Magnetic-Fluctuation-Induced Particle Transport and Density Relaxation in a High-Temperature Plasma

W. X. Ding,[1,3] D. L. Brower,[1,3] G. Fiksel,[2,3] D. J. Den Hartog,[2,3] S. C. Prager,[2,3] and J. S. Sarff[2,3]

[1]Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095, USA
[2]Physics Department, University of Wisconsin-Madison, Madison, Wisconsin 53706, USA
[3]Center for Magnetic Self-Organization in Astrophysical and Laboratory Plasmas, University of Wisconsin-Madison, Madison, Wisconsin 53706, USA

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The first direct measurement of magnetic-fluctuation-induced particle flux in the core of a high-temperature plasma is reported. Transport occurs due to magnetic field fluctuations associated with global tearing instabilities. The electron particle flux, resulting from the correlated product of electron density and radial magnetic fluctuations, accounts for density profile relaxation during a magnetic reconnection event. The measured particle transport is much larger than that expected for ambipolar particle diffusion in a stochastic magnetic field.

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Particle confinement is a critical issue in magnetic fusion plasmas because not only does fusion power increase as the square of density, but also particle transport contributes to momentum and energy transport. In particular, particle transport resulting from fluctuating magnetic fields has been an important and unresolved area in fusion research for many years [1,2]. Fluctuating magnetic fields can arise from global tearing instabilities that often underlie the sawtooth oscillation associated with magnetic reconnection and lead to plasma relaxation [3]. Furthermore, magnetic fluctuations are also generated by energetic particle instabilities associated with noninductive heating and current drive, or inevitably generated in burning plasmas by alpha particles [4]. Conversely, stochastic magnetic fields have been deliberately imposed in tokamak plasmas using external coils (resonant magnetic perturbation) to mitigate the edge-localized modes by relaxing the density profile to achieve heat and particle control in the plasma boundary [5]. A basic understanding of magnetic-fluctuation-induced particle transport processes is thus of great interest and potentially critical to plasma density control and understanding fast particle losses in future burning plasmas like ITER. In earlier work, direct measurements of magnetic-fluctuation-induced particle flux have been made in the edge regions of lower temperature plasmas by probes [1,6–8]. However, progress is largely limited by an inability to measure the magnetic-fluctuation-induced particle flux in the core of hot plasmas.

In this Letter, we report direct measurements of the magnetic-fluctuation-induced electron particle flux in the core of a high-temperature plasma during a period in which the magnetic field is stochastic. We find that the measured magnetic-fluctuation-induced electron flux can account for the rapid particle transport and sudden change in the equilibrium density. The measured particle transport agrees with that predicted for electron stochastic diffusion in the absence of ambipolarity constraints but is much larger than the expected particle diffusion which would arise if the electrons were slowed to the ion diffusion to enforce ambipolarity.

The evolution of electron density (n_e) is simply governed by particle transport and particle sources according to

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma^T_{r,e} = S_e, \qquad (1)$$

where S_e is the electron source and $\Gamma^T_{r,e}$ is the fluctuation-induced transport flux. From this equation we see that a core density reduction ($\frac{\partial n_e(0)}{\partial t} < 0$) can arise only from particle transport since $S_e \geq 0$. The experimental focus of this Letter is the measurement of the two terms on the left side, which then yield the contribution of fluctuation-induced flux to particle transport.

The total radial particle flux driven by fluctuations is described by [1,7]

$$\Gamma^T_{r,\alpha} = \frac{\langle \delta n \delta E_\perp \rangle}{B} + \frac{\langle \delta \Gamma_{\parallel,\alpha} \delta b_r \rangle}{B}. \qquad (2)$$

The first term on the right-hand side results from electrostatic fluctuations, where δn and $\delta E_\perp$ are density and perpendicular electric field fluctuations, respectively. The second term arises from magnetic fluctuations, where $\delta\Gamma_{\parallel,\alpha}$ is the fluctuating particle flux parallel to the magnetic field $\vec{B}$ for species α (electron or ion) and δb_r is the radial magnetic field fluctuation. Brackets $\langle \cdots \rangle$ denote a magnetic surface average. The fluctuating parallel flux results from both density and parallel velocity fluctuations according to the relation $\delta\Gamma_{\parallel,\alpha} = V_{\parallel,\alpha}\delta n + n_e \delta V_{\parallel,\alpha}$. Therefore, we can rewrite the magnetic-fluctuation-induced particle flux as

$$\Gamma_{r,\alpha} = \frac{\langle \delta \Gamma_{\parallel,\alpha} \delta b_r \rangle}{B} = V_{\parallel,\alpha}\frac{\langle \delta n \delta b_r \rangle}{B} + n_e \frac{\langle \delta V_{\parallel,\alpha} \delta b_r \rangle}{B}, \qquad (3)$$

where the first term on the right-hand side is referred to as

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the density fluctuation-induced flux ($\Gamma^{\delta n}_{r,\alpha}$) due to its density fluctuation dependence and the second term is referred to as the velocity fluctuation-induced flux ($\Gamma^{\delta V}_{r,\alpha}$) due to its velocity fluctuation dependence. Each flux term results from correlation with radial magnetic field fluctuations.

Measurements reported herein were carried out on the Madison symmetric torus (MST) reversed field pinch [9] with major radius $R_0 = 1.5$ m, minor radius $a = 0.52$ m, discharge current 350–400 kA, line-averaged electron density $\bar{n}_e \sim 1 \times 10^{19}\ \mathrm{m}^{-3}$, and electron temperature $T_e \sim T_i \sim 300$ eV for a deuterium plasma. A high-speed ($t \sim 4\ \mu$s), laser-based, 11 vertical chord (separation ~ 8 cm) polarimeter (Faraday rotation)–interferometer system is employed to measure the equilibrium and fluctuating density and magnetic field [10–12]. Both the equilibrium and fluctuating electron density gradient are obtained using differential interferometry techniques [13]. At the magnetic axis, the equilibrium magnetic field strength $B \sim 0.3$–0.4 T, the equilibrium current density $J_\parallel \sim 2\ \mathrm{MA/m^2}$, and safety factor $q \sim 1/6$, where q describes the rotational transform of a field line in a torus. The mean electron velocity $V_{\parallel,e}$ is inferred from the measured parallel current density and electron density, $V_{\parallel,e} \approx J_\parallel / n_e e \sim 10^6$ m/s [14]. MST discharges display a sawtooth cycle in many parameters, and the measured quantities are ensemble (flux-surface) averaged over these reproducible sawtooth events.

The equilibrium electron density profile evolution for a standard sawtoothing MST discharge is shown in Fig. 1. Density profile relaxation occurs within $\sim 200\ \mu$s, much faster than the classical collisional diffusion time, indicating that fluctuation-induced particle transport must be active. Sawtooth events in MST are associated with magnetic reconnection driven by resistive tearing instabilities. Electrostatic fluctuation-induced particle flux [the first term in Eq. (2)] measurements, made by a heavy ion beam probe, are found to be negligible in the plasma core on MST [15]. Therefore, a magnetic-fluctuation-induced anomalous particle flux is thought necessary to

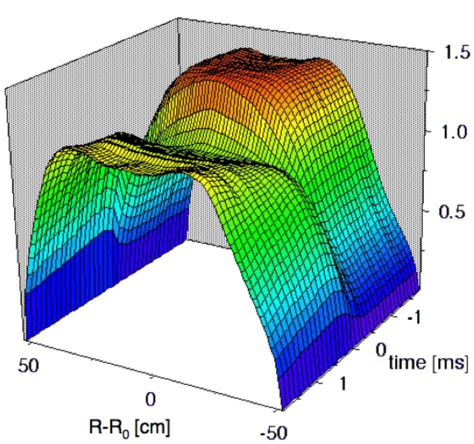

FIG. 1 (color online). Density relaxation during sawtooth crash at $t = 0$. Vertical axis is density ($\times 10^{19}\ \mathrm{m}^{-3}$).

cause density relaxation since tearing-mode-driven magnetic reconnection is strongest at the sawtooth crash.

To quantitatively investigate the effect of magnetic field fluctuations on particle transport and density relaxation, we must measure the magnetic-fluctuation-induced flux and its divergence according to Eqs. (1) and (3). We focus first on the measurement of the density fluctuation-induced particle flux and its derivative for electrons. This requires measurement of (i) the local density fluctuation and its derivative δn, $\partial \delta n / \partial r$, (ii) magnetic field fluctuation δb_r, and (iii) their correlation. In cylindrical coordinates, the expression $\nabla \cdot \Gamma^{\delta n}_{r,e}$ can be simplified since measurements show that the parallel electron velocity, mean magnetic field, and radial magnetic fluctuation profiles are nearly flat in the core [14]. Therefore, we can write $\nabla \cdot \Gamma^{\delta n}_{r,\alpha} = \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma^{\delta n}_{r,\alpha}) \approx 2\frac{V_{\parallel,e}}{B}\langle \frac{\partial \delta n}{\partial r}\delta b_r \rangle$. In the following, measurement of each fluctuating quantity will be described.

First, density fluctuations and their gradient are measured using both conventional and differential interferometry. In general, density fluctuations can be written as $\delta n = \Sigma \delta n_{m,n} \cos(m\theta + n\phi + \Delta(r))$, where m, n, and Δ are the poloidal and toroidal mode numbers and the phase, respectively. In MST, the dominant modes are $m = 1$ and the fluctuating interferometric phase is given by $\delta\phi(x) = c_1 \int \delta n_{m=1}(r) \cos(\theta) dz$, where $c_1 = 1.20 \times 10^{-18}\ \mathrm{m}^2$ for laser wavelength 432 μm. The term $\cos\theta = x/r$, where x is the impact parameter, can be considered as a weighting factor which approaches a delta function as $x \longrightarrow 0$, thereby providing spatial resolution of ~ 10 cm in the core. This leads to a simple relation between phase fluctuations and density fluctuations for chords close to the plasma center, i.e., $\delta n(r) \approx \delta\phi(x)/2ac_1$. The fluctuating line-integrated phase is equivalent to the local density fluctuation amplitude. Differential interferometry, which measures the phase difference between closely spaced ($\Delta x \sim 1$ mm) interferometer chords, provides a line-integrated density gradient measurement, $\partial\phi(x)/\partial x$ [13]. Upon taking the first spatial derivative of line-integrated density, we find $\partial\phi(x)/\partial x = c_1 \int \partial n_e / \partial r \cos\theta dz$, where the cosine term is the same weighting factor obtained earlier. For small impact parameter x, differential interferometry provides a localized measurement of the density gradient and density gradient fluctuations according to $\partial n_e(r)/\partial r \approx 1/2ac[\partial\phi(x)/\partial x]$. From this relation, the differential phase (or its fluctuation) is proportional to the density gradient (or its fluctuation) for measurements made near the magnetic axis. These interferometric approximations offer the great convenience of being able to directly determine the local particle flux without performing any inversion of the line-integrated density measurements [10,13].

The measured electron density and density gradient fluctuations exhibit a significant surge at the sawtooth crash as shown in Figs. 2(a) and 2(b), respectively. These data are for fluctuations at $r/a = 0.11$ associated with the dominant core tearing mode ($m = 1$, $n = 6$) obtained by cross correlation with a specific helical magnetic mode obtained from spatial Fourier decomposition of measurements from an array of 32 wall-mounted magnetic coils at the surface of the plasma. Density fluctuations away from the sawtooth crash are approximately 0.15%, reaching 1% at the crash, while the density fluctuation gradient nearly triples. This is qualitatively consistent with the observation that the largest density change occurs at the sawtooth crash, suggesting that large density fluctuations contribute to density relaxation.

Second, the radial magnetic field fluctuations in the core are obtained by Faraday rotation measurement. Previously, it has been established that radial magnetic fluctuations dominate Faraday rotation fluctuations ($\delta\Psi$) for chords close to the magnetic axis [12,14], leading to $\delta\Psi \sim \int n_e \delta b_r dz$. The measured line-averaged radial magnetic field fluctuation amplitude for the dominant core resonant mode ($m = 1$, $n = 6$) throughout a sawtooth cycle is shown in Fig. 2(c). Maximum amplitude occurs at the crash where the δb_r increases by a factor of 3. For comparison, the poloidal magnetic fluctuation amplitude (dashed line) measured at the wall is also plotted. The local radial magnetic fluctuation profile is obtained by numerically fitting experimental data as described in an earlier work [14]. The radial magnetic field fluctuation amplitude (1%–2% at crash) in the core is approximately 3 times the poloidal magnetic field at the wall, i.e., $\delta b_r(0) \sim 3 \times \delta b_\theta(a)$, consistent with the results from MHD computation.

Finally, the correlated product between δn (or $\partial\delta n/\partial r$) and δb_r is obtained by ensemble averaging. In MST, rotation of the low-n magnetic modes transfers their spatial structure in the plasma frame into a temporal evolution in the laboratory frame. Since the magnetic modes are global, for convenience we correlate δn (or $\partial\delta n/\partial r$) with a specific helical magnetic mode obtained from the spatial Fourier decomposition noted above. After averaging over an ensemble of 400 similar events, we determine the correlated product between δn (or $\partial\delta n/\partial r$) and δb_r for all modes. With combined measurements of density fluctuations, density gradient fluctuations, radial magnetic fluctuations, and their correlated product, the density fluctuation-induced electron particle flux can be determined as shown in Fig. 3(a). Contributions from all large modes ($m = 1$, $n = 6, 7, 8, 9$) have been summed. At the sawtooth crash, δn and δb_r are nearly in phase as the particle flux surges to $\sim 1.5 \times 10^{21}/\text{m}^2\,\text{s}$, concurrent with the core density collapse. In addition, the particle flux divergence also increases significantly, as shown in Fig. 3(b) (solid line), reaching $2.0 \times 10^{22}\ \text{m}^{-3}\,\text{s}^{-1}$.

Fast density profile relaxation (see Fig. 1) implies a surge of dn_e/dt, as indicated in Fig. 3(b) (dashed line). The measured flux divergence ($\nabla \cdot \Gamma_{r,e}^{\delta n}$) shows a similar increase, essentially balancing the density change within experimental errors. One can integrate these traces over time to directly compare the density change and particle flux as shown in Fig. 3(c). Here we see that magnetic field driven particle flux (arising from density fluctuation) can account for the observed core density change at a sawtooth crash. Because of this balance, the electron velocity fluctuation-induced flux ($\Gamma_{r,e}^{\delta V}$) is expected to be small.

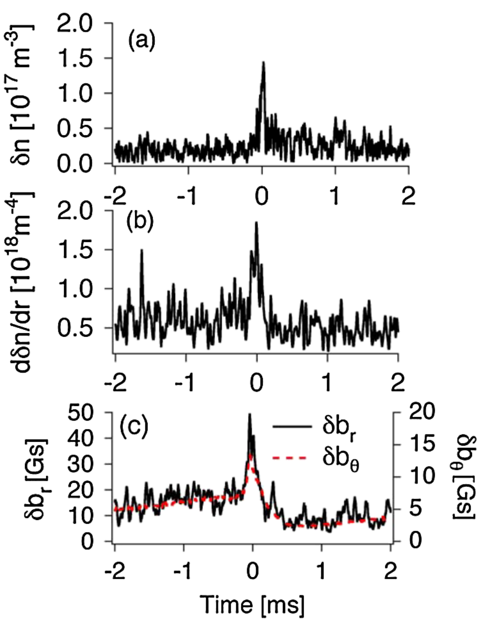

FIG. 2 (color online). (a) Density fluctuation, (b) density fluctuation gradient, and (c) radial magnetic field fluctuation (left vertical axis) during sawtooth cycle (crash at $t = 0$ at $r/a = 0.11$). Dashed line in (c) shows poloidal magnetic field fluctuation amplitude (right vertical axis) as measured at wall.

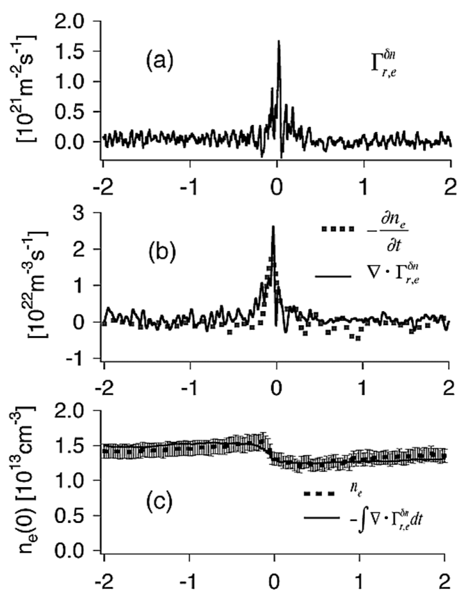

FIG. 3. (a) Electron particle flux, and (b) divergence of density fluctuation-induced electron flux vs time at $r/a = 0.11$ (solid line). Dashed line denotes time derivative of equilibrium density. (c) Time integrated flux divergence (solid line) and electron density (dashed line). The density at $t = -0.475$ ms is taken as an integral constant. All results shown are for a sawtooth cycle (crash occurs at $t = 0$).

Away from the sawtooth crash, the measured density fluctuation-induced particle flux [see Fig. 3(a)] is $\sim 1.0 \times 10^{20}$ m^{-2} s^{-1}, comparable to the particle flux obtained from particle balance [10].

While the measured electron density fluctuation-induced particle transport is significant and can account for the observed density relaxation, the ion density fluctuation-induced particle flux is 2 orders of magnitudes smaller, i.e., $\Gamma_{r,i}^{\delta n} = V_{\parallel,i}\langle \delta n \delta b_r \rangle / B \ll \Gamma_{r,e}^{\delta n} = V_{\parallel,e}\langle \delta n \delta b_r\rangle/B$. This is due to the ion parallel velocity ($\sim 10^4$ m/s) being much less than the electron parallel velocity ($\sim 10^6$ m/s). Therefore, the ion particle flux must be associated with the velocity fluctuation-induced flux ($\Gamma_{r,i}^{\delta V}$). Indeed, the previously measured toroidal (parallel) ion velocity fluctuations increase to (1–2) $\times 10^4$ m/s at the sawtooth crash, and they are in phase with radial magnetic field fluctuations [16]. Thus, the ion velocity fluctuation-induced particle flux can be estimated and found to be approximately (1–2) $\times 10^{21}$ m^{-2} s^{-1} at the sawtooth crash, comparable to the measured electron density fluctuation-induced particle flux.

The magnetic-fluctuation-induced charge flux $\Gamma_q = \langle \delta j_\parallel \delta b_r \rangle / eB = \Gamma_{r,i} - \Gamma_{r,e}$, given by the difference between the electron and ion particle fluxes, was previously obtained by directly measuring the Maxwell stress tensor and found to be small (less than 1% particle flux) [17]. Thus, we have $\Gamma_{r,i}^{\delta n} + \Gamma_{r,i}^{\delta V} - \Gamma_{r,e}^{\delta n} - \Gamma_{r,e}^{\delta V} \approx 0$, where the first term $\Gamma_{r,i}^{\delta n}$ is measured to be negligible and the second term $\Gamma_{r,i}^{\delta V}$ and the third term $\Gamma_{r,e}^{\delta n}$ are measured to be comparable. This implies that the electron velocity fluctuation-induced flux, $\Gamma_{r,e}^{\delta V} = n_e \langle \delta V_{\parallel,e} \delta b_r \rangle / B$, is small compared to the total particle flux (supporting earlier conjecture), likely resulting from a near $\pi/2$ phase between $\delta V_{\parallel,e}$ and δb_r. Electron particle flux originating from electron density fluctuations is comparable to the ion particle flux resulting from ion parallel velocity fluctuations thereby maintaining ambipolarity.

Quasilinear test particle transport in a stochastic magnetic field has been theoretically treated by a few authors [2,18,19]. In the collisionless limit, the test particle diffusivity is $D = V_{\rm th} D_M$, where $V_{\rm th}$ is the particle thermal speed and $D_M = \sum_{m,n} \pi q R_0 (\delta b_r / B)^2$ is the magnetic diffusion coefficient that describes the random walk of stochastic magnetic field lines [6]. Within the self-contained picture of ambipolar particle diffusion in a stochastic field, the diffusivity for both electrons and ions is predicted to be limited to the ion value, $D_i = V_{i,{\rm th}} D_M$ ($V_{i,{\rm th}}$, ion thermal or sound speed), simply because ions stream more slowly along stochastic field lines [7,18,20]. Using measured parameters, we estimate the magnetic diffusivity to be $D_M \approx 2.0 \times 10^{-4}$ m, consistent with direct field line tracing analysis for similar plasma conditions [21]. Using the measured density gradient $\nabla n_e = -2 \times 10^{18}$ m^{-4} in the core, the predicted ambipolar-constrained particle flux is $\Gamma^{\rm QL} = -D_i \nabla n_e = 4.8 \times 10^{19}$ m^{-2} s^{-1}, which is 30 times less than the measured particle flux. Interestingly, the measured flux is within a factor of 2 of the electron value $\Gamma_e^{\rm QL} = -V_{e,{\rm th}} D_M \nabla n_e$ that would occur in the absence of ambipolarity constraints [18], suggesting that electrons can diffuse in the stochastic field at electron diffusion rate. Equally interesting is that the parallel velocity fluctuation-induced ion transport is much larger than the test particle diffusion expectation but matches the electron flux, thereby maintaining quasineutrality. The unexpectedly large particle transport appears to be strongly influenced by nonlinear mode-mode coupling within broad mode spectra in the reversed field pinch [3,14].

In summary, a direct measurement of magnetic-fluctuation-induced particle flux has been made in the core of a high-temperature plasma. The measured electron particle flux can account for density profile relaxation during a sawtooth event. The quasilinear prediction for ambipolar particle transport in a stochastic field underestimates the measured particle transport by at least 1 order of magnitude, emphasizing the need for a more complete theory of magnetic-fluctuation-induced particle transport.

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[1] P. C. Liewer, Nucl. Fusion **25**, 543 (1985).
[2] A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
[3] S. Ortolani and D. D. Schnack, *Magnetohydrodynamics of Plasma Relaxation* (World Scientific, Singapore, 1993); H. P. Furth, J. Killeen, and M. N. Rosenbluth, Phys. Fluids **6**, 459 (1963).
[4] W. W. Heidbrink *et al.*, Phys. Fluids B **5**, 2176 (1993).
[5] T. E. Evans *et al.*, Phys. Rev. Lett. **92**, 235003 (2004).
[6] P. Ghendrih, A. Grosman, and H. Caps, Plasma Phys. Controlled Fusion **38**, 1653 (1996).
[7] M. R. Stoneking, S. A. Hokin, S. C. Prager, G. Fiksel, H. Ji, and D. J. Den Hartog, Phys. Rev. Lett. **73**, 549 (1994).
[8] K. Nagaoka *et al.*, Phys. Rev. Lett. **100**, 065005 (2008).
[9] R. N. Dexter *et al.*, Fusion Technol. **19**, 131 (1991).
[10] N. E. Lanier *et al.*, Phys. Plasmas **8**, 3402 (2001).
[11] D. L. Brower *et al.*, Phys. Rev. Lett. **88**, 185005 (2002).
[12] W. X. Ding *et al.*, Phys. Rev. Lett. **90**, 035002 (2003).
[13] W. X. Ding, D. L. Brower, and T. Y. Yates, Rev. Sci. Instrum. **79**, 10E701 (2008); W. X. Ding, D. L. Brower, B. H. Deng, and T. Yates, Rev. Sci. Instrum. **77**, 10F105 (2006).
[14] W. X. Ding *et al.*, Phys. Plasmas **13**, 112306 (2006); W. X. Ding *et al.*, Phys. Rev. Lett. **93**, 045002 (2004).
[15] J. Lei *et al.*, Phys. Rev. Lett. **89**, 275001 (2002).
[16] D. J. Den Hartog *et al.*, Phys. Plasmas **6**, 1813 (1999).
[17] W. X. Ding *et al.*, Phys. Rev. Lett. **99**, 055004 (2007); W. X. Ding *et al.*, Phys. Plasmas **15**, 055901 (2008).
[18] R. W. Harvey *et al.*, Phys. Rev. Lett. **47**, 102 (1981).
[19] J. M. Finn, P. N. Guzdar, and A. A. Chernikov, Phys. Fluids B **4**, 1152 (1992).
[20] P. W. Terry *et al.*, Phys. Plasmas **3**, 1999 (1996).
[21] T. M. Biewer *et al.*, Phys. Rev. Lett. **91**, 045004 (2003).