Analysis methods for fast impurity ion dynamics data

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A high-resolution spectrometer has been developed and used on the MST reversed-field pinch (RFP) to measure passively impurity ion temperatures and flow velocities with 10 μs temporal resolution. Such measurements of MHD-scale fluctuations are particularly relevant in the RFP because the flow velocity fluctuation-induced transport of current (the "MHD dynamo") may produce the magnetic field reversal characteristic of an RFP. This instrument will also be used to measure rapid changes in the equilibrium flow velocity, such as occur during locking and H-mode transition. The precision of measurements made to date is <0.6 km/s. We are developing accurate analysis techniques appropriate to the reduction of this fast ion dynamics data. Moment analysis and curve-fitting routines have been evaluated for noise sensitivity and robustness. Also presented is an analysis method which correctly separates the flux-surface average of the correlated fluctuations in **u** and **B** from the fluctuations due to rigid shifts of the plasma column. © *1995 American Institute of Physics.*

I. INTRODUCTION

Measurement of the dynamics of ions in a magnetically confined plasma contributes to an understanding of a variety of plasma phenomena including particle transport,[1] H-mode operation,[2] and locked modes.[3] In the reversed-field pinch (RFP), the flow velocity fluctuation-induced transport of current may be the mechanism behind magnetic field reversal. (This current transport is driven by a fluctuation induced electric field, $\mathbf{E}_f=\langle\tilde{\mathbf{u}}\times\tilde{\mathbf{B}}\rangle$, the "MHD dynamo."[4]) These measurements of fast ion dynamics require precise analysis methods because the fluctuations or rapid changes are often of small amplitude. We present here a comparison of moment analysis to traditional curve fitting; the former is elegant and efficient compared to the latter, but is sensitive to noise in the raw data.

Since the primary focus of our work thus far has been the study of the MHD dynamo, the data in this paper is presented in that context. The difficulty in experimentally verifying the existence of the MHD dynamo lies in the fact that $\tilde{\mathbf{u}}$ is expected to be very small. Simulations done with the 3-D MHD code DEBS[5] suggest that $\tilde{\mathbf{u}}$ is on the order of 0.1% of the Alfvén velocity in the MST RFP,[6,7] which implies flow velocity fluctuations of less than 1 km/s. The frequency spectrum of the velocity fluctuations is expected to peak around 20–30 kHz in the lab frame since that is the range in which much of the observed MHD activity takes place in MST.[8] We have developed a specialized spectroscopic diagnostic system with demonstrated measurement precision of <0.6 km/s for flow velocity, with a fluctuation bandwidth of 35 kHz (soon to be increased to 250 kHz with faster digitizers). Only a brief description of the hardware will be given here as this system and its calibration have been described in detail elsewhere.[9] The measurement technique is based on high-speed passive Doppler spectroscopy of C V impurity ions (227.091 nm) which are localized near the hot core of the MST plasma. The chordal views inherent to passive spectroscopy limit the spatial resolution, but this presents no difficulty on MST because the MHD fluctuations of interest are large scale, with wavelengths on the order of the machine size. Our instrument simultaneously records two opposing chordal views of the plasma, each with 16 spectral wavelength channels (a capability that makes it a *duo*-spectrometer).

II. DATA REDUCTION

A. Comparison of analysis routines

We employ two different data reduction techniques, moment analysis[10] and non-linear curve fitting. (Another possible approach to this problem is the "lookup table" technique.[11]) Since there is not widespread use of the moment analysis technique for reduction of spectroscopic data, a brief overview is in order. For a Doppler broadened impurity line, an evaluation of the three lowest-order moments of the spectral line intensity distribution provides good estimates of the flow velocity and temperature:

$$\mathrm{AMP}=\sum_{i=1}^{m} C_i \quad \propto \text{emission intensity}, \tag{1}$$

$$\mathrm{CEN}=\frac{\sum_{i=1}^{m} x_i C_i}{\mathrm{AMP}} \quad \propto \text{flow velocity}, \quad \text{and} \tag{2}$$

$$\mathrm{VAR}=\left(\frac{\sum_{i=1}^{m} x_i^2 C_i}{\mathrm{AMP}}\right)-\mathrm{CEN}^2 \quad \propto \text{ion temperature}, \tag{3}$$

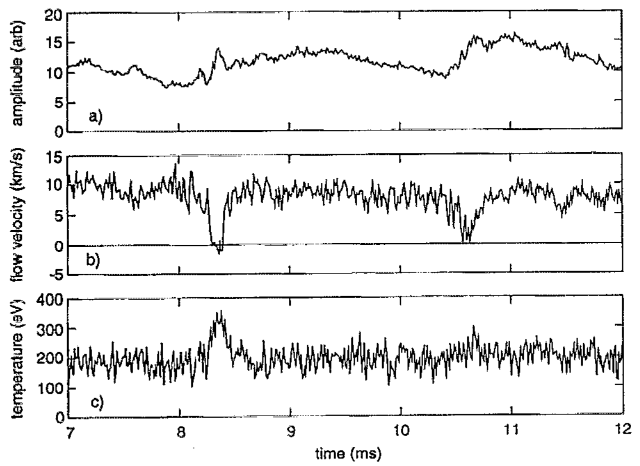

FIG. 1. (a) Signal amplitude (zeroth moment), (b) flow velocity (first moment), and (c) ion temperature (second moment) derived by the moment analysis data reduction technique.

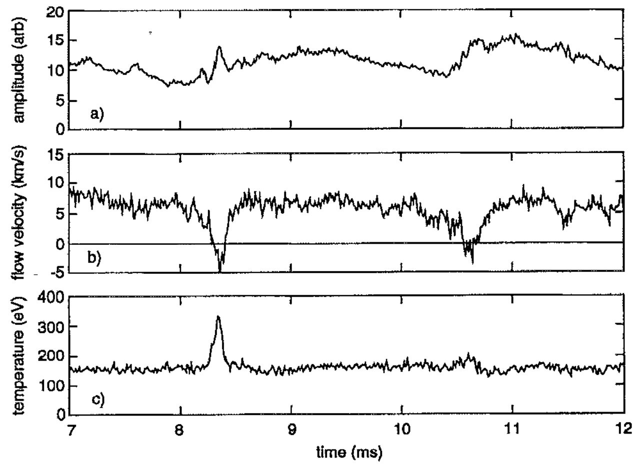

FIG. 2. (a) Signal amplitude, (b) flow velocity, and (c) ion temperature derived with the nonlinear curve-fitting routine CURFIT. The same raw data was analyzed for Figs. 1 and 2.

where m is the total number of wavelength channels, C_i is the intensity in the ith channel, and x_i is the position of the ith channel.

This moment analysis has the advantage that it is strictly correct for any velocity distribution of ions; by definition the first moment of the distribution is the average velocity and the second moment is the average energy. The moment analysis is computationally simple and fast, but is sensitive to noise in the wings of the Doppler broadened spectra. In contrast to this technique there are a variety of nonlinear curve-fitting routines; we use the IDL implementation of CURFIT.[12] The nonlinear routine is computationally intensive, but may be more useful for quantifying fluctuation amplitudes as there seems to be less noise-induced fluctuation of the derived quantities when noise is present in the wings of the line. The curve-fitting routine does require that a specific fitting function be chosen beforehand (e.g., a Gaussian fitting function for an expected Maxwellian velocity distribution).

Figures 1 and 2 show the slightly different results obtained when both moment analysis and CURFIT are applied to the same raw data. The signal amplitudes are almost identical in both cases, although the velocity derived by CURFIT appears to be offset slightly negative compared to the moment analysis velocity (a feature which is not yet understood, but may be related to relative channel sensitivity of the duo-spectrometer). Evident from both analysis methods are the sudden drop in flow velocity and increase in ion temperature that occur coincident with the sawtooth events at 8.3 and 10.7 ms.

It is clear that the velocity and temperature derived by moment analysis have a visibly higher fluctuation level. A substantial part of this difference is a result of the sensitivity of the moment analysis to noise fluctuations in the raw data, especially noise in the wings of the Doppler-broadened spectrum. This can be demonstrated by performing a moment analysis using data from only the central 12 of the 16 channels (being careful to make sure that these outer channels have negligible Doppler-broadened signal); comparison to quantities derived from the full 16-channel moment analysis shows a reduced fluctuation level. The nonlinear fitting routine effectively averages the wing channels by drawing what amounts to a straight line through the data points. Figure 3 illustrates this point by showing the Gaussian function fit to the raw data of typical spectra; such spectra are recorded every 10 μs.

B. Uncertainties

Assuming that the uncertainties in the measured intensities C_i dominate all other sources of uncertainty, the uncertainties of the three lowest-order moments [Eqs. (1)–(3)] can be estimated by taking partial derivatives with respect to each C_i and then propagating the errors in quadrature:

$$\sigma_{\mathrm{AMP}}=\sqrt{\sum_{i=1}^{m}\sigma_i^2}, \tag{4}$$

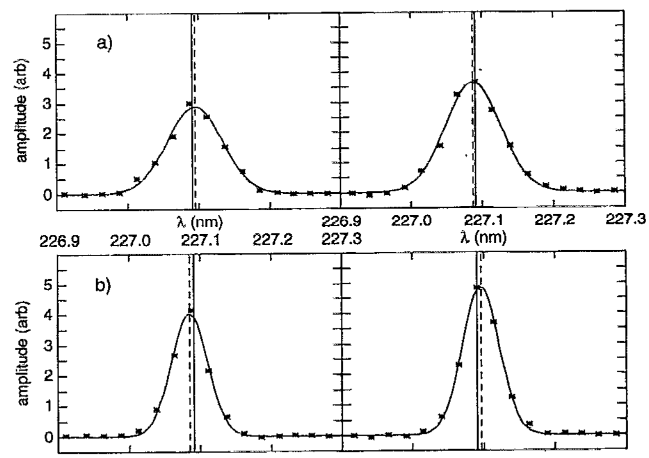

FIG. 3. The Gaussian functions fit to the raw data of typical spectra. (a) Spectra recorded at 8.35 ms (Fig. 2) during the sawtooth event. (b) Spectra recorded at 9.05 ms during an equilibrium period. The two spectra at each timepoint are from the opposing toroidal views of the plasma.

TABLE I. A comparison of the uncertainties and rms fluctuations levels generated by moment analysis and curve fitting. See the text for symbol definitions.

	Velocity (km/s)			Temperature (eV)		
	σ_{unc}	σ_{rms}	Δ	σ_{unc}	σ_{rms}	Δ
Moment analysis	0.60	0.84	+0.24	10	27	+17
CURFIT	0.15	0.57	+0.42	1.4	4.9	+3.5

$$\sigma_{\mathrm{CEN}}=\frac{\sqrt{\sum_{i=1}^{m}(x_i-\mathrm{CEN})^2\sigma_i^2}}{\mathrm{AMP}}, \quad (5)$$

$$\sigma_{\mathrm{VAR}}=\frac{\sqrt{\sum_{i=1}^{m}[(x_i-\mathrm{CEN})^2-\mathrm{VAR}]^2\sigma_i^2}}{\mathrm{AMP}}, \quad (6)$$

where σ_i is the uncertainty of C_i. If the spectrometer is operated in the photon counting regime, then $\sigma_i=\sqrt{C_i}$ and calculation of the uncertainties in the moments is straightforward. However, the light flux at 227.091 nm from the C V ions in MST is generally large enough to insure that the statistical (Poisson) uncertainty is small. The dominant contribution to the uncertainty in C_i appears to come from the fluctuations in the background light signal. This background appears to be a combination of continuum and line radiation that is insufficiently rejected by the duo-spectrometer. This background can be quantified reasonably well because the wing channels of the duo-spectrometer (at 226.915 and 227.283 nm) do not record any Doppler broadened C V light at the temperatures and flow velocities normally seen in MST. The equilibrium component of this background is subtracted from the C_i before analysis, while the fluctuating component is used to estimate σ_i.

As a specific example, the uncertainties (σ_{unc}) calculated with this method and those calculated with CURFIT are listed in Table I. Also shown are the rms fluctuation levels (σ_{rms}) and the difference (Δ) between σ_{unc} and σ_{rms}. All these quantities are an average over the period from 9 to 10 ms for the data shown in Figs. 1 and 2. There is poor quantitative agreement between moment analysis and CURFIT uncertainties, but note that Δ is positive in every case (as it is for other shots that have been examined), indicating that there are nonzero temperature and velocity fluctuations of the C V ions in the MST plasma.

C. Measurement of the MHD dynamo

The two terms of the fluctuating electric field $\mathbf{E}_f$ that drive poloidal current are $\tilde{u}_t\tilde{B}_r$ and $\tilde{u}_r\tilde{B}_t$, both probably of similar magnitude. We have begun by measuring the first term since it is easier to measure the toroidal component of the flow velocity in MST. $\tilde{B}_r$ is measured with a BN shielded coil probe inserted into low current (250 kA) RFP discharges.

Large fluctuations in $\tilde{u}_t$ and $\tilde{B}_r$ occur during a sawtooth event. However, with the current diagnostics, it is hard to separate the part of the fluctuation associated with the rigid shift of the plasma column and that which is truly a flux-surface average of a random fluctuation. As a demonstration of our approach, consider the case where the two measured quantities $u^m(t)$ and $B^m(t)$ can be written as

$$u^m(t)=\bar{u}(t)+u^{\mathrm{st}}(t)+u^r(t),$$

$$B^m(t)=\bar{B}(t)+B^{\mathrm{st}}(t)+B^r(t),$$

where $u^m(t)$ and $B^m(t)$ are the measured values, $\bar{u}(t)$ and $\bar{B}(t)$ are the equilibrium values, $u^{\mathrm{st}}(t)$ and $B^{\mathrm{st}}(t)$ are the sawtooth contribution, and $u^r(t)$ and $B^r(t)$ are everything else: random flux-surface averaged fluctuations and noise.

We pick a time window, either around or between sawtooth events. After some algebraic manipulation and the assumption that each sawtooth is identical (close, but not quite true), we can show that the component of the fluctuation-induced electric field being measured is

$$E_f=\langle u^r(t)B^r(t)\rangle=\langle[u^m(t)-\bar{u}(t)][B^m(t)-\bar{B}(t)]\rangle - \langle u^m(t)-\bar{u}(t)\rangle\langle B^m(t)-\bar{B}(t)\rangle,$$

where the "$\langle\ \rangle$" indicate an average over the ensemble of time-windowed events.

This analysis method has been applied to large ensembles of shots and events. The "sawtooth windowed" data show a burst of fluctuation activity during the sawtooth crash, coupled with what may be an increase in the magnitude of E_f. The "between sawtooth" data show evidence of a nonzero average E_f. In sum, we see possible evidence of the MHD dynamo in this initial investigation. However, since we know from simulations that $\tilde{u}_t$ will be small, we do not consider this result to be definitive and will continue this investigation. Further analysis of existing data and/or upgrades to the duo-spectrometer and new data should allow us to definitely settle the question of the existence of the MHD dynamo.

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