{mn} = 4\sqrt{r_{mn}\tilde{b}_{rmn}/(nB_\theta|q'_{mn}|)}$. The radial magnetic fluctuation amplitude is given by $\tilde{b}_{rmn}$, B_θ is the poloidal magnetic field, and q'_{mn} is the gradient in the q profile at the rational surface ($q = rB_\phi/RB_\theta$, where r is the minor radius, R is the major radius, and B_ϕ is the toroidal magnetic field.) In the limit of $s \gg 1$, RR assumed the electron heat flux to be diffusive, obeying Fourier's law $q_e = -n_e\chi_{\rm RR}\nabla T_e$, and put forth that

$$\chi_{\rm RR} = v_{T_e} D_{M,\rm RR} = v_{T_e}\pi L_{\rm eff}\frac{\tilde{b}_n^2}{B^2}. \tag{3}$$

Here n_e is the electron density, T_e is the electron temperature, v_{T_e} is the electron thermal velocity, B is the total magnetic field, and $L_{\rm eff}^{-1} = L_{\rm ac}^{-1} + \lambda_{\rm mfp}^{-1}$ is the effective autocorrelation length (for electrons) given by the inverse sum of the autocorrelation length and the electron mean-free-path length.

Toroidal plasmas operating with strong magnetic relaxation and reconnection, as can occur in the reversed-field pinch (RFP) and spheromak configurations, are believed to have a stochastic magnetic field as a consequence of magnetic fluctuations associated with relaxation. The magnetic fluctuations in the RFP are a broad spectrum of internally resonant tearing modes driven by gradients in the current density profile. The RFP configuration is expected to be a good test bed for studying magnetic fluctuation induced transport since the overlapping magnetic tearing mode islands create a large radial region in which the magnetic flux surfaces are destroyed and field lines wander stochastically [6]. Recent efforts to control the current profile (magnetic stability free energy) lead to reduced stochasticity and greatly improved confinement in the RFP [7].

In this Letter, the coefficients of electron particle and heat transport have been determined from power balance analysis across the profile of the Madison Symmetric Torus (MST) reversed-field pinch [8] and are compared to theoretical predictions of transport in a stochastic magnetic field. Measurements of the magnetic equilibrium

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and modeling of the internal magnetic fluctuations (constrained by measurements on the plasma boundary) are used to numerically estimate the stochastic wandering of field lines in the plasma core (D_M); the measured transport coefficients are consistent (within experimental uncertainty) with the predictions from the model [Eq. (1)]. Moreover, both the experiment and simulation show strong reduction in the thermal transport in the edge region where magnetic field line diffusion is small.

This analysis has been made possible for the first time on the MST [9] by a number of new, time-resolved profile diagnostics. In particular, an upgraded Thomson scattering diagnostic [10] has been used to measure the electron temperature profile evolution. Additionally, the time evolving current density profile has been determined from equilibrium reconstructions constrained by measurements of Faraday rotation [11] and on-axis magnetic field strength [12]. The equilibrium reconstructions provide two critical components for the analysis. First, they provide the q profile and hence the location of each rational surface. Second, the time sequence of equilibriums can be used to determine the internal parallel electric field profile [13]; this information has provided a new technique for estimating the Ohmic power deposition profile that does not rely upon modeling of the plasma resistivity.

The measurements presented in this Letter are representative of transport in standard MST RFP plasmas which exhibit a sawtooth cycle of MHD relaxation [14,15]; these plasmas do not exhibit enhanced confinement as observed during pulsed poloidal current drive [7]. The transport analysis presented is from a phase of the sawtooth cycle between crashes (i.e., 1.25 ms before the next crash). The sawtooth period is ~6 ms. Typical pulse lengths for the experiments reported here were 70 ms, with electron temperature $T_e \sim 325$ eV, plasma current $I_p \sim 385$ kA, and a line-averaged electron density $n_e \sim 1.1 \times 10^{19}$ m^{-3}. The MST is a large sized reversed-field pinch experiment, with a major radius of 1.5 m and a minor radius of 0.52 m. The mode amplitudes in the MST are resolved up to $n = 15$ by a 32 element toroidal magnetic probe array.

The equilibrium field and the Ohmic power deposition profile have been determined from equilibrium reconstruction techniques. The MSTFIT code solves the Grad-Shafranov equation in the toroidal geometry of the MST, calculating a least-χ^2 fit to the experimentally measured data. Shown in Fig. 1 are the flux surface-mapped, least-χ^2-spline fits to the electron temperature and density measurements. The fitting technique is discussed extensively in Ref. [16]. The 6 ms sawtooth cycle is divided for analysis into 12 time slices, at 0.5 ms intervals. At each time slice, the plasma equilibrium magnetic field profiles are determined from solutions to the Grad-Shafranov equation.

Utilizing finite time difference of sequences of neighboring equilibriums, time derivatives of the toroidal and poloidal flux are determined which provide the poloidal and toroidal electric field profiles and the time rate of change of the stored magnetic energy. From the electric and magnetic field profiles, along with the change in the magnetic energy, the Ohmic input power profile is calculated using Poynting's theorem ($\mathbf{E}\cdot\mathbf{J}$). Calculating the power deposition profile in this way partially accounts for the MHD dynamo mechanism in that any dynamo-generated current is included in the measured current density profile. In the MST the dynamo EMF, $\langle\tilde{\mathbf{v}}\times\tilde{\mathbf{B}}\rangle$, is measured to be small between sawtooth crashes [17]; hence, dissipation associated with dynamo-related turbulence ($\tilde{\mathbf{E}}\cdot\tilde{\mathbf{j}}$) is expected to be small between crashes. Electromagnetic energy transfer within the plasma volume caused by the dynamo is also small between crashes.

From the Ohmic input power deposition profile the electron thermal conductivity is calculated through local power balance considerations. The Ohmic input power is by far the dominant term in the local power balance, but collisional losses of energy to both the ions and impurities are taken into account, as well as radiative losses and ambipolar losses from electron movement in the radial electric field [18]. From the total electron heat flux the measured convective heat flux ($\frac{5}{2}\Gamma_e T_e$) is subtracted, yielding the conductive heat flux. From Fig. 1(c) it is clear that the convective heat flux is a small correction, accounting for less than 10% of the total heat flux at all radii. The electron thermal conductivity profile is found using Fourier's law, dividing the conductive heat flux by the product of the measured density and the measured temperature gradient. Previous measurements have shown the heat loss at the edge of the plasma to be primarily convective [19]; however, there are significant differences between the two experiments (e.g., this experiment was free of external probes, it was at nearly double the plasma current, and the particle confinement time for the plasmas studied here is measured to be 4 times longer.)

To investigate the magnetic structure of the MST, the field stochasticity was numerically simulated using the nonlinear DEBS code. DEBS solves the nonlinear MHD equations in doubly periodic, cylindrical geometry to produce the evolving magnetic fluctuation eigenfunctions [20]. The q-profile representation of the equilibrium magnetic field profiles is shown in Fig. 2(a). For these calculations, DEBS is initialized with the measured resistivity profile from the experiment, and run at a Lundquist number of $S \sim 10^6$. The experimental Lundquist number is $S \sim 3 \times 10^6$. The DEBS simulations reproduce several of the observed experimental features, including the spectrum of dominant modes and the sawtooth period. The toroidal and poloidal eigenfunction amplitudes are compared at the perfectly conducting wall to the value of the wall-measured toroidal and poloidal mode amplitudes. This normalization factor (which is ~1.7 and varies from mode to mode) can then be applied to the radial eigenfunctions, shown in Fig. 2(b), to estimate the radial mode amplitudes in the experiment. Figure 2(c) is a puncture

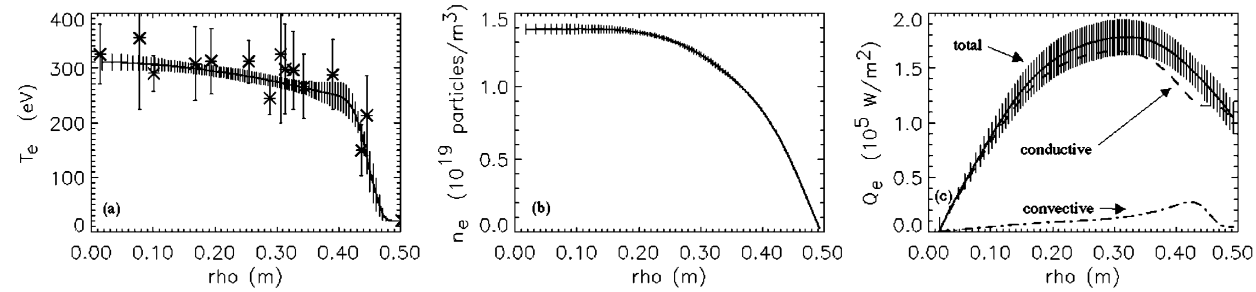

FIG. 1. Profiles −1.25 ms away from the sawtooth crash of (a) measured electron temperature, (b) measured electron density, and (c) calculated heat and particle flux. The sawtooth period is ~6 ms.

plot representation of magnetic field lines traced from 20 equally spaced radii in the simulated MST plasma. The puncture plot is generated using the MAL code [21], which utilizes the magnetic fluctuation eigenfunctions calculated by the DEBS code, and normalized by the experimentally measured mode amplitudes. A measure of the magnetic field diffusivity, and hence thermal conductivity, can be found directly from these simulations [22] by ensemble averaging the square of the radial excursion Δr over the field line length ΔL, $\chi_{\mathrm{MAL}} = v_{T_e}\langle \Delta r^2/2\Delta L\rangle$. This is a numerical simulation of the RR analytical expression for stochastic diffusion. The result of this simulation is in very good agreement with the measured thermal conductivity as shown in Fig. 3(a). Moreover, where the field stochasticity is high, the relation $\chi_{\mathrm{RR}} = \chi_{\mathrm{MAL}}$ should hold, which is supported by Fig. 3(a). This relation is violated when the stochasticity is low, in accordance with the derivation of χ_{RR}.

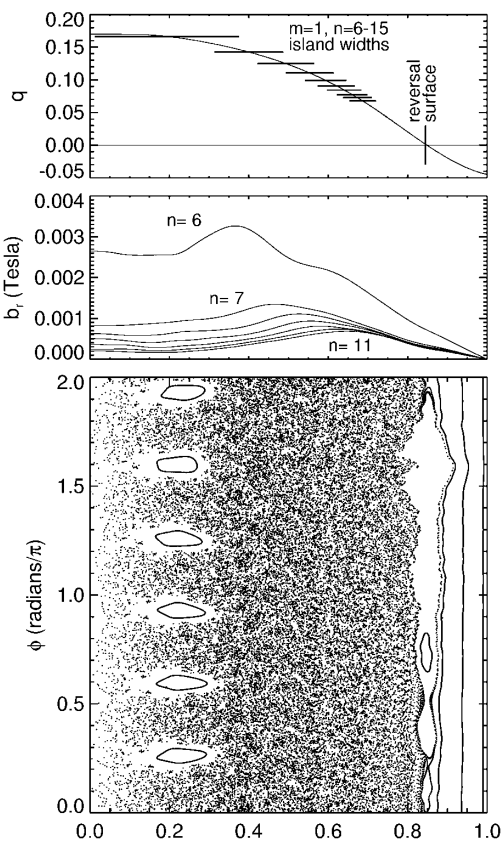

FIG. 2. (a) The experimental q profile away from a sawtooth crash including calculated magnetic tearing mode island widths, (b) the eigenfunctions of radial magnetic fluctuations as calculated by DEBS modeling at $S \sim 10^6$, and (c) the puncture plot that results from field line tracing applying the MAL code to the DEBS simulation. The field structure outside of the reversal surface is not captured in this figure, since only $0 < n < 33$ modes are plotted in this case.

Figure 2(a) shows the q-profile representation of the experimental equilibrium fields in an MST plasma, along with the locations and widths of the dominant tearing mode islands as calculated from the experimentally normalized radial fluctuation eigenfunctions from DEBS. From Figs. 2(a) and 2(c) it is clear that in the core of the plasma there is a large magnetic island ($m = 1$, $n = 6$), which is relatively isolated, due to the local q shear, from the next large island ($m = 1$, $n = 7$). In the midradius region, however, the location of resonant islands ($m = 1$, $n = 10, 11, 12, \ldots$) are packed closely together. Even though the mode amplitudes (and islands) are small in this region the overlap among islands is large. The field line tracing calculations confirm the experimental measurements that though there is a large variation in the size of the magnetic fluctuations, the thermal transports in the core and midradius regions are approximately equal. This suggests that field stochasticity, and not fluctuation amplitude, is a more important indicator of magnetic fluctuation induced transport. In the edge of the plasma, good flux surfaces are again restored. The DEBS simulation confirms that the edge field is not stochastic, consistent with the fact that the dominantly unstable tearing modes are nonresonant at the edge, and that the radial field is forced to zero at the perfectly conducting boundary.

In summary, transport in a stochastic magnetic field, caused by overlapping adjacent islands in the RFP, has been measured and compared to theory. These measurements have been made in standard, i.e., no external current drive, MST plasmas. A numerical simulation of the stochastic transport (at Lundquist number similar to the experiment) is in good agreement with the measured values, though the error bars are large. In the edge of

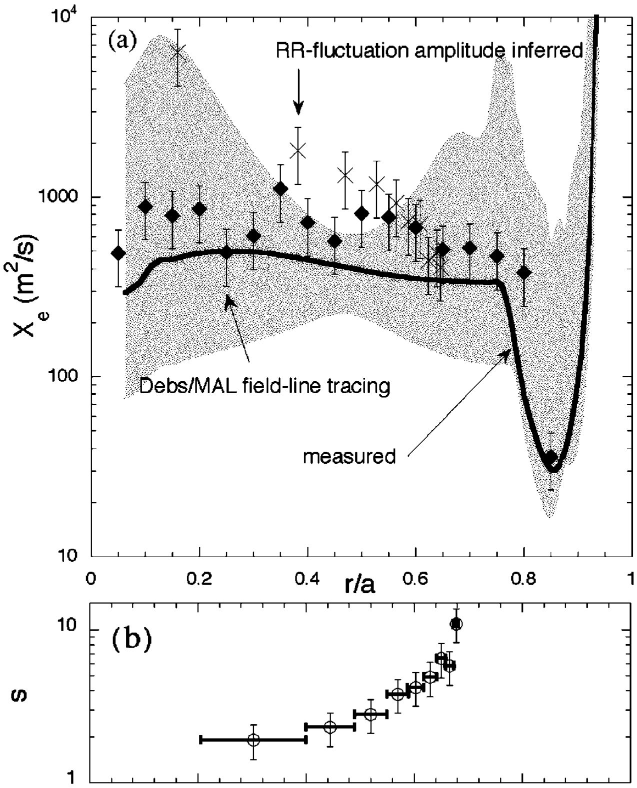

FIG. 3. (a) The measured electron thermal conductivity profile (solid line) compared with numerical calculation of thermal conductivity from DEBS/MAL modeling (diamonds), and the analytic model of Rechester-Rosenbluth (crosses). The uncertainty in the measured χ_e (shown in grey) is extremely sensitive to the measured value and statistical uncertainty of ∇T_e, which is greatest near the core. (b) The stochasticity parameter, s, shows the core is only mildly stochastic since it is dominated by a single mode, while the midradius region ($0.4 < r < 0.8$) has strong island overlap.

the MST, both experiment and simulation indicate that there are favorable transport properties. All of these points are consistent with recent calculations for the RFP [6,23]. Whereas the MST is afflicted by magnetic fluctuation induced transport inside the reversal surface, the presence of a transport barrier at the edge salvages the overall confinement of the plasma. Reducing the overall stochasticity within the midradius can result in reducing the electron thermal conductivity and transport, and there is strong evidence that this can be achieved through current profile control [7]. This suggests that progress towards development of the RFP, and in general all toroidal magnetic confinement systems, relies on good understanding and control of symmetry-breaking magnetic instabilities and field errors.

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[1] A. B. Rechester and T. H. Stix, Phys. Rev. Lett. **36**, 587 (1976).
[2] J. D. Callen, Phys. Rev. Lett. **39**, 1540 (1977).
[3] B.V. Chirikov, Phys. Rep. **52**, 264 (1979).
[4] J. B. Taylor, Rev. Mod. Phys. **58**, 741 (1986).
[5] A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
[6] F. D'Angelo and R. Paccagnella, Phys. Plasmas **3**, 2353 (1996).
[7] B. E. Chapman *et al.*, Phys. Plasmas **9**, 2061 (2002).
[8] R. Dexter, D. Kerst, T. Lovell, S. Prager, and J. Sprott, Fusion Technol. **19**, 131 (1991).
[9] R. Bartiromo, V. Antoni, T. Bolzonella, A. Buffa, L. Marrelli, P. Martin, E. Martines, S. Martini, and R. Pasqualotto, Phys. Plasmas **6**, 1830 (1999).
[10] T. M. Biewer, D. J. DenHartog, D. J. Holly, and M. R. Stoneking, Rev. Sci. Instrum. **74**, 1649 (2003).
[11] D. Brower, W. Ding, S. Terry, J. Anderson, T. Biewer, D. Craig, C. Forest, S. Prager, and J. Sarff, Phys. Rev. Lett. **88**, 185005 (2002).
[12] D. Craig, D. J. DenHartog, G. Fiksel, V. Davydenko, and A. A. Ivanov, Rev. Sci. Instrum. **72**, 1008 (2001).
[13] C. B. Forest, K. Kupfer, T. Luce, P. Politzer, L. Lao, M. Wade, D. Whyte, and D. Wroblewski, Phys. Rev. Lett. **73**, 2444 (1994).
[14] R. G. Watt and R. A. Nebel, Phys. Fluids **26**, 1168 (1983).
[15] G. A. Wurden, Phys. Fluids **27**, 551 (1984).
[16] J. K. Anderson, C. B. Forest, T. M. Biewer, J. S. Sarff, and J. C. Wright, Nucl. Fusion (to be published).
[17] J. T. Chapman, Ph.D. thesis, University of Wisconsin–Madison, 1998.
[18] T. M. Biewer, Ph.D. thesis, University of Wisconsin–Madison, 2002.
[19] G. Fiksel, S. C. Prager, W. Shen, and M. Stoneking, Phys. Rev. Lett. **72**, 1028 (1994).
[20] D. D. Schnack, D. C. Barnes, Z. Mikic, D. S. Harned, and E. J. Caramana, J. Comput. Phys. **70**, 330 (1987).
[21] G. Fiksel, B. Hudson, and Y. Tsidulko, under development at UW–Madison (2002).
[22] A. H. Boozer and R. B. White, Phys. Rev. Lett. **49**, 786 (1982).
[23] P.W. Terry *et al.*, Phys. Plasmas **3**, 1999 (1996).