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Influence of energetic ions on tearing modes in a reversed field pinch

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Abstract

Tearing mode stability analysis in the presence of circulating energetic ions (CEI) is studied in the reversed field pinch (RFP) magnetic configuration. In contrast to the minimal effect of precessional drift of CEI on tearing modes in tokamaks, the effect of precessional drift of CEI on tearing modes is important in the RFP. It is found that the effects of CEI on tearing modes in RFP depend on their toroidal circulating direction, and have a strong relation to the pressure gradient of CEI. For co-CEI, tearing modes can be stabilized if the pressure gradient of energetic ions is sufficiently large, which is qualitatively consistent with experimental results in the Madison Symmetric Torus.

Keywords: energetic ions, tearing modes, reversed field pinch

(Some figures may appear in colour only in the online journal)

1. Introduction

Tearing modes represent a class of magnetohydrodynamic (MHD) instabilities that can negatively impact the performance of magnetically confined plasmas. These instabilities alter the magnetic field topology, lead to the formation of magnetic islands, increase the local radial transport and degrade plasma confinement. This is particularly important in reversed field pinch (RFP) devices [1]. In the RFP, tearing modes are responsible for the transformation between poloidal and toroidal magnetic flux, maintaining the reversal of toroidal magnetic field. They also play an important role in the anomalous transport of particles and energy in the core region of RFP plasmas. Hence, understanding the physics of tearing modes in the RFP is important.

Energetic particles are inevitably produced in burning plasmas and their confinement is important for self-sustained heating and maintaining the integrity of the first wall in fusion reactors. Significant effort has been devoted to investigating the interaction of energetic particles and MHD instabilities [2, 3]. However, the study on the interaction between tearing modes and energetic particles has only recently begun. Both theory and experiment have shown that the interaction between tearing modes and energetic particles can be effective [4–10]. Some experiments [4–6] have shown that neoclassical tearing modes (NTMs) can enhance the loss of energetic particles, and energetic particles can stabilize NTMs in turn. Recently, the effects of energetic ions on linear tearing modes have been studied by theory and simulation [8–10]. It was found that energetic ions affect the stability of tearing modes mainly through interaction with the ideal outer region [9, 10], and the effects of circulating energetic ions (CEI) on tearing modes depend on the magnitude of energetic ion pressure and their toroidal circulating direction. Previous studies on the interaction between tearing modes and energetic particles focused on the tokamak geometry. However, energetic particles are also abundant in RFPs with neutral beam injection. Recent experiment in Madison Symmetric Torus (MST) RFP showed that the amplitude of tearing modes is decreased with co-neutral beam injection [11], namely tearing mode is stabilized by co-CEI. Due to the unique RFP characteristics, such as low q(q is the safety factor), the physics of the effect of energetic ions on tearing modes in RFP maybe different from that in tokamak. Energetic ions in the experiment in MST with tangential neutral beam injection are primarily circulating and thus we will investigate the effect of CEI on tearing modes in this article.

In section 2, a heuristic picture is presented. In section 3, the ideal MHD equation in the outer region including the effects of energetic ions for tearing modes is derived, and the stability criterion is calculated based on the parameters in MST. Finally, the conclusion is given in section 4.

2. Physical picture

In this section, to understand the underlying physics, a heuristic interpretation is described. The physics is similar to that provided for the tokamak [9], except for the inclusion of energetic ion precession drift, due to low $q(q < 1)$ and high $n(n \gg 1)$ in the RFP. Due to the drift orbit of energetic ions being much larger than the resistive layer width of the linear tearing mode, the effect of energetic ions on tearing modes is mainly through the interaction between energetic ions and the ideal outer region of tearing modes. Namely, energetic ions can change the value of the stability criterion Δ' (the jump discontinuity in the logarithmic derivative of the perturbed magnetic poloidal flux at the resonance surface), which determines the stability of tearing modes. Δ' is obtained from the ideal MHD equation in the outer region of tearing mode. In the outer region of tearing mode without energetic ions, $\mathbf{b}\cdot\nabla J_\parallel = 0$ (where $\mathbf{b} = \mathbf{B}/B$ is the direction of magnetic field, $J_\parallel$ is the parallel current) in the low-β plasmas, where the diamagnetic current due to the pressure of background plasma is not considered. Then, $J_\parallel = J_\parallel(\Omega)$ is a flux function, where $\Omega = Q(\psi) + \delta\psi$, $\mathrm{d}Q/\mathrm{d}\psi = 1 - q/q_s$ (ψ and $\delta\psi$ is the equilibrium and perturbed poloidal function, respectively. $q_s = m/n$ is the value of q on resonant surface, m and n are the poloidal and toroidal mode number, respectively), satisfying $\mathbf{b}\cdot\nabla\Omega = 0$. Then the perturbed current can be obtained as $\delta J_\parallel = -(\mathrm{d}J_{\parallel 0}/\mathrm{d}r)(1 - q/q_s)^{-1}\delta\psi$, which is the ideal MHD equation in outer region of tearing mode. With energetic ions, the equation becomes $\mathbf{b}\cdot\nabla J_\parallel + \nabla\cdot\mathbf{J}_{\perp h} = 0$ ($\mathbf{J}_{\perp h}$ is perpendicular current of energetic ions), $J_\parallel$ is no longer a flux function. One can separate $J_\parallel = J_{\parallel,c} + J_{\parallel,h}$, satisfying $\mathbf{b}\cdot\nabla J_{\parallel,c} = 0$, $\mathbf{b}\cdot\nabla J_{\parallel,h} + \nabla\cdot\mathbf{J}_{\perp,h} = 0$, where $J_{\parallel,c}$, $J_{\parallel,h}$ are the parallel current of background plasma and energetic ions, respectively. The behavior of background plasma is not changed, so the parallel current perturbation of background plasma $\delta J_{\parallel,c} = -(\mathrm{d}J_{\parallel,c0}/\mathrm{d}r)(1 - q/q_s)^{-1}\delta\psi$, where $J_{\parallel,c0}$ is the background plasma equilibrium current. For energetic ions, $f_h = f_h(Q_d + a^2\delta\psi)$ can be obtained, where f_h is the distribution of energetic ions, $\mathrm{d}Q_d/\mathrm{d}\psi_d = 1 - q/q_s - \omega_d/(q_s\omega_t)$, $a^2 \leqslant 1$ denotes the orbit average effect, ω_d, ω_t are the precessional frequency and transit frequency, respectively. Thus, one can obtain the perturbation current of energetic ions $\delta J_{\parallel,h} = -\langle Zev_\parallel(\mathrm{d}f_h/\mathrm{d}r)a^2\delta\psi/(1 - q/q_s - \omega_d/(q_s\omega_t)\rangle$, where $\langle\ldots\rangle$ denotes the integration over velocity space, Z is the particle charge number. Hence, the total perturbation of plasma current

$$\begin{aligned}\delta J_\parallel &= \delta J_{\parallel,c} + \delta J_{\parallel,h}\\ &\simeq -\frac{\mathrm{d}J_{\parallel,0}}{\mathrm{d}r}\frac{\delta\psi}{1-q/q_s} + (1-a^2)\frac{\mathrm{d}J_{\parallel,h0}}{\mathrm{d}r}\frac{\delta\psi}{1-q/q_s}\\ &\quad -\left\langle Zev_\parallel\frac{\mathrm{d}f_{h0}}{\mathrm{d}r}\frac{\omega_d/(q_s\omega_t)}{\left(1-q/q_s-\omega_d/(q_s\omega_t)\right)\left(1-q/q_s\right)}a^2\delta\psi\right\rangle,\end{aligned} \tag{1}$$

where the first term on the right hand of equation (1) results from the MHD effect, the second term on the right hand of equation (1) denotes the orbit average effect on the perturbation current of energetic ions. It tends to reduce the perturbation current of energetic ions. Thus, given the equilibrium currrent, the orbit average effect of energetic ions tends to stabilize tearing modes for co-CEI, while destabilize tearing modes for counter-CEI. This effect is well described in the [9, 10]. The last term on the right hand of equation (1) represents the perturbation current due to the precessional drift of particles. If the precessional frequency is very small, satisfying $|\omega_d/((q_s - q)\omega_t)| \ll 1$ near the resonance surface, it reduces to the normal diamagnetic current. This is always satisfied in tokamak or for thermal particles. On the other hand, if the precessional frequency is large, $|\omega_d/((q_s - q)\omega_t)| \gg 1$ near the resonance surface, it means that CEI have a net drift after the bounce average, which can produce parallel perturbed current. It also depends on the toroidal circulating direction of CEI, which is similar to the orbit average effect. In tokamaks, the effect of precessional drift of CEI is neglected due to $|\omega_d/((q_s - q)\omega_t)| \ll 1$, except very close to resonant surface, as done in [9]. However, in the RFP, $|\omega_d/((q_s - q)\omega_t)| \sim O(1)$ in some region near the resonant surface due to $q < 1$ and $n > 1$, which was pointed out in [12]. Thus, the effect of precessional drift of tearing modes is important in the RFP.

3. Stability criterion of tearing modes

In this section, based on the above physical picture, the detailed calculation will be shown. In the outer region, the linearized equations are

$$-\nabla\delta p_c - \nabla\cdot\delta\mathbf{p_h} + \frac{1}{c}\delta\mathbf{J}\times\mathbf{B} + \frac{1}{c}\mathbf{J}\times\delta\mathbf{B} = 0, \tag{2}$$

$$\mathbf{B}\cdot\nabla\delta p_c + \delta\mathbf{B}\cdot\nabla p_c = 0, \tag{3}$$

$$\delta\mathbf{p_h} = \delta p_{\perp,h}\mathbf{I} + \left(\delta p_{\parallel,h} - \delta p_{\perp,h}\right)\mathbf{bb}, \tag{4}$$

where δp_c is the perturbed pressure of background plasma, which is assumed to be isotropic and incompressible. $\delta\mathbf{p}_h$ is the pressure tensor of energetic ions, satisfying the Chew–Goldberger–Low [13] pressure tensor form. $\mathbf{J}$, $\delta\mathbf{J}$ are the equilibrium and perturbed currents, respectively. $\mathbf{B}$, $\delta\mathbf{B}$ are the equilibrium and perturbed magnetic field, respectively. $\mathbf{I}$ is the unit tensor, $\mathbf{b} = \mathbf{B}/B$ denotes the direction of equilibrium magnetic field. Making the operation $\nabla\cdot[\mathbf{B}/B^2\times(\ldots)]$ on equation (2), one can obtain

$$\mathbf{B}\cdot\nabla\frac{\delta J_\parallel}{B}+\delta\mathbf{B}\cdot\nabla\sigma-\mathbf{J}\cdot\nabla\frac{\delta B_\parallel}{B}+c\nabla\cdot\left(\frac{\mathbf{B}\times\nabla\delta p_c}{B^2}+\frac{\mathbf{B}\times\nabla\cdot\delta\mathbf{p}_h}{B^2}\right)=0, \tag{5}$$

where $\sigma = J_\parallel/B$. The equilibrium magnetic field can be expressed as

$$\mathbf{B}=I\nabla\zeta+\nabla\zeta\times\nabla\psi, \tag{6}$$

where the toroidal geometry is assumed to be axisymmetric. The symmetric coordinate, (ψ, θ, ζ) are chosen as flux coordinates. θ and ζ are the poloidal and toroidal angle, respectively. For tearing modes, the perturbed magnetic field can be written as

$$\delta\mathbf{B}=\nabla\times\left(\delta A_\parallel\mathbf{b}\right). \tag{7}$$

For simplicity, single helicity is considered, namely the form of perturbation can be expressed as $\delta A_\parallel=\delta\widehat{A}_\parallel(r)\exp(\mathrm{i}m\theta-\mathrm{i}n\zeta-\mathrm{i}\omega t)$. Then, making the integral $\oint\mathrm{d}\theta\, J\exp(-\mathrm{i}m\theta+\mathrm{i}n\zeta)$ on equation (5), one can obtain

$$\begin{aligned}(m-nq)\frac{\delta\widehat{J}_\parallel}{B}&+\left(mI+n\frac{r}{R_0}\frac{\partial\psi}{\partial r}\right)\frac{\mathrm{d}\sigma}{\mathrm{d}\psi}\frac{\delta\widehat{A}_\parallel}{B}\\&-\frac{qR_0^2}{I}\left(\frac{m}{r}J_\theta-\frac{n}{R_0}J_\zeta\right)\frac{\delta\widehat{B}_\parallel}{B}\\+c\oint J\nabla\cdot\left(\frac{\mathbf{B}\times\nabla\delta\widehat{p}_c}{B^2}\right)\mathrm{d}\theta&+c\oint J\nabla\cdot\left(\frac{\mathbf{B}\times\nabla\cdot\delta\mathbf{p}_h}{B^2}\right)\exp\\(-\mathrm{i}m\theta+\mathrm{i}n\zeta)\,\mathrm{d}\theta&=0,\end{aligned} \tag{8}$$

where $J = (\nabla\psi \times \nabla\theta\cdot\nabla\zeta)^{-1}$ is the Jacobian. Then, following the procedure of [14], equation (8) can be derived as

$$L\delta\psi+\frac{4\pi\mathrm{i}rB_\theta B^2}{G}\delta K=0, \tag{9}$$

$$\begin{aligned}L=F&\left\{\frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{r}{m^2+n^2\epsilon^2}\frac{\mathrm{d}}{\mathrm{d}r}\right]\right.\\&\left.-\left[\frac{1}{r}-\frac{r\sigma^2}{m^2+n^2\epsilon^2}+\frac{2n\epsilon(m\sigma+n\epsilon g)}{(m^2+n^2\epsilon^2)^2}\right]\right\}\\&-\frac{1}{m^2+n^2\epsilon^2}\left[rG\frac{\mathrm{d}\sigma}{\mathrm{d}r}-r\frac{\mathrm{d}(gF)}{\mathrm{d}r}-rG\sigma g+\frac{2mn\epsilon Gg}{m^2+n^2\epsilon^2}\right.\\&\left.+\frac{2n^2\epsilon^2gG^2}{F(m^2+n^2\epsilon^2)}\right],\end{aligned} \tag{10}$$

$$\delta K=\oint J\nabla\cdot\left(\frac{\mathbf{B}_0\times\nabla\cdot\delta\mathbf{p}_h}{B_0^2}\right)\exp\left(-\mathrm{i}m\theta+\mathrm{i}n\zeta\right)\mathrm{d}\theta, \tag{11}$$

where $F = m B_\theta - n\,\epsilon\, B_\zeta$, $G = m B_\zeta + n\,\epsilon\, B_\theta$, $\delta\,\psi = G\,\delta A_\parallel/B$, $g = (\mathrm{d}\, p_c/\mathrm{d}\, r)/B^2$, $\epsilon = r/R_0$ is the aspect ratio. Without energetic ions, equation (9) reduces to equation (17) in [14]. Next, to obtain the expression of $\delta\,\mathbf{p}_h$, it is necessary to solve the perturbed distribution of energetic ions. The linearized drift kinetic equation [15] is

$$\delta f_h=\frac{\partial F_h}{\partial P_\zeta}\frac{Ze}{\omega}\delta A\frac{1}{JB_\theta^2}\left(mI+n\frac{r}{R}\frac{\partial\psi}{\partial r}\right)+Ze\left(\delta\phi-\delta A\right)\frac{\partial F_h}{\partial E}+\delta h, \tag{12}$$

$$\frac{\mathrm{d}\delta h}{\mathrm{d}t}=Q\left(\delta A-\delta\phi-\mathbf{v}_d\cdot\nabla\,\delta A\frac{1}{i\omega}\right), \tag{13}$$

where $P_\zeta = -\,Z\,e\,\psi/c + v_\parallel I/B_0$ is the toroidal momentum, $E = v^2/2$ is the kinetic energy, $Q = (-\mathrm{i})\,Z\,e\,(\omega - n\,\omega_*)(\partial F_h/\partial E)$, $\omega_* = (\partial F_h/\partial P_\zeta)(\partial F_h/\partial E)^{-1}$, $\mathbf{v}_d$ is the magnetic drift velocity of particles, $\delta\,\phi$ is the perturbed electrostatic potential, δA satisfies $\delta A_\parallel = c\,\mathbf{b}\cdot\nabla\delta A/(\mathrm{i}\,\omega)$. For ideal MHD, the perturbed electric field $\delta E_\parallel = 0$, so $\delta A = \delta\,\phi$. To solve δh, it is convenient to introduce a transform coordinate (r_d, θ_d, α), satisfying $r_d = r - q\,\rho_h\cos\theta$, $\theta_d \simeq \omega_t\, t$, $\alpha = q\,\theta - \zeta \simeq \omega_d\, t$ for well circulating particles, where $\omega_t = v_\parallel/(J\,B)$, $\omega_d = -\,v_d\, B_\theta/(r\,B_0)$, $v_d = v_\parallel^2/(\Omega_c R)$, $\rho_h = v_\parallel/\Omega_c$, $q\,\rho_h$ is the orbit width of particles, Ω_c is the gyro-frequency, B_θ is the equilibrium poloidal magnetic field. In this coordinate, equation (13) can be rewritten as

$$\begin{aligned}\frac{\mathrm{d}\delta h_d}{\mathrm{d}t}=&\left[k_\theta\frac{2RB_\theta^2}{rB^2}+\left[\mathrm{e}^{\mathrm{i}\theta}\left(k_\theta-\mathrm{i}k_r\frac{B_\zeta}{B}\right)+\mathrm{e}^{-\mathrm{i}\theta}\left(k_\theta+\mathrm{i}k_r\frac{B_\zeta}{B}\right)\right]\right]\\&\frac{Qv_d}{2\omega}\delta\widehat{A}_d\\&\times\sum_l i^l\mathrm{e}^{\mathrm{i}l\theta}J_l(\lambda)\,\mathrm{e}^{\mathrm{i}(m-nq)\theta+\mathrm{i}n\omega_d t-\mathrm{i}\omega t},\end{aligned} \tag{14}$$

where the subscript d denotes the variables defined in the r_d, $k_\theta = (m\,B_\zeta + n\,r\,B_\theta/R)/(r\,B)$, $k_r = -\,\mathrm{i}\,(\partial\delta A/\partial r)/\delta A$, $J_l(\lambda)$ is the Bessel function, $\lambda = k_r\,q\,\rho_h$. Here, the terms with $O(q\,\rho_h/a)$ are neglected, while the effect of $O(q\,\rho_h/\Delta_m)$ (Δ_m is the mode width) is kept. Then, integrating equation (14) over t, one can obtain

$$\begin{aligned}\delta h_d=&\frac{Qv_d}{\omega}\delta\widehat{A}_d\mathrm{e}^{\mathrm{i}(m-nq)\theta+\mathrm{i}n\omega_d t-\mathrm{i}\omega t}\\&\left\{k_\theta\frac{RB_\theta^2}{rB^2}\sum_l\frac{i^{l-1}\mathrm{e}^{\mathrm{i}l\theta}J_l}{(l+m-nq)\,\omega_t+n\omega_d-\omega}\right.\\&\left.+\sum_l\frac{i^l\mathrm{e}^{\mathrm{i}l\theta}}{(l+m-nq)\,\omega_t+n\omega_d-\omega}\left(\mathrm{i}k_r\frac{lJ_l}{\lambda}-k_\theta\frac{\mathrm{d}J_l}{\mathrm{d}\lambda}\right)\right\}.\end{aligned} \tag{15}$$

Making the reversed transform to the coordinate (r, θ, ζ), equation (15) can be derived as

$$\begin{aligned}\delta h=&\frac{Qv_d}{\omega}k_\theta\frac{RB_\theta^2}{rB^2}\sum_{l,l'}\frac{i^{l+l'1}\mathrm{e}^{\mathrm{i}(l+l')\theta}J_lJ_{-l'}}{(l+m-nq)\,\omega_t+n\omega_d-\omega}\delta A\\&+\frac{Qv_d}{\omega}\sum_{l,l'}\frac{i^{l+l'}\mathrm{e}^{\mathrm{i}(l+l')\theta}}{(l+m-nq)\,\omega_t+n\omega_d-\omega}\\&\left(\mathrm{i}k_r\frac{lJ_l}{\lambda}-k_\theta\frac{\mathrm{d}J_l}{\mathrm{d}\lambda}\right)J_{-l'}\delta A.\end{aligned} \tag{16}$$

Now, the perturbed distribution of CEI is obtained. Based on the expressions (12) and (16), and using the definition $\delta p_{\parallel h} = \int\mathrm{d}^3 v v_\parallel^2\delta f_h$, equation (11) can be derived as

$$\delta K=\int\mathrm{d}^3vv_\parallel^2\left(\delta\widehat{K}_f+\delta\widehat{K}_k\right), \tag{17}$$

$$\delta\widehat{K}_f=\mathrm{i}k_\theta^2\frac{B_\theta}{B^4}\frac{\partial F_h}{\partial r}\frac{\delta\widehat{A}}{\omega}, \tag{18}$$

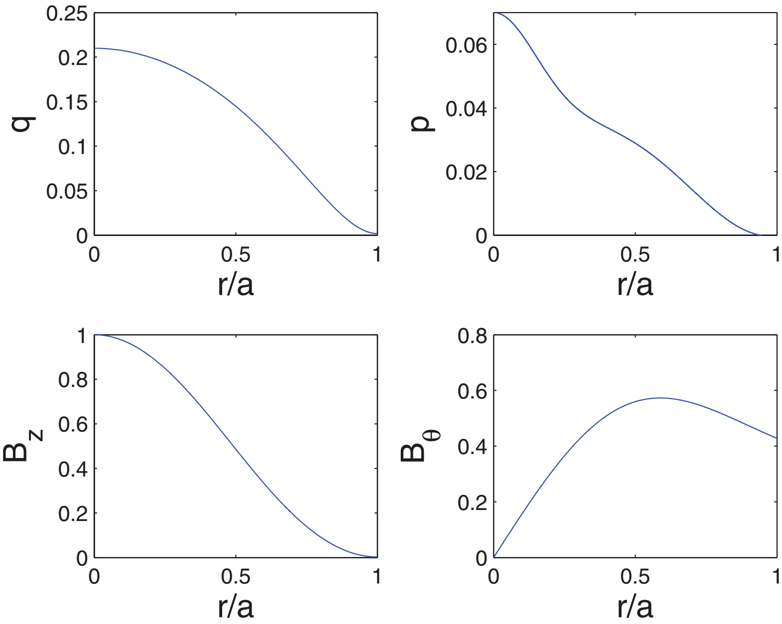

Figure 1. The equilibrium profiles with $\beta\,(0) = 0.07$, $\beta_{\text{frac}} = 0.5$ ($\beta_{\text{frac}} = \beta_h(0)/\beta\,(0)$).

$$\delta\widehat{K}_k = -\frac{Q}{\omega}\frac{r}{R_0 B B_\theta}\delta\widehat{A}\left[\frac{v_d}{(m-nq)\,\omega_t + n\omega_d - \omega}\left(k_\theta\frac{R_0B_\theta^2}{rB^2}\right)^2 J_0^2(\lambda) - 2k_\theta\frac{R_0B_\theta^2}{rB^2}\left(1 - J_0^2(\lambda)\right) - \frac{1}{q\rho_h}\left(m - nq + \frac{n\omega_d - \omega}{\omega_t}\right)\frac{B_\zeta}{B}\left(1 - J_0^2(\lambda)\right)\right], \tag{19}$$

where $\delta\widehat{K}_f$, $\delta\widehat{K}_k$ represent the adiabatic and kinetic contributions of CEI, respectively. Here, the high harmonic $|l| \geqslant 1$ terms are neglected, and large aspect ratio is assumed.

To proceed, it is assumed that the equilibrium distribution $F = \sum_\nu F_{h,0}^\nu$ satisfies slowing-down model for a population of formed CEI by a purely co-CEI ($\nu = +$) component or a purely counter-CEI ($\nu =-$) component, as

$$F_{h0}^\nu = \frac{p_h^\nu}{2^{3/2}\pi m_h B_0 E_m^\nu}E^{-3/2}\delta\,(\lambda)\,H\,(E - E_m)\,, \tag{20}$$

where $\lambda = \mu/E$ denotes the pitch angle, E_m is the maximal energy, $H(x)$ is the step function. As done in [9], it is assumed that $J_0^2 \simeq \overline{\alpha}$. $\overline{\alpha}<1$ is a constant coefficient. Then equation (17) can be reduced to

$$\delta K = -\sum_\nu \frac{i}{B^2}\frac{\mathrm{d}p_h^\nu}{\mathrm{d}r}\delta A_\| \left(\frac{B_\theta G^2}{rB^3F}\overline{\alpha}\,(CI-1) + \frac{2rB}{RB_\theta^2}\frac{n}{\rho_{hm}^\nu}(1-\overline{\alpha})\right), \tag{21}$$

where ω is neglected for tearing modes, $CI = 1 + 2\,\chi + 2\,\chi^2 \ln[(1 - \chi)/(-\chi)]$, $\chi = (m - nq)\,R_0\,/\left(n\rho_{hm}^\nu\right)$, $\rho_{hm}^\nu = \nu\sqrt{2E_m}\,/\,\Omega_c$, $\omega_d\,/\,\omega_t = \rho_h^\nu\,/\,R_0$ is employed. The physics of δK is similar to that in equation (1). The first term on the right hand of equation (21) represents the effect of precessional drift effect, and the last term denotes the orbit average effect on the perturbed current of energetic ions. If $|\chi| \gg 1$, the precessional frequency is very small, the first term in δK reduces to the usual diamagnetic drift current. For tokamaks, $|\chi| \gg 1$ is always satisfied, except very narrow region close to the resonant surface, so the effect of precessional drift can be neglected, as done in [9]. For RFP, $|\chi| \sim O\,(1)$ for energetic ions in some region near the resonant surface, due to the $q < 1$ and $n > 1$ in the RFP, which is pointed out in [12]. Thus, the effect of precessional drift on tearing modes can be important in the RFP. Substituting equation (21) into equation (9), one can obtain

$$\widehat{L}\delta\psi + \delta\psi\sum_\nu \frac{1}{\widehat{B}^2}\frac{\mathrm{d}\beta_h^\nu}{\mathrm{d}r}\left[\frac{\widehat{B}_\theta^2}{2\widehat{F}}\overline{\alpha}\,(CI-1) + \frac{r\epsilon\widehat{B}^4}{\widehat{B}_\theta\widehat{G}^2}\frac{n}{\widehat{\rho}_{hm}^\nu}(1-\widehat{\alpha})\right] = 0, \tag{22}$$

where the parameters are normalized as follows: $r \to a\ r$, $B \to B\,(0)\,\widehat{B}$, $\beta_h^\nu = 8\pi p_h^\nu\,/\,B^2\,(0)$. Here, the imaginary part of δK is not considered, since it induces a real frequency, which is not important for tearing mode in outer region as the inertia term. From equation (22), it can be known that co-CEI plays a stable role on tearing modes, while counter-CEI plays an unstable role on tearing modes with $\mathrm{d}\beta_h/\mathrm{d}\,r < 0$. This is consistent with the physical picture in section 2. If the precessional effect is neglected, equation (22) is equivalent to that in [9] in tokamak geometry. Thus, based on equation (22), the stability criterion Δ' for tearing mode can be calculated.

For typical RFP like MST, the main parameters $B_0 = 0.3$ T, $a = 0.5\ m$, $R_0 = 1.5$ m. The profiles of equilibrium current and pressure are $\sigma = \sigma_0\,(1 - r^\mu)$, $\sigma_0 = 2\,a/(q\,(0)\,R_0)$, $\beta_c = \beta_{c0}\,(1.04 + 0.52\,r^2 - 6.78\,r^4 + 7.68\,r^6 - 2.47\,r^8)$ ($\beta_c = 8\,\pi\,p_c/B^2(0)$, p_c is the background pressure), $\beta_h = \beta_{h0}\,(3.5*\exp(-r^2/0.044))$, where β_{c0}, β_{h0} are used to modify the weight of β_c, β_h, respectively. Plasma beta at axis $\beta\,(0) = \beta_c(0) + \beta_h(0)$. Considering $m = 1$, $n = 5$ mode, $\beta\,(0) = 7\%$ and 25 keV ions formed by CEI, the equilibrium profiles can be seen in figure 1. Based on

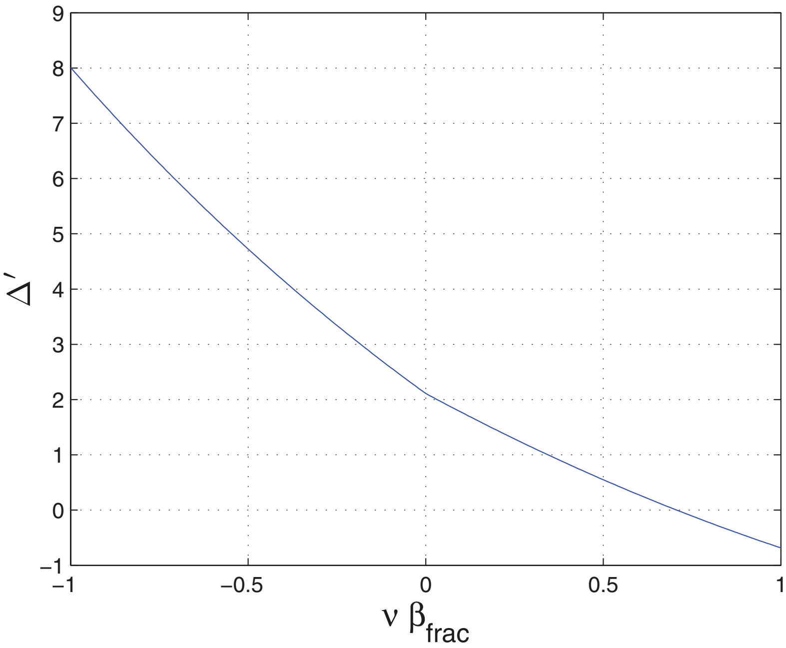

Figure 2. The stability criterion of tearing modes against the fraction of CEI $\nu\ \beta_{\mathrm{frac}}$, where $\nu = -1$ and $\nu = 1$ denote counter-CEI and co-CEI, respectively.

equation (22), the stability criterion Δ' for tearing modes can be calculated, which can be seen in figure 2. It can be found that the effect of CEI on tearing modes is dramatic, and depends on the toroidal direction of CEI. For counter-CEI, as the fraction β_{frac} of counter-CEI increases, Δ' increases. For co-CEI, as β_{frac} increases, Δ' decreases, and becomes negative if β_{frac} is sufficiently large. Thus, counter-CEI tends to destabilize tearing modes, while co-CEI stabilizes tearing modes. This is consistent with the physics picture pointed out in section 2, and it is also qualitatively consistent with the recent experimental results in MST [11], where it was shown that the amplitude of tearing modes was reduced when co-NBI was injected.

4. Conclusion

In this article, the effects of CEI on tearing modes in RFP have been studied. In contrast to little precessional drift effect of CEI on tearing modes in tokamak, the precessional effect of CEI on tearing modes is important in RFP, since $\omega_d/((m - n\ q)\ \omega_t) \sim O\ (1)$ near the resonant surface due to low q and high n in RFP. It is found that the effects of CEI on tearing modes have a great relation to the magnitude of β_h, and depend on the toroidal circulating direction of CEI. For co-CEI, energetic ions can reduce the value of stability criterion Δ' of tearing modes, and stabilize tearing modes, even suppress tearing modes. For counter-CEI, energetic ions can increase the value of stability criterion, and destabilize tearing modes. For the balanced tangential neutral beam injection, energetic ions have no or little effect on tearing modes. The result is qualitatively consistent with the recent experimental result in MST, where it was shown that tearing modes can be stabilized by CEI. Thus, our analysis suggests that tearing modes can be suppressed by co-CEI in RFP with appropriate β_h.

Here, it is necessary to pointed out that the effects of energetic ions on Δ' is only considered, which is valid for orbit width of energetic ions larger than island width. If the orbit width of energetic ions and island width are comparable, the effects of energetic ions in the resistive inner region needs to be considered.

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References

[1] Robinson D C 1978 *Nucl. Fusion* **18** 939
[2] Chen L, White R B and Rosenbluth M N 1984 *Phys. Rev. Lett.* **52** 1122
[3] Porcelli F 1991 *Plasma Phys. Control. Fusion* **33** 1601
[4] Poli E *et al* 2008 *Phys. Plasmas* **15** 032501
[5] García-Muñoz M *et al* 2009 *Nucl. Fusion* **49** 085004
[6] Buttery R J *et al* 2008 *Phys. Plasmas* **15** 056115
[7] Hegna C C and Bhattacharjee A 1989 *Phys. Rev. Lett.* **63** 2056
[8] Takahashi R, Brennan D P and Kim C C 2009 *Phys. Rev. Lett.* **102** 135001
[9] Cai H, Wang S, Xu Y, Cao J and Li D 2011 *Phys. Rev. Lett.* **106** 075002
[10] Cai H and Fu G 2012 *Phys. Plasmas* **19** 072506
[11] Anderson J K *et al* 2013 *Phys. Plasmas* **20** 056102
[12] Cai H, Cao J and Li D 2014 *Nucl. Fusion* **54** 032001
[13] Chew G F, Goldberger M L and Low F F 1956 *Proc. R. Soc.* **236** 112
[14] Zonca P and Sattin F 2003 *Plasma Phys. Control. Fusion* **45** 1
[15] Porcelli F, Stankiewicz R, Kerner W and Berk H L 1994 *Phys. Plasmas* **1** 470