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Influence of energetic ions on tearing modes in a reversed field pinch

Huishan Cai¹, Liang Lin², Weixing Ding², J K Anderson¹ and D L Brower²

¹ CAS Key Laboratory of Geospace Environment, Department of Modern Physics, University of Science and Technology of China, Hefei 230026, People’s Republic of China
² Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095-7099, USA
³ University of Wisconsin, Madison, WI 53706, USA

E-mail: hscai@mail.ustc.edu.cn

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Abstract

Tearing mode stability analysis in the presence of circulating energetic ions (CEI) is studied in the reversed field pinch (RFP) magnetic configuration. In contrast to the minimal effect of precessional drift of CEI on tearing modes in tokamaks, the effect of precessional drift of CEI on tearing modes is important in the RFP. It is found that the effects of CEI on tearing modes in RFP depend on their toroidal circulating direction, and have a strong relation to the pressure gradient of CEI. For co-CEI, tearing modes can be stabilized if the pressure gradient of energetic ions is sufficiently large, which is qualitatively consistent with experimental results in the Madison Symmetric Torus.

Keywords: energetic ions, tearing modes, reversed field pinch

(Some figures may appear in colour only in the online journal)

1. Introduction

Tearing modes represent a class of magnetohydrodynamic (MHD) instabilities that can negatively impact the performance of magnetically confined plasmas. These instabilities alter the magnetic field topology, lead to the formation of magnetic islands, increase the local radial transport and degrade plasma confinement. This is particularly important in reversed field pinch (RFP) devices [1]. In the RFP, tearing modes are responsible for the transformation between poloidal and toroidal magnetic flux, maintaining the reversal of toroidal magnetic field. They also play an important role in the anomalous transport of particles and energy in the core region of RFP plasmas. Hence, understanding the physics of tearing modes in the RFP is important.

Energetic particles are inevitably produced in burning plasmas and their confinement is important for self-sustained heating and maintaining the integrity of the first wall in fusion reactors. Significant effort has been devoted to investigating the interaction of energetic particles and MHD instabilities [2, 3]. However, the study on the interaction between tearing modes and energetic particles has only recently begun. Both theory and experiment have shown that the interaction between tearing modes and energetic particles can be effective [4–10]. Some experiments [4–6] have shown that neoclassical tearing modes (NTMs) can enhance the loss of energetic particles, and energetic particles can stabilize NTMs in turn. Recently, the effects of energetic ions on linear tearing modes have been studied by theory and simulation [8–10]. It was found that energetic ions affect the stability of tearing modes mainly through interaction with the ideal outer region [9, 10], and the effects of circulating energetic ions (CEI) on tearing modes depend on the magnitude of energetic ion pressure and their toroidal circulating direction. Previous studies on the interaction between tearing modes and energetic particles focused on the tokamak geometry. However, energetic particles are also abundant in RFPs with neutral beam injection. Recent experiment in Madison Symmetric Torus (MST) RFP showed that the amplitude of tearing modes is decreased with co-neutral beam injection [11], namely tearing mode is stabilized by co-CEI. Due to the unique RFP characteristics, such as low $q$ ($q$ is the safety factor), the physics of the effect of energetic ions on tearing modes in RFP maybe different from that in

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tokamak. Energetic ions in the experiment in MST with tangential neutral beam injection are primarily circulating and thus we will investigate the effect of CEI on tearing modes in this article.

In section 2, a heuristic picture is presented. In section 3, the ideal MHD equation in the outer region including the effects of energetic ions for tearing modes is derived, and the stability criterion is calculated based on the parameters in MST. Finally, the conclusion is given in section 4.

2. Physical picture

In this section, to understand the underlying physics, a heuristic interpretation is described. The physics is similar to that provided for the tokamak [9], except for the inclusion of energetic ion precession drift, due to low $q(q < 1)$ and high $n(n \gg 1)$ in the RFP. Due to the drift orbit of energetic ions being much larger than the resistive layer width of the linear tearing mode, the effect of energetic ions on tearing modes is mainly through the interaction between energetic ions and the ideal outer region of tearing modes. Namely, energetic ions can change the value of the stability criterion $\Delta'$ (the jump discontinuity in the logarithmic derivative of the perturbed magnetic poloidal flux at the resonance surface), which determines the stability of tearing modes. $\Delta'$ is obtained from the ideal MHD equation in the outer region of tearing mode. In the outer region of tearing mode without energetic ions, $b \cdot V J_{\parallel} = 0$ (where $b \equiv B/B$ is the direction of magnetic field, $J_{\parallel}$ is the parallel current) in the low-$\beta$ plasmas, where the diamagnetic current due to the pressure of background plasma is not considered. Then, $J_{\parallel} = J_{\parallel}(\Omega)$ is a flux function, where $\Omega = Q(q_{s} + \delta \psi), dQ/d\psi = 1 - q/q_{s} (\psi$ and $\delta \psi$ is the equilibrium and perturbed poloidal field, respectively, $q_{s} = mn$ is the value of $q$ on resonant surface, $m$ and $n$ are the poloidal and toroidal mode number, respectively), satisfying $b \cdot \nabla \Omega = 0$. Then the perturbed current can be obtained as $\delta J_{\parallel} = -(dJ_{\parallel}/d\psi)\cdot d(1 - q/q_{s})^{-1}\delta \psi$, which is the ideal MHD equation in outer region of tearing mode. With energetic ions, the equation becomes $b \cdot V J_{\parallel} + \nabla \cdot J_{\parallel,h} = 0 (J_{\parallel,h}$ is perpendicular current of energetic ions), $J_{\parallel,h}$ is no longer a flux function. One can separate $J_{\parallel} = J_{\parallel,c} + J_{\parallel,h}$, satisfying $b \cdot V J_{\parallel,c} = 0, b \cdot V J_{\parallel,h} + \nabla \cdot J_{\parallel,h} = 0$, where $J_{\parallel,c}$, $J_{\parallel,h}$ are the parallel current of background plasma and energetic ions, respectively. The behavior of background plasma is not changed, so the parallel current perturbation of background plasma $\delta J_{\parallel,c} = -(dJ_{\parallel,c}/d\psi)\cdot d(1 - q/q_{s})^{-1}\delta \psi$, where $J_{\parallel,c}$ is the background plasma current. For energetic ions, $J_{\parallel,h} = f_{h}(Q_{h} + a^{2}\delta \psi)$. For energetic ions, $dQ/d\psi = 1 - q/q_{s} - \omega_{d}(q_{s}/q_{h})$, $a^{2} \ll 1$ denotes the orbit average effect on the perturbed current, $\omega_{d}$, $\omega_{p}$ are the precessional frequency and transit frequency, respectively. Thus, one can obtain the perturbation current of energetic ions $\delta J_{\parallel,h} = -(Z_{e} e_v \cdot (dJ_{\parallel}/d\psi)\cdot a^{2}\delta \omega/(1 - q/q_{s} - \omega_{d}(q_{s}/q_{h}))) > \delta \psi$, where $< \ldots >$ denotes the integration over velocity space, $Z$ is the particle charge number. Hence, the total perturbation of plasma current

$$\delta J_{\parallel} = \delta J_{\parallel,c} + \delta J_{\parallel,h} = \delta J_{\parallel,c} - \omega_{d}(q_{s}/q_{h}) \frac{\delta \psi}{(1 - q/q_{s} - \omega_{d}(q_{s}/q_{h}))} a^{2}\delta \psi.$$  

(1)

where the first term on the right hand of equation (1) results from the MHD effect, the second term on the right hand of equation (1) denotes the orbit average effect on the perturbation current of energetic ions. It tends to reduce the perturbation current of energetic ions. Thus, given the equilibrium current, the orbit average effect of energetic ions tends to stabilize tearing modes for co-CEI, while destabilize tearing modes for counter-CEI. This effect is well described in the [9, 10]. The last term on the right hand of equation (1) represents the perturbation current due to the precessional drift of particles. If the precessional frequency is very small, satisfying $[\omega_{d}(q_{s}/q_{h} - \omega_{p})] \ll 1 \text{ near the resonance surface, it reduces to the normal diamagnetic current.}$

3. Stability criterion of tearing modes

In this section, based on the above physical picture, the detailed calculation will be shown. In the outer region, the linearized equations are

$$-\nabla \cdot \delta \rho \_L - \nabla \times \delta \rho \_B + \frac{1}{c} \delta J \times B + \frac{1}{c} J \times \delta B = 0,$$  

(2)

$$b \cdot \nabla \cdot \delta \rho \_L + \delta B \cdot \nabla \cdot p = 0,$$  

(3)

$$\delta p_{h} = \delta p_{L,h} + \delta B \cdot p = 0,$$  

(4)

where $\delta \rho_{L}$ is the perturbed pressure of background plasma, which is assumed to be isotropic and incompressible. $\delta p_{h}$ is the pressure tensor of energetic ions, satisfying the Chew–Goldberger–Low [13] pressure tensor form. $J$, $\delta J$ are the equilibrium and perturbed currents, respectively. $B$, $\delta B$ are the equilibrium and perturbed magnetic field, respectively. $I$ is the unit tensor, $b = B/B$ denotes the direction of equilibrium magnetic field. Making the operation $\nabla \cdot [B/B^{2} \times (\ldots)]$ on equation (2), one can obtain
\[
\mathbf{B} \cdot \nabla \frac{\delta \mathbf{B}_h}{B} + \mathbf{B} \cdot \nabla \sigma - J \cdot \nabla \frac{\delta \mathbf{B}_h}{B} + c \nabla \left( \frac{\mathbf{B} \times \nabla \delta \mathbf{p}_h}{B^2} + \frac{\mathbf{B} \times \nabla \cdot \mathbf{p}_h}{B^2} \right) = 0, \quad (5)
\]

where \( \sigma = J/B \). The equilibrium magnetic field can be expressed as
\[
\mathbf{B} = i \nabla \zeta + \nabla \chi \times \nabla \psi, \quad (6)
\]

where the toroidal geometry is assumed to be axisymmetric. The symmetric coordinate, \((\psi, \theta, \zeta)\) are chosen as flux coordinates. \(\theta\) and \(\zeta\) are the poloidal and toroidal angle, respectively. For tearing modes, the perturbed magnetic field can be written as
\[
\delta \mathbf{B} = \nabla \times \left( \delta \mathbf{A}_\parallel \right). \quad (7)
\]

For simplicity, single helicity is considered, namely the form of perturbation can be expressed as \(\delta \mathbf{A}_\parallel = \delta \mathbf{A}_\parallel (r) \exp (im \theta - in \zeta - ist)\). Then, making the integral \(i \delta \theta / J \exp(-i m \theta + in \zeta)\) on equation (5), one can obtain
\[
\frac{(m-nq)}{B} \delta \mathbf{B} + \left( \frac{m l + n \partial \psi}{R_0} \right) \frac{d}{dr} \sigma \frac{\delta \mathbf{A}_\parallel}{B} + \frac{qR_0^2}{I} \left( \frac{m \partial \psi - n \partial \phi}{R_0} \right) \frac{\delta \mathbf{B}_h}{B} + c \int J \mathbf{V} \cdot \frac{\mathbf{B} \times \nabla \delta \mathbf{p}_h}{B^2} \exp(-im \theta + in \zeta) \frac{d \theta}{d \psi} = 0, \quad (8)
\]

where \(J = (\nabla \psi \times \nabla \theta \nabla \zeta)^{-1}\) is the Jacobian. Then, following the procedure of [14], equation (8) can be derived as
\[
L \delta \psi + \frac{4 \pi r B_0 R^2}{G} \delta K = 0, \quad (9)
\]
\[
L = F \left\{ \frac{d}{dr} \left[ \frac{r}{m^2 + n^2 \epsilon^2} \right] - \frac{1}{r} \frac{m^2 + n^2 \epsilon^2}{(m^2 + n^2 \epsilon^2)^2} \left[ \frac{r \sigma}{dr} - \frac{r d(E \sigma)}{dr} - \frac{r G \sigma}{m^2 + n^2 \epsilon^2} + \frac{2 m n \epsilon G \sigma}{m^2 + n^2 \epsilon^2} \right] \right\}, \quad (10)
\]
\[
\delta K = \int J \mathbf{V} \cdot \left( \frac{B_0 \times \nabla \cdot \delta \mathbf{p}_h}{B_0^2} \right) \exp(-im \theta + in \zeta) d \theta, \quad (11)
\]

where \(F = m B_0 - n \in B_0, G = m B_2 + n \in B_0, \partial \psi = G \partial \mathbf{A}_\parallel / B_0, g = (d \rho / d r) / B_0^2, \epsilon = r / R_0\) is the aspect ratio. Without energetic ions, equation (9) reduces to equation (17) in [14]. Next, to obtain the expression of \(\delta \mathbf{p}_h\), it is necessary to solve the perturbed distribution of energetic ions. The linearized drift kinetic equation [15] is
\[
\frac{d \delta h}{dt} = Q \left( \delta \lambda - \delta \phi - \frac{v_a}{c} \nabla \cdot \delta A \frac{1}{i \omega} \right), \quad (13)
\]

where \(P_c = -Z e \psi c + v_i B_0\) is the toroidal momentum, \(E = v_i^2/2\) is the kinetic energy. \(Q = (i) Z c / (\omega - n \omega) (\partial F_c / \partial E), \omega_c = (\partial F_c / \partial P_c)(\partial F_c / \partial E), \epsilon_d\) is the magnetic drift velocity of particles, \(\delta \phi\) is the perturbed electrostatic potential, \(\delta \lambda\) satisfies \(\delta \lambda = c \cdot B \nabla \lambda / (i \omega)\). For ideal MHD, the perturbed electric field \(\delta E = 0\), so \(\delta \lambda = \delta \phi\). To solve \(\delta \lambda\), it is convenient to introduce a transform coordinate \((r_d, \theta_d, \alpha)\), satisfying \(r_d = r - q \rho_i \cos \theta, \theta_d \approx \alpha, \alpha \approx \theta - \zeta \approx \alpha_d t\) for well circulating particles, where \(\omega_c = \psi / (J B), \alpha_d = -v_d B_d \rho / (r B_d), v_d = \psi / (i \Omega_d), \rho_i = \psi / (\Omega_d), \rho_i\) is the orbit width of particles, \(\Omega_d\) is the gyro-frequency, \(B_d\) is the equilibrium poloidal magnetic field. In this coordinate, equation (13) can be rewritten as
\[
\frac{d \delta h_d}{dt} = \left[ k_d \frac{2 R B^2}{r B^2} + \left( e^{i \theta} k_d - i k_d B \right) \right], \quad (14)
\]

where the subscript \(d\) denotes the variables defined in the \(r_d\).

Making the reversed transform to the coordinate \((r, \theta, \zeta)\), equation (15) can be derived as
\[
\delta h = \frac{Q_{\psi}}{\omega} k_r \frac{2 R B^2}{r B^2} \sum_{l}(l + m - nq) \omega_l + no_d - \omega + \sum_{l} \left( \frac{i k_d B_0}{\lambda} - k_d \frac{\delta A}{\delta \lambda} \right) \left( i k_d J - k_d f_{\psi} \frac{\delta F}{\delta A} \right) J_{-l}\delta A. \quad (16)
\]

Now, the perturbed distribution of CEI is obtained. Based on the expressions (12) and (16), and using the definition \(\delta \mathbf{F}_{\psi} = \int d^3 v \psi \delta \mathbf{F}_{\psi}\), equation (11) can be derived as
\[
\delta K = \int d^3 v \psi \delta \mathbf{K}_T + \delta \mathbf{K}_I, \quad (17)
\]

\[
\delta \mathbf{K}_I = i k_d^2 B_0 \frac{\partial E}{\partial r} \frac{\delta A}{\delta \lambda}, \quad (18)
\]
\[
\delta K_\ell = -\frac{Q}{\omega} \frac{r}{R_0 B_0} \delta \lambda
\]
\[
\left[ \frac{q_f}{m_0 m_e r B_0} \left( k_0^2 \frac{R_0 B_0^2}{r B^2} \right) - 2k_0 \frac{R_0 B_0^2}{r B^2} (1 - J_0^2 (\lambda)) \right] \left( 1 - J_0^2 (\lambda) \right),
\]

where \(\delta K_\ell\) and \(\delta \lambda\) represent the adiabatic and kinetic contributions of CEI, respectively. Here, the high harmonic \(|\lambda| \geq 1\) terms are neglected, and large aspect ratio is assumed.

To proceed, it is assumed that the equilibrium distribution \(F = \sum \overline{F}_\ell \delta \lambda\) satisfies slowing-down model for a population of formed CEI by a purely co-CEI \((\nu = +)\) component or a purely counter-CEI \((\nu = -)\) component, as

\[
\overline{F}_\ell = \frac{\rho_{\ell}^b}{2\pi m_0 r E_m} E^{-3/2} \delta (\lambda) H(E - E_m),
\]

where \(\lambda = \mu E\) denotes the pitch angle, \(E_m\) is the maximal energy, \(H(x)\) is the step function. As done in [9], it is assumed that \(J_0^2 \approx \sigma, \sigma < 1\) is a constant coefficient. Then equation (17) can be reduced to

\[
\delta K = - \sum_i \frac{d \rho_i^b}{d r} \delta A_i \left( \frac{B_\ell^2 G^2}{r B^2 F} (\sigma (CI - 1) - 2 r B_\ell^2 \frac{n}{B_\ell^2 \rho_{\ell m}} (1 - \sigma)) \right),
\]

where \(\sigma\) is neglected for tearing modes, \(CI = 1 + 2 \chi + 2 \chi^2 \ln(1 - \chi) / (\ln(1 - \chi)), \chi = (m - q_0) R_0 / (m \rho_{\ell m}), \rho_{\ell m} = \sqrt{2 E_m / \Omega}, \omega_f / \omega_1 = \rho_{\ell f} / R_0\) is employed. The physics of \(\delta K\) is similar to that in equation (1). The first term on the right hand side of equation (21) represents the effect of precessional drift effect, and the last term denotes the orbit average effect on the perturbed current of energetic ions. If \(|\lambda| \gg 1\), the precessional frequency is very small, the first term in \(\delta K\) reduces to the usual diamagnetic drift current. For tokamaks, \(|\lambda| \gg 1\) is always satisfied, except very narrow region close to the resonant surface, so the effect of precessional drift can be neglected, as done in [9]. For RFP, \(|\lambda| \sim O(1)\) for energetic ions in some region near the resonant surface, due to the \(q < 1\) and \(n > 1\) in the RFP, which is pointed out in [12]. Thus, the effect of precessional drift on tearing modes can be important in the RFP. Substituting equation (21) into equation (9), one can obtain

\[
\bar{L} \delta \mu + \delta \nu \sum_i \frac{1}{B^2} \frac{d \rho_i^b}{d r} \left[ \frac{\rho_{\ell f}^b}{2 F} (CI - 1) - \frac{r B_\ell^4 n}{2 B_\ell G^2 \rho_{\ell m}^b} (1 - \sigma) \right] = 0,
\]

where the parameters are normalized as follows: \(r \rightarrow a r, B \rightarrow B (0) \bar{B}, \beta_{\ell f}^b = 8 \pi p_{\ell f}^b / B^2 (0)\). Here, the imaginary part of \(\delta K\) is not considered, since it induces a real frequency, which is not important for tearing mode in outer region as the inertia term. From equation (22), it can be known that co-CEI plays a stable role on tearing modes, while counter-CEI plays an unstable role on tearing modes with \(d \rho_i / d r < 0\). This is consistent with the physical picture in section 2. If the precessional effect is neglected, equation (22) is equivalent to that in [9] in tokamak geometry. Thus, based on equation (22), the stability criterion \(\delta'\) for tearing mode can be calculated.

For typical RFP like MST, the main parameters \(B_0 = 0.3\ T, a = 0.5\ m, R_0 = 1.5\ m\). The profiles of equilibrium current and pressure are \(\sigma = \sigma_0 (1 - r^2), \sigma_0 = 2 \omega (q (0) R_0), \beta_\ell = \beta_{\ell 0} (1.04 + 0.52 r^2 - 6.78 r^4 + 7.68 r^6 - 2.47 r^8) (\beta_\ell = 8 \pi p_{\ell r} (B^2 (0), p_\ell is the background pressure), \beta_{\ell 0} = \beta_{\ell 0} (3.5 \exp(-r^2/0.044)), where \(\beta_{\ell 0}\) are used to modify the weight of \(\beta_c, \beta_h\), respectively. Plasma beta at axis \(\beta (0) = \beta_h (0) + \beta_c (0)\). Considering \(m = 1, n = 5\) mode, \(\beta (0) = 7\%\) and 25 keV ions formed by CEI, the equilibrium profiles can be seen in figure 1. Based on
equation (22), the stability criterion $\Delta'$ for tearing modes can be calculated, which can be seen in figure 2. It can be found that the effect of CEI on tearing modes is dramatic, and depends on the toroidal direction of CEI. For counter-CEI, as the fraction $\chi_{\text{frac}}$ of counter-CEI increases, $\Delta'$ increases. For co-CEI, as $\chi_{\text{frac}}$ increases, $\Delta'$ decreases, and becomes negative if $\chi_{\text{frac}}$ is sufficiently large. Thus, counter-CEI tends to destabilize tearing modes, while co-CEI stabilizes tearing modes. This is consistent with the physics picture pointed out in section 2, and it is also qualitatively consistent with the recent experimental results in MST [11], where it was shown that the amplitude of tearing modes was reduced when co-NBI was injected.

4. Conclusion

In this article, the effects of CEI on tearing modes in RFP have been studied. In contrast to little precessional drift effect of CEI on tearing modes in tokamak, the precessional effect of CEI on tearing modes is important in RFP, since $\frac{\omega_d}{(m - n q) \omega_t} \sim O(1)$ near the resonant surface due to low $q$ and high $n$ in RFP. It is found that the effects of CEI on tearing modes have a great relation to the magnitude of $\beta_h$, and depend on the toroidal circulating direction of CEI. For co-CEI, energetic ions can reduce the value of stability criterion $\Delta'$ of tearing modes, and stabilize tearing modes, even suppress tearing modes. For counter-CEI, energetic ions can increase the value of stability criterion, and destabilize tearing modes. For the balanced tangential neutral beam injection, energetic ions have no or little effect on tearing modes. The result is qualitatively consistent with the recent experimental result in MST, where it was shown that tearing modes can be stabilized by CEI. Thus, our analysis suggests that tearing modes can be suppressed by co-CEI in RFP with appropriate $\beta_h$.

Here, it is necessary to pointed out that the effects of energetic ions on $\Delta'$ is only considered, which is valid for orbit width of energetic ions larger than island width. If the orbit width of energetic ions and island width are comparable, the effects of energetic ions in the resistive inner region needs to be considered.

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References