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Tearing mode dynamics and locking in the presence of external magnetic perturbations

R. Fridström,1,a) S. Munaretto,2 L. Frassinetti,1 B. E. Chapman,2 P. R. Brunsell,1 and J. S. Sarff2

1Department of Fusion Plasma Physics, School of Electrical Engineering, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden
2Department of Physics, University of Wisconsin-Madison, 1150 University Avenue, Madison, Wisconsin 53706, USA

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In normal operation, Madison Symmetric Torus (MST) [R. N. Dexter et al., Fusion Technol. 19, 131 (1991)] reversed-field pinch plasmas exhibit several rotating tearing modes (TMs). Application of a resonant magnetic perturbation (RMP) results in braking of mode rotation and, if the perturbation amplitude is sufficiently high, in a wall-locked state. The coils that produce the magnetic perturbation in MST give rise to RMPs with several toroidal harmonics. As a result, simultaneous deceleration of all modes is observed. The measured TM dynamics is shown to be in qualitative agreement with a magnetohydrodynamical model of the RMP interaction with the TM [R. Fitzpatrick, Nucl. Fusion 33, 1049 (1993)] adapted to MST. To correctly model the TM dynamics, the electromagnetic torque acting on several TMs is included. Quantitative agreement of the TM slowing-down time was obtained for a kinematic viscosity in the order of $\nu_{\text{kin}} \approx 10-20 \text{ m}^2/\text{s}$. Analysis of discharges with different plasma densities shows an increase of the locking threshold with increasing density. Modeling results show good agreement with the experimental trend, assuming a density-independent kinematic viscosity. Comparison of the viscosity estimates in this paper to those made previously with other techniques in MST plasmas suggests the possibility that the RMP technique may allow for estimates of the viscosity over a broad range of plasmas in MST and other devices. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4953438]

I. INTRODUCTION

Resonant magnetic perturbations (RMPs) have several important plasma control applications in fusion machines. For example in tokamaks, RMPs have been used to mitigate edge-localized modes (ELMs)1,2 and the method is planned to be used in ITER.3,4 Another method utilizes RMPs to orient a neo-classical tearing mode (NTM) in an optimal position for stabilization by electron cyclotron current drive (ECCD).5 A non-disruptive method to detect the intrinsic error field makes use of rotating RMPs to find the phase of the error field.6,7 In stellarators, RMPs have been used to stabilize detached/radiative divertor plasmas, thereby reducing divertor heat load by a factor of 3–10.8 In reversed-field pinches (RFPs), RMP application has been used to, for example, induce slow rotation to reduce localized plasma wall interaction9 and to control the orientation of 3D equilibria.10 The interaction between a resonant error field and a tearing mode (TM) can cause braking of the TM itself and the plasma rotation.11–13 This can eventually lead to a wall locked mode and consequently reduced confinement and possibly, in tokamaks, a disruption.14–17 Therefore, the interaction between plasma and an RMP, simulating an error field, continues to be studied both theoretically12,18–21 and experimentally.13–16,22–25

The interaction of a static RMP with a fast rotating TM of the same poloidal and toroidal mode numbers causes an electromagnetic (EM) torque, which is proportional to (I) the RMP amplitude, (II) TM amplitude, and (III) sine of the TM-RMP relative phase difference.12,18,19 On average, the EM torque brakes the TM and plasma rotation at the resonant surface. The local change in plasma velocity results in a differential rotation, where the plasma layer at the resonant surface is rotating slower than the surrounding plasma layer. This gives rise to a viscous torque that acts to equate the rotation between the two layers. The result is a restored rotation at the resonant surface and a reduced rotation of the bulk plasma. If the RMP amplitude is above a “locking-threshold,” the EM torque overcomes the viscous torque and the TM locks relative to the RMP phase. At the time of locking, the velocity reduction is peaked at the resonant surface.12,18,26 Thereafter, the velocity reduction spreads via viscosity to cause a relaxation of the whole plasma rotation. This significantly reduces the viscous (restoring) torque at the resonant surface. To release a locked TM, the RMP amplitude (and thereby the EM torque) has to be reduced significantly below the “locking-threshold,” so that the reduced viscous torque can overcome the EM torque.12,18,19,26 Hence, there is hysteresis in the locking and unlocking of a TM. An increase of EM torque and a related deepening of the hysteresis are caused by the increase in amplitude of the locked TM.19,26 The increase in TM amplitude is due to the loss of the stabilizing effect of the conducting shell, which is dependent on TM rotation.

Braking and locking of a tearing mode caused by an RMP has been studied both in tokamaks13–15,22–24,27 and

a)richard.fridstrom@ee.kth.se
RFPs. In EXTRAP T2R reversed-field pinch, TM dynamics and velocity braking due to a single-harmonic RMP were found to be in qualitative agreement with theory. Furthermore, after TM wall-locking, hysteresis caused by reduction of viscous torque and increase in the tearing mode amplitude has been experimentally observed.

Previous works in RFPs have mainly focused on the interaction between a single harmonic RMP and a TM of corresponding resonance. However, in tokamaks, the number of RMP coils that can be placed outside the machine-wall is limited by other equipment, such as ports for neutral beam injection (NBI) and radio frequency (RF) heating. This leads to a broad spectrum of harmonics, which can potentially be resonant with more than one TM. For example, to describe the effect of magnetic perturbations on one TM island in ASDEX, the torques on all TM islands had to be included. In the Madison Symmetric Torus (MST) reversed-field pinch, RMP coils cannot be distributed outside the vacuum vessel for another reason: it is a relatively thick aluminum shell that would effectively stop the RMP-field from penetrating it and entering the plasma. However, the MST shell has two relatively narrow gaps (cuts): one poloidal (vertical) and one toroidal (horizontal). They are a source of radial magnetic error fields. The error field at the poloidal gap is corrected for by external saddle coils in a feedback system. In addition to the error field correction, the feedback system can be used to produce RMPs with a pre-set amplitude and poloidal mode number \( m \). Due to the coils limited toroidal extent, they produce a broad spectrum in toroidal mode number \( n \). MST has several TMs that are naturally rotating together with the plasma fluid. Hence, MST is a suitable device to study the simultaneous braking of several TMs, due to a multi-harmonic RMP.

In this work, the RMP effect on the tearing mode dynamics is experimentally studied in MST. The plasmas in this study, called multiple helicity (MH) RFP plasmas, exhibit several tearing modes of comparable amplitudes. The main unstable TMs have poloidal mode number \( m = 1 \) and toroidal mode numbers \( n \approx 2R_0/a \) (\( \approx 6 \) in MST), where \( R_0 \) is the major radius and \( a \) is the minor radius. The locking of multiple resonant modes due to RMPs has previously been observed in MST, but the RMP system was relatively crude. The more recently installed radial-field feedback system allows the production of almost arbitrary waveforms and a cleaner spectrum, minimizing poloidal \( m \) sidebands. In addition, theory has been updated to a general cylindrical equilibrium, to include the dispersion relation for the MST shell, multi-harmonic RMP, and improved TM amplitude evolution. It is therefore of interest to exploit the new system and compare the experimental results with the theoretical models. To this end, the feedback control coils at the poloidal gap have been used to produce a magnetic perturbation (MP) with poloidal harmonic \( m = 1 \) and a spectrum of several (resonant and non-resonant) \( n \) harmonics with similar amplitudes.

In addition to the TM interaction with an RMP, TM braking can also be produced by eddy currents induced in the wall. Because of the finite conductivity of the wall, the eddy currents have a phase that lags the tearing mode’s phase. The phase difference leads to an EM torque that brakes the TM, and its magnitude is proportional to the square of the TM amplitude. Thus, the braking effect is most prominent in \textit{quasi-single helicity} (QSH) RFP plasmas, which exhibit one dominant TM. Tearing mode braking due to “wall-torque” has been observed in MST, where a model of the TM interaction with the wall showed both qualitative and quantitative agreement with the experimental data. In the present paper, the modeling in Ref. is extended to include the effect of the multi-harmonic RMP acting on several TMs \( (m = 1, n) \). Due to both the high amplitude of the RMP and relatively low TM amplitudes in MH plasmas, the wall-torque turns out to be negligible compared to the RMP-torque. The EM coupling between different tearing modes is not included. In MST, this coupling is the largest at the time of a sawtooth crash. Between crashes, this coupling is usually weak, and theoretically, the coupling effect is expected to be small compared to the RMP that acts on all \( m = 1 \) modes, as discussed in Section V. The plasmas in the present study exhibit sheared velocity profiles prior to the RMP application, which further motivates that the torque due to mode coupling is relatively small.

In the present modeling, the only input parameter that is not experimentally measured is the kinematic viscosity. It is instead chosen to fit the experimental TM braking evolution. In this way, the model provides a prediction of the experimental kinematic viscosity. The predicted value is compared with previous measurements in MST hydrogen plasmas. The previous method was based on inserable biased probes applied to temporarily alter the plasma momentum profile. Since this is an invasive technique, it can only be applied to plasmas with a modest energy-density. In general, the lack of experimental measurements of the kinematic viscosity is a problem for, e.g., validation of visco-resistive magnetohydrodynamical (MHD) codes. This further motivates the present study; the modeling of TM braking due to an RMP might be used (as a non-invasive technique) to provide an estimate of the experimental kinematic viscosity in a wide range of fusion plasmas (those with rotating tearing modes).

The paper is organized as follows: In Section II, MST, the feedback system and the diagnostics are presented. Section III describes the equation of fluid motion, TM phase evolution, and the Rutherford equation modified to include the RMP effect, which together models the TM dynamics. Section IV presents the experimental TM data measured in MST and a comparison with the model of the TM time evolution. Section V contains discussion and Section VI gives conclusion. The Appendix presents a brief derivation and explanation of the model equations.

## II. MST DEVICE AND DIAGNOSTICS

The experimental data in this work were measured in the MST reversed-field pinch. Essential for this study is the magnetic feedback system, which is used to perturb the plasma. The resonant part of the perturbation interacts with the tearing modes, whose dynamics are measured by poloidal magnetic field pick-up coils.
MST has a thick toroidal aluminum shell that has the role of both vacuum vessel and conducting shell. Some of its properties are listed in Table I.

The shell time constant, $\tau_b = \mu_0 \sigma_b \delta_b r_b \approx 0.8$ s, is about ten times longer than the typical discharge duration, where $\mu_0 = 4\pi \times 10^{-7}$ N/A². Thus, the amplitudes of the rotating tearing modes are negligible outside the shell. This motivates the use of the thick-shell approximation in the modeling (Section III). Though the shell is mainly uniform, it has port-holes and two 1.3 cm-wide gaps. One gap extends toroidally, while the other extends poloidally. Their main function is to let the corresponding component of the magnetic flux enter the vessel. At the poloidal gap, the radial component of the error field is measured and corrected for by magnetic coils in a feedback system (32 sensors and 38 actuators).

The feedback system was used to apply magnetic perturbations with poloidal harmonics $m = 1$ and several toroidal $n$. The $m = 1$ radial component of the applied field is measured by the sensors, but the $n$ spectrum cannot be resolved due to the coil geometry. However, the poloidal gap is too small compared to the torus circumference that it is basically a spatial delta function in toroidal angle. Calculations that consider the geometry of the RMP-coils result in a relatively broad $n$ spectrum, where the amplitude of each harmonic ($m = 1, n$) is approximately the same for all $n$ of interest (in this study $n = 6 - 15$). The calculation method is quite rough and does not consider the exact MST geometry. Therefore, the relative contribution to the amplitude of each $n$ was estimated by a combination of vacuum measurements and calculation of the Newcomb solution in vacuum. MST is equipped with a toroidal array of magnetic field pick-up coils, which is located at the shell inner surface, $r = r_0$. The coil array comprises 32 coil pairs. One coil in each pair measures the time derivative of the poloidal field ($b_0$), and the other measures the time derivative of the toroidal field ($b_\phi$). Using these coils, the poloidal $m = 1$ and toroidal $n$ spectrum can be resolved for the $b_0$ and $b_\phi$ field components. The measured contribution to each $b_0 = l = 1$, $n$ was approximately 1/100 of the total applied $m = 1$ RMP amplitude. The solution to Newcomb’s equation provides the relative amplitude of the poloidal, toroidal, and radial components of the perturbed magnetic field. This theoretical result is combined with the measurements to estimate the radial component $b_\phi = l = 1$, $n$ of the applied RMP-field. Comparing with the total radial component of the $m = 1$ RMP amplitude, it is found that the relative contribution to each toroidal harmonic $n$ is $b_\phi = l = 1$, $n \approx 0.014 \times b_\phi = l = 1$. When comparing the experimental tearing mode data with the model described in Section III, this estimate is used to approximate the applied perturbation amplitude for each $n$. Since the RMP is applied through a gap, its interaction with the shell can be neglected in the modeling.

During the RMP experiments, tearing mode dynamics are measured by the array of poloidal magnetic field pick-up coils, described above. The perturbed magnetic field of a single $(m, n)$ mode can be expressed as

$$ b(r, t) = b_0^{m,n}(r) \exp[i(m\theta - n\phi + x_0^{m,n}(t))], $$

where $x_0^{m,n}(t) = -\int_0^t \omega_0^{m,n}(t')dt'$ describes the time variation of the phase. The angular phase velocity can be divided into the poloidal and toroidal components

$$ \omega_0^{m,n}(t) = m\omega_0^{m,n}(t) - n\omega_0^{m,n}(t). $$

In the MST core, the rotation is primarily toroidal and therefore, it is the only velocity component considered in the present paper. By neglecting the poloidal rotation, the TM angular velocity is approximately

$$ \omega_0^{m,n}(t) \approx -n\omega_0^{m,n} = -\frac{v_{\phi}^{m,n}}{R}, $$

where $v_{\phi}^{m,n}$ is the toroidal phase velocity of the $(m, n)$ TM and $R$ is the major radius. Measurements of toroidal flow velocity of Carbon V ions are in approximate agreement with the measured toroidal TM velocity $v_{\phi}^{m,n}$. Therefore, co-rotation of the TM islands and plasma is assumed at each corresponding resonant radius ($r^{m,n}_0$). This is often called the no-slip condition. Due to the many resonant tearing modes inside the MST plasma, the TM velocity measurements provide an estimation of the plasma velocity radial profile. Thus, assuming that the no-slip condition is valid, the plasma toroidal angular velocity at the resonant surface is

$$ \Omega_\phi(r^{m,n}_0, t) = \frac{v_{\phi}^{m,n}}{R}. $$

The plasma toroidal angular velocity reduction due to the RMP is $\Delta \Omega_\phi(r, t) = \Omega_\phi(r, t) - \Omega_\phi(r, t_0)$, where $\Omega_\phi(r, t_0)$ is the initial (or natural) rotation at a time ($t_0$) before the application of the RMP. The measured velocity reduction will be compared with predictions by the theoretical model, described in Sec. III.

### III. MODELING OF TEARING MODE EVOLUTION

This section describes the TM island time evolution model, which is compared with experimental TM measurements in Section IV. It is based on a theoretical model that describes the mode’s braking due to electromagnetic (EM) interaction with the thick shell in MST. The original model is extended by including the EM interaction between RMPs and multiple TMs with harmonics $(m = 1, n)$. The RMP effect on the TM amplitude is also included through the modified Rutherford equation. The Appendix describes the details of the adaptation to the MST’s thick shell and RMP field at the poloidal gap.

As described in Section II, the plasma and TM rotation in the MST core are mainly in the toroidal direction. Furthermore, the ratio of toroidal to poloidal EM torques is $T_{EM,\phi}/T_{EM,0} = -n/m$. This implies that for the inner TM

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**TABLE I. MST shell parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius, $R_0$</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Plasma minor radius, $a$</td>
<td>0.51 m</td>
</tr>
<tr>
<td>Shell inner radius, $r_b$</td>
<td>0.52 m</td>
</tr>
<tr>
<td>Shell thickness, $\delta_b$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>Shell conductivity, $\sigma_b$</td>
<td>$2.5 \times 10^7$ $\Omega^{-1}$ m$^{-1}$</td>
</tr>
</tbody>
</table>
\(m = 1, n = 6\), the magnitude of the toroidal torque component is six times larger than the poloidal one. For these two reasons, only the toroidal component is included in the model and in comparison with the experimental results.

It is assumed that each TM rotates together with the plasma at the corresponding resonant radius \(r_n^m\).\(^{12}\) Hence, when the TM velocity is changed (due to, e.g., an external EM torque), the fluid velocity at \(r_n^m\) will be changed accordingly. The surrounding plasma tends to oppose this change through a viscous torque. In the present model, the external torques are EM torques \((R \times J \times B)\) caused by the TMs interaction with a multi-harmonic RMP and with eddy currents induced in the shell by the rotating TMs.

### A. Model equations

The model describes the TM island evolution through three coupled equations: (I) the equation of fluid motion including the EM torques,\(^{12,20,33,36,44,45}\) (II) the no-slip condition\(^{12,19,46}\) that ensures the co-rotation of the fluid and TM island, and (III) the Rutherford equation (TM amplitude time evolution) modified to include the effect of the RMP\(^{34,47}\) and thick shell\(^{33,36}\).

\[
\frac{\rho(r) \partial \Delta \Omega_{\phi}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu_{\phi} \frac{\partial \Delta \Omega_{\phi}}{\partial r} \right) + \sum_n \frac{\tau_{EM}(t)}{4 \pi^2 \tau^3_0} \delta(r - r_n^m), \tag{5}
\]

\[
\frac{d\varepsilon_{m,n}}{dt} = n [\Omega_{\phi}(r_n^m, t_0) + \Delta \Omega_{\phi}(r_n^m, t)], \tag{6}
\]

\[
\frac{d[\Psi_{s,m}^n(t)]}{dt} = \frac{C_{s,m}^n}{\Delta c_{s,m}} \left[ F_{s,m}^n + D_{s,m}^n(t) \right] \sqrt{\Psi_{s,m}^n} + \frac{[\Psi_{s,m}^n]}{\sqrt{\Psi_{s,m}^n}} \left[ F_{s,m}^n \cos(\Delta \varepsilon_{m,n}(t)) \right] - \frac{\Lambda}{G_{s,m}^n} (\varepsilon_{s,m}^n)^2 \ln \left[ \frac{C_{s,m}^n}{\sqrt{\Psi_{s,m}^n}} \right]. \tag{7}
\]

The equation of fluid motion (5) is discretized in space using method-of-lines, which allows solving the full system (5)–(7) as a set of coupled ordinary differential equations. The number of equations depends on the number of modes \((m, n)\) included. In this paper, the poloidal mode number is \(m = 1\) and the number of toroidal modes \((n)\) is varied from one to eight. This means (6) and (7) constitute \(n\) equations each, i.e., the phase and amplitude evolution of each included TM. The complex parameter \(\Psi_{s,m}^n\) determines TM amplitude and phase at the resonant surface. The RMP amplitude and phase are determined by \(\Psi_{c,m}^n\), as described in the Appendix.

Equation (5) describes how the plasma inertia (left hand side) is balanced by the viscous torque (1st term right hand side (rhs)) and EM torques (2nd term rhs). On the left hand side, \(\rho(r)\) is the plasma mass density and \(\Delta \Omega_{\phi}(r, t)\) is the plasma toroidal angular velocity reduction profile, defined in Section II. The viscous torque is proportional to the second derivative of \(\Delta \Omega_{\phi}(r, t)\) and the perpendicular dynamic viscosity \(\mu_{\phi}(r)\). Dynamic viscosity is related to the kinematic viscosity \(\nu_{\phi}(r)\) through \(\mu_{\phi}(r) = \rho(r) \nu_{\phi}(r)\). The delta function indicates that the EM torque acts locally at the resonant surface of each TM. The EM torque is divided into one contribution from the RMP \((T_{RMP})\) and one from the wall \((T_{wall})\), which are calculated from basic principles in the Appendix. The torque due to the RMP is proportional to the TM amplitude, RMP amplitude, and negative sine of the relative phase difference between the RMP and TM:

\[T_{RMP} \propto -|\Psi_{s,m}^n| |\Psi_{c,m}^n| \sin[\Delta \varepsilon_{m,n}(t)],\]

where \(|\Psi_{s,m}^n|\) and \(|\Psi_{c,m}^n|\) are proportional to the TM and RMP amplitude, respectively. In the experiments, the RMP phase is constant (zero radians) and the TM phase \((\varepsilon_{m,n})\) is changing in accordance with the TM rotation, i.e., \(\Delta \varepsilon_{m,n}(t) = \varepsilon_{m,n}(t) - \Delta \varepsilon_{RMP} = \varepsilon_{m,n}(t)\). The sign of \(T_{RMP}\) changes due to \(-\sin[\Delta \varepsilon_{m,n}(t)]\) dependence, and the RMP produces oscillations in the velocity of a rotating TM. However, on average, the TM tends to stay longer in the relative phase where TM braking is produced. The wall torque on the other hand is always braking the TM. However, in the present RMP experiments, \(T_{RMP}\) is significantly larger than \(T_{wall}\). This is manifested by the experiments, where the TMs are rotating unhindered or at least with no clear braking when the RMP is not applied.

In the no-slip condition (6), the phase is related to the TM angular velocity via \(d\varepsilon_{m,n}/dt = -\Omega_{m,n}(t) \approx \nu_{m,n}(t)\), as described in Section II. The form in (6) assumes negligible poloidal rotation, as described in Equations (1)–(4).

In Rutherford equation (7), the RMP produces oscillations of the TM amplitude according to the cosine term \((\cos[\Delta \varepsilon_{m,n}(t)]\)) and the other terms are described in the Appendix.

### B. Model input parameters

Input parameters to the model are the measured plasma equilibrium, the MST machine dimensions, initial amplitude and velocity of each tearing mode, and the applied RMP amplitude.

It is assumed that the plasma density profile is flat in the central region with the following radial profile, \(\rho = \rho_0(1 - r/a)^3\). The value of \(\rho_0\) is estimated from the experimental interferometer measurements for each shot. It is assumed that the dynamic viscosity has no radial variation \(\mu_{\phi} = \rho \nu_{\phi} \). The kinematic viscosity \(\nu_{\phi}\) is chosen to match the locking time of the experimental \(n = 6\) TM velocity \((\nu_{\phi,6} = 0\). The model fit to the whole braking curve (time evolution) will show if the model can qualitatively describe the observed TM dynamics. In case of a good fit, the model-required kinematic viscosity might be a good estimation of the experimental value in MST. This will be further discussed in Section V.

The parallel current profile \(\sigma(r)\), used for solving Newcomb’s equation (A5), is calculated by the MSTFit code\(^{48}\) using the experimentally measured data. The safety factor, \(q(r)\), is also obtained from a fit\(^{48}\) to experimental measurements. Pressure is included in the calculation of \(q(r)\),
which proved necessary to find the $n = 6$ TM resonant inside the plasma. The resistivity used in the model is $\eta = 10^{-7}$ $\Omega$ m in agreement with the experimental MSTFit value. The corresponding resistive time (Equation (A23)) is then on the order of $0.1 - 1$s, depending on the location of the resonant radius.

The applied perturbation amplitude ($b_r^{m=1}$) is read into the model from the experimental measurement at the poloidal gap. The RMP coils are positioned outside the gap at minor radius $r \approx 0.636$ m. The sensing coils, measuring the radial perturbation at the gap, are positioned at the inner shell surface $r = r_b \approx 0.52$ m. Since the shell diffusion time is long compared to the shot and RMP duration, the perturbation enters mainly through the poloidal gap. The gap affects the toroidal $n$ spectrum inside the machine, since the gap toroidal extent ($\approx 1$ cm) is smaller than the width of the RMP coils. For the above reasons, it is assumed that the RMP is applied with coils of the same width as the gap, positioned just outside the sensing coils at $r_c \approx 0.53$ m. This assumption will affect the value of the parameter $E_w$ (Equation (A11)). A change in $r_c$ of 1 cm changes the value of $E_w$ less than 3%, and an increase of $r_c$ by 10 cm increases $E_w$ about 13%. Uncertainty in how the RMP field enters the machine, together with uncertainties in other model input, leads to an uncertainty of model-prediction of the (free parameter) kinetic viscosity. However, the main conclusions of the paper will not be affected.

The initial conditions for the model are $\Delta \Omega_0 (r,t = 0) = 0$, $\Delta x^n(0) = 0$, and the initial TM amplitude is similar to the experimental one. The boundary conditions for the toroidal angular velocity reduction are $\left|d\Delta \Omega_0 (r,t)/dr\right|_{r = 0} = 0$ and $\Delta \Omega_0 (r = a,t) = 0$.

For comparison with the experimental measurements, the complex parameter that determines TM amplitude and phase at the resonant surface ($\Psi^{m,n}_s$) is related to the measured signals at the shell ($b_0(r_b)$) using Equations (A2) and (A9) and Equations (64) and (65) in Ref. [36]

$$\Psi^{m,n}_s = -ib_0 \frac{m^2 + n^2 \epsilon^2}{m} \frac{r}{E_{bs}} \bigg|_{r = r_b},$$

where $\epsilon = r/R_0$ and $E_{bs}^{m,n}$ is defined in Equation (A11). The formula above was also used in the TM amplitudes as model-input instead of using the Rutherford equation, as shown later.

### IV. TEARING MODE DYNAMICS IN MST—COMPARISON WITH MODEL

This section presents the experimental results of TM dynamics under the influence of a multi-harmonic RMP. The TM braking dependence on plasma density is also investigated. Experimental results are compared with the TM evolution model described in Section III.

Discharges analyzed are multiple helicity RFP plasmas with typical experimental plasma parameters as in Table II. The discharges were fueled with deuterium ($D_2$). The toroidal magnetic field reversal at the edge is relatively high, as quantified by the reversal parameter $F = B_\phi (a)/\langle B_\phi \rangle \approx -0.3$, where $B_\phi (a)$ is the toroidal magnetic field at the plasma edge and $\langle B_\phi \rangle$ is the poloidal-cross-section average. The pinch parameter is defined as $\Theta = B_\phi (a)/\langle B_\phi \rangle$, where $B_\phi (a)$ is the poloidal magnetic field at the edge. The central TM is the harmonic $m = 1$, $n = 6$.

Two types of waveforms were used for the RMP: a ramp and a square. The RMP was applied during the flat-top phase of the plasma current ($\approx 20$ ms into the discharge), and its maximum amplitude was varied from shot-to-shot.

In MST, sawtooth crashes occur periodically in which the central rotation drops significantly within 100 ms.[37] During this short phase, there is transfer of momentum due to the coupling between the central $m = 1$ TMs and the $m = 0$ TMs resonant at the reversal surface, located in the plasma edge. To avoid the effects of the crash, shots that exhibit a sawtooth crash during the RMP phase were not included in the comparison with the model.

### A. Time evolution—Single shot

The effect of RMPs on the $(m = 1, n = 6)$ TM in MST is described in Figure 1. Figure 1(a) shows the waveform of the applied $m = 1$ perturbation, and Figures 1(b)–1(e) show the corresponding behavior of the TM dynamics. The magnetic perturbation is ramped up to 130 G within 6 ms. When the perturbation has reached an amplitude of about 80 G, a clear braking of the TM is observed (at $t \approx 22$ ms in 1(c)). The braking increases significantly when the perturbation amplitude has reached about 125 G ($t \approx 23.5$ s) and at $t \approx 24$ ms wall-locking occurs. After the RMP is ramped down to zero, the TM remains locked. This is probably due to machine error fields, which can sustain the locked-state because the plasma viscous torque is reduced (due to the lower plasma rotation surrounding the mode). The TM amplitude (Figure 1(b)) fluctuates in the interval $4 - 10$ G. The fluctuations are correlated to fluctuations in the plasma equilibrium ($F$) and are not related to the RMP. The EM torque (A12) is proportional to the TM amplitude, so the fluctuations should affect the TM velocity evolution according to theory. However, no clear effect is visible in Figure 1(c). This possible effect will be further investigated when examining other TMs ($n > 6$) and in the comparison with the model. The TM amplitude does not increase significantly after wall-locking, contrary to, for example, EXTRAP TZR.[26] This is probably related to the long shell time constant in MST that prevents full penetration of the shell. Finally, in Figures 1(d) and 1(e) are the short time scale behavior of the TM velocity ($\nu^{m=1,n=6}_s$) and the TM-RMP relative phase ($\Delta \phi^{m=1,n=6}_s$), respectively. It is zoomed in on the time interval
Thus, the TM is decelerated in the phase interval between the TM and RMP, i.e., $\Delta T_{\text{RMP}}$ is proportional to the negative sine of the phase difference $\Delta \alpha$. Theoretically, this is explained by the fact that the EM torque on one mode ($D$) related to the fact that $\Delta \alpha$ tends to spend more time between $0 < \Delta \alpha < 1$ than between $-1 < \Delta \alpha < 0$. Theoretically, this is explained by the fact that the EM torque is proportional to the negative sine of the phase difference between the TM and RMP, i.e., $T_{\text{RMP}} \propto -\sin[\Delta \alpha(t)]$. Thus, the TM is decelerated in the phase interval ($0 < \Delta \alpha < 1$) and accelerated in the interval ($-1 < \Delta \alpha < 0$), which means that the minimum TM velocity should be reached at $\Delta \alpha = \pi$ rad and the maximum at $\Delta \alpha = 0$ rad. The picture is slightly more complicated due to the viscous torque that tends to oppose the change in TM velocity ($v_{\phi}^{1,6}$). This will slightly change the angles at which the minimum/maximum $v_{\phi}^{1,6}$ occurs. The evolution of the experimentally measured phase is in approximate agreement with the theoretical prediction. Furthermore, the experimental locking occurs in phase with the RMP, as predicted by the theoretical model.19

For the time intervals indicated by gray areas in Figure 1, the time-averaged velocities of the ten most central TMs ($m = 1, n = 6, 7, ... , 15$) are plotted at each corresponding resonant radius in Figure 2(a). In the braking time interval ($t_3$), all TMs have a reduced velocity compared to the initial time interval ($t_1$). The reduction is similar for all TMs, except the outer TMs that are already locked. At the time interval of the wall-locking of the $n = 6$ TM, $t = t_3$, all modes are wall-locked. Wall-locking starts from the outer modes and goes inward until the inner mode is locked. The large (1σ) error bars outside $r/a \approx 0.5$ are due to the low amplitudes (low signal-to-noise ratio) of the $n > 9$ tearing modes. The main model-experiment comparison will be done for the $n = 6$ mode, since it has the highest amplitude and signal-to-noise ratio. Figure 2(b) shows the velocity reduction compared to the initial rotation for each mode, which will be compared with modeled velocity reduction profiles.

The model described in Section III is used to simulate the TM evolution of the plasma shot (1150310065) described in Figure 2. The experimental data are from shot 1150310065. The modeled data are calculated by solving Equations (5)–(7), including the EM torque on one mode ($D$) as used as model-input), (b) TM amplitude, and (c) TM velocity. In frames (b) and (c), data are smoothed in a 0.5 ms time window. Frame (d) is the unsmoothed TM velocity during the braking phase and (e) is the phase difference between TM and RMP. The blue lines are experimental TM data and the magenta dotted-dashed lines are modeled TM data. The gray areas indicate time intervals $t_1$, $t_2$, and $t_3$, which are used for the $v_{\phi}^{1,6}$ and $\Delta v_{\phi}^{1,6}$ radial profiles in Figure 2. The experimental data are from shot 1150310065.

The time evolution of (a) experimental perturbation amplitude (which is also used as model-input), (b) TM amplitude, and (c) TM velocity. In frames (d) and (e), data are smoothed in a 0.5 ms time window. Frame (d) is the unsmoothed TM velocity during the braking phase and (e) is the phase difference between TM and RMP. The blue lines are experimental TM data and the magenta dotted-dashed lines are modeled TM data. The gray areas indicate time intervals $t_1$, $t_2$, and $t_3$, which are used for the $v_{\phi}^{1,6}$ and $\Delta v_{\phi}^{1,6}$ radial profiles in Figure 2. The experimental data are from shot 1150310065. The modeled data are calculated by solving Equations (5)–(7), including the EM torque on one mode ($n = 6$).
above. The measured central line-averaged electron density is \( n_e \approx (0.65 \pm 0.04) \times 10^{19} \text{ m}^{-3} \) (average and standard deviation in the time window of the RMP application, 18.0 – 25.0 ms). The resulting density in the center is \( n_e \approx (0.85 \pm 0.05) \times 10^{19} \text{ m}^{-3} \) for the assumed profile. Solving Newcomb’s equation (Appendix) and Equation (A11) gives the input-parameters for the model (Table III). The viscosity has been chosen to have the best match between experimental and modeled locking time, as discussed in Section III B.

At first, only the torque acting on the most central mode \((n = 6)\) is included in the model. The experimental and modeled \(m = 1\) RMP amplitude is represented by the same signal in Figure 1(a). The corresponding experimental and modeled TM evolution is shown in Figures 1(b)–1(e). In frame (b) and (c), the modeled and experimental data are time-averaged in a 0.5 ms time-window. The modeled TM amplitude (dashed line in Figure 1(b)) is almost constant in the time interval. Whereas, the smoothed experimental signal (blue line) shows oscillations, on a longer time scale than the inverse TM rotation frequency, which are not predictable with the model (i.e., the Rutherford equation). As already mentioned, these experimental oscillations coincide with small fluctuations in \( \Theta \) and \( F \), much smaller than the fluctuations during a sawtooth crash. The TM velocity, Figure 1(c), is well reproduced by the model using a kinematic viscosity \( \nu_{\text{kin}} = 9.4 \pm 1.0 \text{ m}^2/\text{s} \), where the error estimation only considers the \( (1\sigma) \) standard deviation in the measured density. The short time scale behavior of the TM velocity, Figure 1(d), agrees quite well with the experiment. The TM velocity is the derivative of the TM-RMP relative phase \( \Delta \Phi^{1-6,m-n} \) (Figure 1(e)). During the braking phase, the RMP modulates \( \Delta \Phi^{1-6} \) in a similar way to the experimentally measured phase modulation. At \( t \approx 24 \) ms, the predicted \( \Delta \Phi \) is included in the model. The phase-locking also occurs around \( \Delta \Phi^{1-6} \approx 0 \), but then the phase drifts towards \( \Delta \Phi^{1-6} \approx 0.3 \).

In general, the theory states that the locking always occurs in a 0.5 ms time-window. The modeled TM amplitudes are calculated by solving Equations (5)–(7), including the EM torque on one mode \((n = 6)\). The shaded areas show the \( 1\sigma \) standard deviation of the modeled velocity reduction, within each time window. During the braking phase \((t_z)\), the relatively large standard deviation at the resonant surface is a consequence of the acceleration and deceleration produced by the RMP–TM interaction (see Figure 1(d)). The model underestimates the braking of the outer tearing modes \((n > 6)\), both for the velocity reduction during the braking \((v_\alpha(t_z) - v_\alpha(t_1))\) and after wall-locking \((v_\alpha(t_1) - v_\alpha(t_i))\). A likely explanation is that in the experiment there is an EM torque on each mode, due to the multi-harmonic RMP. Whereas in this case, the model only includes the \( n = 6 \) RMP torque.

To examine the importance of the outer tearing modes, the model was run including the RMP torque on eight modes, \( n = 6, 7, \ldots, 13 \), in Equation (5). For each TM, the corresponding no-slip condition \((6)\) and Rutherford equation \((7)\) were solved to model simultaneously the velocity, phase, and amplitude of all the considered TMs. Figure 4 shows a comparison of simulated TM time evolution with experimental data (same shot as in Figures 1–3) for the three most central modes \((n = 6, 7, \text{ and } 8)\). Frame (a) compares experimental mode amplitudes with those modeled by the Rutherford equation. It is obvious that the Rutherford equation misses the fluctuations in mode amplitude. However, the average experimental TM amplitude is only slightly higher than the modeled one. The modeled TM velocities are plotted against the experimental for two cases; in Figure 4(b), the Rutherford equation was used to model the TM amplitude, and in Figure 4(c), the experimentally measured TM amplitude was used as model input (see Equation (8)). Both model cases replicate the \( n = 6 \) experimental TM velocity quite well since the viscosity is changed to obtain a good fit. However, the model-required kinematic viscosities were similar in both cases: \( \nu_{\text{kin}} = 11.5 \pm 1.2 \text{ m}^2/\text{s} \) in Figure 4(b).

![FIG. 3. Radial profiles of the modeled plasma velocity reduction and the experimental velocity reduction of each TM. Velocity reductions are calculated at time intervals \( t_z \) and \( t_i \) compared to the initial velocity \( v_\alpha \). Time intervals are indicated in Figure 1. The shaded areas show the standard deviation \((1\sigma)\) of the simulated velocity reduction, within each time window. Experimental data are from shot 1150310065. The modeled data are calculated by solving Equations (5)–(7), including the EM torque on one mode \((n = 6)\). Inside the \( n = 6 \) resonant surface \((r/a < 0.3)\), profiles are dependent on the viscosity and on the boundary condition \([\Delta v_\alpha(r,t)/dr]_{r=0} = 0\).](image)

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<th>( n = 8 )</th>
<th>( n = 9 )</th>
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and \( \nu_{\text{kin}} = 7.0 \pm 1.0 \text{ m}^2/\text{s} \) in Figure 4(c). Comparing the other mode velocities \( n = 7 \) and \( n = 8 \) in Figure 4, the agreement is better using the experimental mode amplitude. Hence, fluctuations in mode amplitude observed in the experiment are of importance for the TM velocity evolution.

In Figure 5, the modeled plasma velocity reduction profiles for the “experimental TM amplitude case” are compared with the experimental velocity reduction of TMs \( n = 6, 7, \ldots, 15 \) (Figure 2(b)). The large error bars of the experimental TM velocity outside \( r/a \approx 0.5 \) \( (n > 9) \) are a consequence of the relatively low TM amplitudes (low signal-to-noise ratio). Therefore, the comparison inside \( r/a \approx 0.5 \) is more relevant. In the braking phase \( (t_2) \), the modeled standard deviation shows that the EM torques produce large fluctuations over a broad plasma region, in contrast to the more local interaction in the single mode case (Figure 3). Compared to the model with one mode, the agreement with the experimental velocity reduction (radial profile) is better in the case of eight modes. Hence, it is important to include the EM torque on many modes to describe the deceleration of the TMs and plasma.

FIG. 4. Experimental and modeled TM data for modes \( n = 6, 7, \) and \( 8, \) in blue, green, and red color, respectively. (a) The amplitude of each TM, where \( n = 6 \) (blue) has the highest amplitude and \( n = 8 \) (red) has the lowest. The modeled TM velocities are shown for two cases: in (b), the modeled amplitude (Equation (7)) of each TM was used, and in (c), the experimental TM amplitudes were used as model input applying Equation (8). The model includes the EM torque on eight modes \( (n = 6, 7, 8, \ldots, 13) \) in Equation (5), the no-slip condition (6) for each TM and the amplitude of each TM (Equation (7)) in (b) and Equation (8) in (c). The RMP amplitude is the same as in Figure 1. Experimental data are from shot 1150310065.

FIG. 5. Modeled plasma velocity reduction radial profile and experimental TM velocity reduction. Velocity reduction is calculated at time intervals \( t_2 \) and \( t_3 \) compared to the initial velocity \( (t_1) \). Time intervals are indicated in Figures 1 and 4. The shaded areas show the standard deviation (1\( \sigma \)) of the simulated velocity reduction. Experimental data are from shot 1150310065. The modeled data (same simulation as in Figure 4(c)) are calculated by solving Equations (5), (6), and (8), including the EM torque on eight modes \( (n = 6–13) \). Inside the \( n = 6 \) resonant surface \( (r/a < 0.3) \), profiles are dependent on the viscosity and on the boundary condition \( [\Delta \nu_{\text{EM}}(r,t)/dr]_{r=0} = 0 \).

In the cases modeled with the Rutherford equation, the required kinematic viscosity was higher when 8 TMs are included \( (\nu_{\text{kin}} = 11.5 \pm 1.2 \text{ m}^2/\text{s}) \) than in the case of 1 TM \( (\nu_{\text{kin}} = 9.4 \pm 1.0 \text{ m}^2/\text{s}) \). This is due to the higher total (braking) torque when more modes are included, which means a higher (restoring) viscous torque is needed to keep the balance. The modeled viscosity dependence on the number of modes included is shown in Figure 6. There is a relatively strong braking at the \( n = 6 \) resonant surface due to the viscous coupling to the neighboring \( (n = 7) \) TM and a weaker dependence on more distant modes \( (n > 7) \). Hence, model-predicted kinematic viscosity for shot 1150310065 is \( \nu_{\text{kin}} = 11.5 \pm 1.2 \text{ m}^2/\text{s} \). However, because of the viscous

FIG. 6. Model-required kinematic viscosity for best fit to (shot 1150310065) experimental locking time (i.e., when \( v_{\text{TM}} = 0 \text{ km/s} \)) versus number of modes included in the model (Equations (5)–(7)). The dashed line shows the trend of the simulated results (blue stars). For the error bars, the lower/upper value corresponds to the best fit when the modeled plasma density is changed according to the experimentally measured standard deviation, \( n_{\text{e0}} = (0.85 \pm 0.05) \times 10^{19} \text{ m}^{-3} \).
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coupling between TMs, the effect of viscosity is not trivial when RMPs are acting on multiple TMs. On one hand, an increase of viscosity leads to an increase in viscous restoring torque locally (at one TM). But on the other hand, it leads to a higher viscous transfer of braking that has occurred at adjacent TMs, which the RMP is also acting on. Depending on which of the two processes dominates, the effect of a higher viscosity can lead to faster or slower braking of the individual TM. In the case where the torques on several TMs are included, the \( n = 6 \) viscous coupling to other TMs causes the \( n = 6 \) TM velocity to brake faster. To match the experimental \( n = 6 \) TM braking curve, the viscosity had to be increased, which increases the local viscous restoring torque. Thus, in the present case the local effect of viscosity dominates the slowing-down time. The opposite result might be expected if braking is higher at adjacent modes, for example, if the \( n = 7 \) RMP amplitude (EM torque) would be significantly higher than the \( n = 6 \) RMP amplitude (EM torque).

B. Tearing mode wall-locking for different plasma densities

In the equation of fluid motion (5), the amount of braking of the plasma velocity that an electromagnetic torque produces is dependent on the plasma inertia, thereby the plasma density. However, a change in density might change other plasma parameters, such as the kinematic viscosity, that affect the braking. To investigate the plasma and TM braking dependence on plasma density, the experiment was run with a different plasma density from shot-to-shot and the results were compared with the model, assuming a constant (density-independent) kinematic viscosity.

Several shots with similar equilibria and natural (pre-RMP) TM rotation velocities were analyzed. In each shot, a square waveform was used for the RMP and the amplitude was stepped up to a constant (pre-set) value. The amplitude was changed from shot-to-shot to find the locking threshold (i.e., the amplitude required to lock the \( n = 6 \) TM within a certain time). Figure 7 shows tearing mode velocity versus applied \( (n = 1) \) perturbation amplitude for a number of shots. The shots are divided into three groups, depending on the electron density around the time the RMP is applied. The groups are low density \( \langle n_e \rangle = 0.35 - 0.5 \times 10^{19} \) m\(^{-3}\) (red circles), middle density \( \langle n_e \rangle = 0.5 - 0.8 \times 10^{19} \) m\(^{-3}\) (blue squares), and high density \( \langle n_e \rangle = 0.8 - 1.25 \times 10^{19} \) m\(^{-3}\) (green diamonds). The \( n = 6 \) TM velocity is averaged between 2.00 and 2.25 ms into the RMP phase. The experimental data in Figure 7 indicate that the locking threshold increases with increasing electron density. This is not a surprise, considering the higher inertia for the high density cases. However, it might be possible that other plasma properties, important for the braking mechanism, are changed due to the different density. Therefore, it is interesting to vary the density in the model (keeping the other parameters constant) and compare with the experimental measurements.

First, the model was applied to a discharge (shot 115030048) in the middle density range, with a measured central line-averaged density \( \langle n_e \rangle \approx (0.57 \pm 0.03) \times 10^{19} \) m\(^{-3}\). This results in the central density \( n_{io} \approx (0.76 \pm 0.04) \times 10^{19} \) m\(^{-3}\) for the assumed profile. The parameters \( E_{\phi} \) required by the model are calculated from the Newcomb solution (A1)–(A11) with input from the measured equilibrium signals (in the same way as for shot 1150310065). The Rutherford equation was used to model the amplitude of each TM. The model-required kinematic viscosity was determined for two different cost functions. At first, the agreement with the experimental \( (n = 6) \) locking was optimized, i.e., a fit to \( v_{\phi}^0 = 0 \) (similar to Figure 1). This resulted in \( v_{kin} = 17.0 \pm 1.5 \) m\(^2\)/s, where the error bars consider the (1\( \sigma \)) standard deviation in the measured \( \langle n_e \rangle \). The locking occurs 3.50 ms into the RMP phase. However, in Figure 7, an earlier time interval is considered (2.00 – 2.25 ms into the RMP phase). Therefore, a second estimation of the viscosity was made to match the braking in this interval. The resulting value is \( v_{kin} = 20.0 \pm 1.5 \) m\(^2\)/s. The match to the locking time produces a slightly better fit with the whole experimental time evolution (since it matches the large velocity fluctuations prior to locking). The two estimates do not overlap, but note that the estimated error bars only consider the uncertainty in the density. In the following simulations, we have assumed a value in-between the two estimates: \( v_{kin} = 18.5 \) m\(^2\)/s. (The model-prediction for shot 1150300948 is higher than the value for shot 1150310065 in Figure 6; \( v_{kin} = 11.5 \pm 1.2 \) m\(^2\)/s). Possible reasons for this will be addressed in Sec. V.

For the three groups of discharges in Figure 7, the modeled density was set to: \( \langle n_e \rangle = 0.41 \times 10^{19} \) m\(^{-3}\) (red curve), \( \langle n_e \rangle = 0.64 \times 10^{19} \) m\(^{-3}\) (blue curve), and \( \langle n_e \rangle = 0.90 \times 10^{19} \) m\(^{-3}\) (green curve). The modeled RMP amplitude was varied by a constant factor in each simulation and \( v_{\phi}^0 \) is averaged between 2.00 and 2.25 ms into the perturbation phase, the same as for the experimental data. This results in
the braking curves in Figure 7. Comparing with experimental data, the model can produce the same trend of increasing TM locking threshold with increasing density. The modeled TM velocity brakes approximately an equal amount as in the experimental discharges for each density group, respectively. The locking occurs when \( b_r(a) / B_0(a) \approx 7\% \) for low \( \langle n_e \rangle \) and \( b_r(a) / B_0(a) \approx 12\% \) for middle \( \langle n_e \rangle \). For the high density discharges, the maximum applied RMP amplitude was too low to lock the TM within 2.0–2.25 ms. However, the modeled braking curve (for high \( \langle n_e \rangle \)) shows modest braking at \( b_r(a) / B_0(a) \approx 11\% \), which is consistent with the experimental TM braking. The above locking-thresholds \( b_r(a) / B_0(a) \approx 7\% – 12\% \) consider the total \( m = 1 \) perturbation amplitude. According to Section II, the \( n = 6 \) fraction is about 1.4\% of the total amplitude, i.e., \( b_r^6(a) / B_0(a) \approx 0.10\% – 0.17\% \). The kinematic viscosity was kept constant in the model, which indicates that it is not dependent on the plasma density (since the TM braking is well reproduced by the model).

Unlike for the density, for a given natural TM rotation range, no strong trend in locking threshold was observed with different pre-RMP values of \( \Theta, F, \) or \( I_g \).

V. DISCUSSION

The island evolution equations subject to a multi-harmonic RMP show qualitative agreement with experimental tearing mode dynamics in MST. In the model, the kinematic viscosity was chosen to fit the experimental \( n = 6 \) TM velocity.

The present experiments were performed in deuterium (D\(_2\)) plasmas with \( I_p = 350 \text{ kA}, F = -0.3, \) and \( \langle n_e \rangle \approx 0.3 \times 10^{19} \text{ m}^{-3} \). The kinematic viscosity \( (\nu_{\text{kin}}) \) was estimated in two discharges (shot 1150310065 and 1150309048) that have similar equilibrium parameters. The main part of the simulations in the present paper applies the Rutherford equation to model the TM amplitudes and use the match with the \( (n = 6) \) locking time to estimate \( \nu_{\text{kin}} \). In these cases, we can compare the \( \nu_{\text{kin}} \) estimations between the two discharges. One of the main differences between the two discharges is the RMP waveform, which is a ramp in shot 1150310065 and a square in shot 1150309048. For the RMP-ramp-case, in Figures 1–6, the model-predicted kinematic viscosity is \( \nu_{\text{kin}} \approx 11.5 \pm 1.2 \text{ m}^2/\text{s} \). For the RMP-square-case, model prediction is \( \nu_{\text{kin}} \approx 17.0 \pm 1.5 \text{ m}^2/\text{s} \).

The reason for the difference in model-predicted \( \nu_{\text{kin}} \) between the two shots is not clear. A possibility is that the difference in RMP waveforms (ramp and square) could result in a difference in the plasma–wall interaction.\(^{39}\) This interaction could lead to a torque at the edge, which is not included in the model. However, the plasma braking due to the RMP is mainly in the core-region, where the dominant TMs \( (n = 6 – 10) \) are located. Therefore, the torque that arises from plasma–wall interaction should not (to a large extent) affect the torque balance between electromagnetic and viscous torques at the corresponding resonant surfaces.

Another possibility is that there is actually a shot-to-shot variation in the kinematic viscosity. The ramp waveform case (shot 1150310065) has an Enhanced Confinement (EC) period during the whole RMP phase. The EC period is characterized by, e.g., small bursts in the toroidal flux that are spaced in time.\(^{49}\) In previous EC-plasmas, the energy confinement time is roughly three times the MST standard.\(^{49}\) In other MST plasmas,\(^{39}\) the energy confinement time is comparable to the momentum confinement time \( (\tau_M) \), which is proportional to the viscous diffusion time scale \( (\tau_M \propto \tau_v \approx \gamma^2 / \nu_{\text{kin}}) \) and thereby inversely proportional to the kinematic viscosity, i.e., \( \nu_{\text{kin}} \propto 1 / \tau_M \propto 1 / \tau_E \).\(^{39}\) Hence, a higher \( \tau_E \) should correspond to a lower kinematic viscosity. The square waveform case (shot 1150309048) has a much shorter EC period that does not extend over the full RMP application. In this case, a higher (lower) average value of \( \nu_{\text{kin}} \) during the RMP phase is expected compared with the ramp case, in agreement with the model-prediction. Thus, the difference in EC periods might explain the different model-predicted kinematic viscosities for the two discharges. However, it should also be mentioned that the shots used for the density scan (Figure 7) are a mix of EC and non-EC (i.e., the RMP phase is either fully within, partly within, or outside an EC period). No strong trend is observed when the data in Figure 7 are sorted on EC/no-EC periods. However, a possible (weak) trend might be obscured due to strong density dependence and the relatively large shot-to-shot difference in density (that occurs even within each density-group).

Previously, the kinematic viscosity in MST has been estimated by observations of the slowing-down time after increasing the plasma velocity using a biased probe.\(^{39}\) The experiments were performed in standard-confinement (i.e., without EC periods)\(^{50}\) hydrogen (H\(_2\)) plasmas with \( I_p = 200 \text{ kA}, F = -0.15, \) and \( \langle n_e \rangle \approx 0.8 \times 10^{19} \text{ m}^{-3} \). The results were a core kinematic viscosity \( \nu_{\text{kin}} \approx 55 \text{ m}^2/\text{s} \) and edge kinematic viscosity \( \nu_{\text{kin}} \approx 45 \text{ m}^2/\text{s} \).\(^{39}\) The results are about two orders of magnitude higher than Braginskii\(^{51}\) classical kinematic viscosity. This suggests that the viscosity in MST is anomalous. In the core, the results by Almagri \( et\ al.\)\(^{39}\) were consistent with magnetically driven momentum transport.

Another estimation of the kinematic viscosity in MST is made by Chapman \( et\ al.\)\(^{36}\) by fitting the modeled TM braking (due to wall currents) to the experimental braking curves. The plasmas in that study included both hydrogen and deuterium plasmas with standard-confinement time. The results showed a good agreement with the biased probe experiments\(^{39}\) in the case of H\(_2\) plasmas. However, in the case of D\(_2\) plasmas, the model-required kinematic viscosity was reduced about 30\% compared to the H\(_2\) plasmas.\(^{36}\) That the kinematic viscosity is lower in the case of D\(_2\) than H\(_2\) is consistent with experimental expectation.\(^{36}\) The charge exchange of plasma ions with neutrals results in a reduction of the momentum confinement time \( (\tau_M) \). Measurements of the central neutral particle density indicate a lower value in the case of D\(_2\) compared to H\(_2\) plasmas.\(^{52}\) Thus, \( \tau_M \) is expected to be larger in D\(_2\) plasmas, i.e., a lower kinematic viscosity \( (\nu_{\text{kin}} \propto 1 / \tau_M) \). For D\(_2\) plasmas, the results showed no significant change in the model-required kinematic viscosity with a changed equilibrium \( (I_p \approx 280 \text{ kA}, F = 0.0 \) versus \( I_p \approx 380 \text{ kA}, F = -0.2) \).
In the present study (D₂ plasmas), the model-predicted kinematic viscosity is about a quarter of the experimental value for H₂ plasmas in Ref. 39. However, the difference in the value of \( \nu_{\text{kin}} \) is expected due to the difference in fuel isotopes and energy confinement regimes, as previously discussed.

The reasonable agreement with the previous experimental measurements of \( \nu_{\text{kin}} \) is encouraging. The method of modeling TM braking due to an RMP might, with more validation, be used for predictions of \( \nu_{\text{kin}} \). The kinematic viscosity is an important parameter in the visco-resistive magnetohydrodynamical (MHD) codes. It is however, in general, not well-known in fusion plasmas. In some cases, \( \nu_{\text{kin}} \) can be measured with biased probes. The probes have to be in contact with the hot plasma and can only be used in low-energy-density plasmas. Thus, development of the RMP-technique in combination with modeling can provide a useful (non-invasive) experimental measurement of \( \nu_{\text{kin}} \).

The modeling is quite complex, in the sense that it includes several inputs from the experiment and Newcomb’s equation, and each input has an uncertainty. The model-predicted \( \nu_{\text{kin}} \) might also have a systematic error due to the neglect of other torques than the resonant toroidal components. Inclusion of, for example, poloidal EM torque or neoclassical toroidal viscosity (NTV) torque might lead to a slightly higher estimation of the kinematic viscosity.

From a theoretical point of view, the magnetic perturbation (MP) should produce a reduction of plasma rotation via the NTV-torque (\( T_{\text{NTV}} \)). For non-resonant harmonics, the torque acts globally across the plasma. In contrast, for resonant harmonics, the \( T_{\text{NTV}} \) is peaked around the resonant radius. The NTV-torque magnitude, relative to the EM-torque, should be quantified in future work. However, we make two observations that suggest that the EM-torque is the dominant braking torque at the \( n = 6 \) resonant surface. First, since only the locking time is optimized in Figure 1(c), the reasonable fit with the whole experimental time evolution indicates that the experimental braking torque scales as the modeled EM-torque. Second, the agreement with the experimental anharmonic oscillations prior to the locking, Figures 1(d) and 1(e), suggests that the EM torque is important. This does not mean that the NTV-torque is negligible, but indicates that it is not large relative to the EM-torque.

The present modeling does not include the EM coupling between TMs. In previous MST experiments, this interaction is weak in periods without sawtooth crashes in MST. In contrast to the RMP–TM interaction, the EM mode-coupling only redistributes momentum between the different tearing modes. The effect is an equilibration of the mode velocities. The torque is dependent on the product of the interacting mode amplitudes. Therefore, it is expected that the most relevant coupling for this paper would be between modes \( (m, n) = (1, 6), (1, 7), \) and \( (0, 1) \). Prior to the RMP application, the \( m = 0 \) modes are either stationary or rotate in opposite direction to the central \( m = 1 \) modes. Thus, if this interaction was strong, the \( m = 1 \) modes would decelerate and the phase velocities of the \( (1, 6) \) and \( (1, 7) \) mode would equate to produce a flat velocity profile. This is in contrast to the sheared velocity profiles observed experimentally (Figure 2(a)). During the RMP application, the profile remains sheared during the braking phase, until the locking time. This suggests that the electromagnetic torque due to EM-coupling (\( T_{\text{VM-}w} \)) is small relative to \( T_{\text{RMP}} \). Previously, the mode-locking due to a resonant error-field and mode-coupling has been theoretically investigated in MST. In that theoretical model it was assumed that the core modes are phase locked and move together as a rotor and that the \( m = 0 \) modes are stationary (wall-locked), which is not exactly the case in the present experiment. However, as previously discussed, the core modes rotate fast relative to the \( m = 0 \) modes, so we can use this model as a first approximation. The relative contribution of the two electromagnetic torques is derived for the inertial and viscous limits (Equations (58) and (59) in Ref. 38). In the present work, the RMP has the same role as the resonant error-field. If we only consider the dominant mode \( (n = 6) \), the relative contribution in the inertial limit (IL) is

\[
\left( \frac{T_{\text{RMP}}}{T_{\text{VM-}w}} \right)_{\text{IL}} = \frac{4r_i^2}{r\Delta_{\text{a}}\sqrt{|q'|}} \frac{b_{\text{TM}}^{6,1}}{b_{\text{TM}}^{6,1}} \sqrt{\frac{B}{B_{\text{TM}}}} \approx 40,
\]

where the experimentally measured values are used for the approximation. The \( n = 1 \) mode is difficult to measure since it is locked or slowly rotating, but from the measurements, we estimate \( b_{\text{TM}}^{6,1} \approx 2 \). The reversal surface is at \( r_i \approx 0.8a \), the maximum value of \( \sqrt{q'|q|} \approx 1 \), the extent of the core region is assumed to stop at the reversal surface (\( \Delta \approx 0.8a \)), the average core radius is \( r_i \approx 0.4a \), the \( n = 7 \) mode amplitude is \( b_{\text{TM}} \approx 4 \), the RMP amplitude is close to its maximum value \( b_{\text{RMP}} \approx 1 \), and the equilibrium field is in the order of \( B \approx 0.1 T = 1000 G \). In the viscous limit (VL), the RMP-contribution is larger than in the inertial limit by a factor of \( n \), i.e., \( (T_{\text{RMP}}/T_{\text{VM-}w})_{\text{VL}} \approx 240 \). In the experiment, both the viscous and inertial effects are important, and the plasma should be somewhere in-between the two limits. Through this theoretical estimation and the experimentally observed sheared velocity profile, we conclude that the mode coupling is a high-order effect compared to the RMP–TM interaction.

In all cases, the outer tearing modes lock earlier in MST than in the modeling. There are several possible reasons for this behavior. For example, the RMP amplitude seen by the modes might be higher for the outer modes due to a lower screening effect of the rotating plasma. Another possibility is that the viscosity could be lower in the outer plasma region, which would result in a lower restoring viscous torque on the outer modes. There might be significant contributions from torques not modeled, such as the poloidal component of the resonant torque, which might be more important for the outer modes. For example, in the EXTRAP T2R reversed-field pinch, the ratio of poloidal to toroidal rotation \( \left( \nu_p/\nu_b \right) \) increases with minor radius. The increase in plasma-wall interaction during the RMP application, previously observed in MST, is another possible source of plasma braking at the edge.

In the shots where the central TM \( (n = 6) \) locks, the locked state remains even when the RMP is turned off. The
hysteresis in the locking-unlocking in MST is also predicted by the theoretical model (see Figure 1). The only observed cases of TM unlocking are in relation to a large sawtooth crash, where the mode amplitude is decreased and energy is redistributed between the central $m=1$ modes and $m=0$ modes at the reversal surface. Hysteresis in the locking-unlocking of a TM to a static RMP has previously been observed in EXTRAP T2R reversed-field pinch\textsuperscript{26,30} and in tokamaks.\textsuperscript{37,58} In RFX-mod, the hysteresis is avoided by feedback suppression of the TM amplitude at the shell surface.\textsuperscript{59} This is not in contradiction to the model in Section III that allows for a reversible process if the RMP amplitude is decreased before: (1) the TM amplitude is increased and (2) the velocity reduction spreads to reduce the viscous torque. In MST, condition (1) might be possible due to the long shell time constant, but condition (2) is not likely because of the plasma braking at multiple resonant surfaces. The plasma velocity reduction profile (Figure 5) relaxes already in the braking phase, which reduces the restoring viscous torque (Equation (5)).

### VI. CONCLUSION

Resonant magnetic perturbations (RMPs) are important for the control of many unstable modes in fusion devices. A possible consequence is that RMPs can cause the tearing mode (TM) to lock to the wall. Wall-locked TMs reduce energy confinement and can, in tokamaks, cause a plasma disruption. It is therefore important to avoid wall-locked modes.

With the application of a multi-harmonic RMP, we have experimentally investigated the TM dynamics and unlocking threshold in MST. Observations showed that the locking threshold was increased with increasing plasma density. A model, describing the TM interaction with the wall and an external multi-harmonic RMP, showed qualitative agreement with the experimental data. The model was used to estimate the kinematic viscosity ($\nu_{\text{kin}}$) and $\nu_{\text{kin}} \approx 10-20 \text{ m}^2/\text{s}$ resulted in a quantitative agreement with experimental slowing-down time of the $n=6$ TM. It is shown that the EM torque acting on multiple resonant surfaces has to be modeled to describe the evolution of other TMs $n > 6$ (Figures 4 and 5). Since the TM rotation is connected to the plasma rotation, the inclusion of multiple modes is also important to describe the viscous torque and estimate the kinematic viscosity.

### SUPPLEMENTARY MATERIAL

See supplementary material for the MST data shown in this paper.\textsuperscript{60}

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### APPENDIX: THEORETICAL MODEL—TM DYNAMICS UNDER INFLUENCE OF RMP

This appendix describes the theoretical model of TM island evolution. The present approach is based on Fitzpatrick’s work.\textsuperscript{12,33,34} The model is adapted to MST thick shell and RMP field at the poloidal gap. Only the toroidal component of the velocity is included, as discussed in Section III.

#### 1. Geometry and equilibrium

The model assumes a large aspect-ratio and zero pressure. These are good first order approximations for MST, which has an aspect-ratio of $R_0/a \approx 3$ and poloidal beta $\beta_p \approx 7\%$ in standard discharges.\textsuperscript{36} With these assumptions, the equilibrium is well approximated by a periodic cylinder, and cylindrical polars coordinates are adopted $(r, \theta, z)$. Toroidal angle is defined by $\phi = z/R_0$. The force-balance for the equilibrium magnetic field $B = [0, B_\theta, B_\phi]$ is $\nabla \times B = \sigma(r) B$, where $\sigma(r)$ is the parallel current profile. The equilibrium profile of $\sigma(r)$ is calculated by MSTFit (a non-linear, fixed boundary Grad–Shafranov solver),\textsuperscript{48} using experimental measurements as input.

#### 2. Perturbations

For a single $(m, n)$ mode, the perturbed magnetic field can be expressed as

$$b(r, t) = b^{m,n}(r, t) \exp[i(m\theta - n\phi)], \quad (A1)$$

where

$$b_r^{m,n} = \frac{i\psi_{m,n}}{r}, \quad (A2)$$

$$b_\theta^{m,n} = -\frac{m}{m^2 + n^2 c^2} \frac{d\psi_{m,n}}{dr} + \frac{nc}{m^2 + n^2 c^2} \psi_{m,n}, \quad (A3)$$

$$b_\phi^{m,n} = \frac{nc}{m^2 + n^2 c^2} \frac{d\psi_{m,n}}{dr} + \frac{m c}{m^2 + n^2 c^2} \psi_{m,n}, \quad (A4)$$

and $\epsilon = r/R_0$. The linearized magnetic flux function, $\psi_{m,n}(r, t)$, satisfies Newcomb’s equation\textsuperscript{43}

$$\frac{d}{dr} \left[ f^{m,n} \frac{d\psi_{m,n}}{dr} \right] - g^{m,n} \psi_{m,n} = 0, \quad (A5)$$

where

$$f^{m,n}(r) = \frac{r}{m^2 + n^2 c^2}, \quad (A6)$$

and

$$g^{m,n}(r) = \frac{1}{r} + \frac{r(ncB_\theta + nB_\phi)}{(m^2 + n^2 c^2)(mB_\theta - ncB_\phi) \frac{d\sigma}{dr}} + \frac{2mc\epsilon}{(m^2 + n^2 c^2)^2} - \frac{r a^2}{m^2 + n^2 c^2}. \quad (A7)$$

Equation (A5) is singular at the TM resonant surface, $r = r_s^{m,n}$, which satisfies

$$q(r_s^{m,n}) = \frac{r_s^{m,n} B_\theta(r_s^{m,n})}{R_0 B_\phi(r_s^{m,n})} = \frac{m}{n}. \quad (A8)$$
Since the resonant torque is considered to arise from the helical currents flowing inside the tearing mode and its interaction with currents in the shell and currents in the RMP-coil, it is natural to divide the flux function into three corresponding parts: 21,33

\[ \psi_{s,m,n}(r,t) = \psi_{s,m,n}^{s}(r,t) + \Psi_{b,m,n}^{s}(r,t) + \Psi_{c,m,n}^{s}(r,t), \]  

where \( \psi_{s,m,n}^{s}(r,t), \Psi_{b,m,n}^{s}(r,t), \) and \( \Psi_{c,m,n}^{s}(r,t) \) are complex numbers containing the phase and amplitude of the perturbation at the resonant radius \( r = r_{m,n} \), the shell \( r = r_{b} \), and RMP coil \( r = r_{c} \), respectively. The eigenfunctions \( \psi_{s,m,n}^{s}(r), \Psi_{b,m,n}^{s}(r), \) and \( \Psi_{c,m,n}^{s}(r) \) are solutions to Equation (A5), using the following boundary conditions:

\[ \psi_{s,m,n}^{s}(0) = 0, \psi_{s,m,n}^{s}(r_{s}) = 1, \psi_{s,m,n}^{s}(r_{b}) = 0, \]
\[ \psi_{b,m,n}^{s}(r_{s}) = 0, \psi_{b,m,n}^{s}(r_{b}) = 1, \]
\[ \psi_{c,m,n}^{s}(r_{s}) = 0, \psi_{c,m,n}^{s}(r_{c}) = 1. \]

The radial structures of the eigenfunctions are shown in Figure 8.

The discontinuities in flux function can be represented by

\[ \Delta \Psi_{m,n} = \frac{d \psi_{m,n}^{O}}{dr} \bigg|_{r_{c}}, \]  

and

\[ E_{ij}^{m,n} = \frac{d \psi_{m,n}^{O}}{dr} \bigg|_{r_{c}}, \]  

where indices \( i \) and \( j \) can be \( s, b, \) or \( c \) which represents, respectively, the TM islands position, the shell position, and coils position. For more details, see Refs. 33 and 36.

3. Electromagnetic torques

Here follows a brief derivation of the EM-torque terms for MST. Using Equations (A9)–(A11) and the dispersion relation for the thick shell defined in Ref. 33, the toroidal component of electromagnetic torque acting on the TM is given by

\[ T_{EM}^{m,n}(t) = C_{m,n} \text{Im}\{\Delta \Psi_{m,n}^{s}(\Psi_{m,n}^{s})^{*}\} = T_{wall}^{m,n}(t) + T_{RMP}^{m,n}(t), \]  

where

\[ T_{wall}^{m,n}(t) = -\frac{|\Psi_{m,n}^{s}(t)|^{2} \sqrt{K_{m,n}(t)E_{bs}^{m,n}E_{bs}^{m,n}}}{K_{m,n}(t) - 2K_{m,n}(t)E_{bs}^{m,n} + (E_{bs}^{m,n})^{2}} \times C_{m,n}, \]  

and

\[ K_{m,n}(t) = n(\Omega_{s}^{m,n}(t) + \Omega_{s}^{m,n}(t)) \times \sin[\Delta \omega_{m,n}(t)], \]  

where \( T_{wall} \) is the torque caused by the interaction with eddy currents induced in MST shell and \( T_{RMP} \) is the torque due to the RMP–TM interaction. Furthermore, \( \omega_{s}^{m,n}(t) \) is the toroidal angular velocity of the TM, \( x_{m,n}(t) \) is the phase of the TM, and \( \Delta \omega_{m,n}(t) \) is its phase difference to the RMP. Note that \( E_{bs}^{m,n} \) depends on both the RMP and TM amplitude through \( |\Psi_{m,n}^{s}| \) and \( |\Psi_{m,n}^{s}| \), respectively. The \( E_{ij}^{m,n} \) constants are calculated according to (A11). Except \( E_{bs}^{m,n} \), that is, \( E_{bs}^{m,n} = -E_{bs}^{m,n}E_{bs}^{m,n} / (E_{bs}^{m,n}(r_{b}) - E_{bs}^{m,n}(r_{c})) \), where \( E_{bs}^{m,n}(r_{b}) \) and \( E_{bs}^{m,n}(r_{c}) \) are the tearing stability index assuming a perfectly conducting shell at the wall \( r = r_{b} \) and at \( r = r_{c} + \infty \), respectively.

4. Equation of fluid motion

The modification of the fluid velocity due to an RMP is determined by two competing torques: the viscous torque \( T_{visc} \) that acts to oppose changes in the plasma rotation and the electromagnetic (EM) torque \( T_{EM} \) that acts on the TM island located at \( r_{m,n}^{s,12,19,20} \). The equation of fluid motion can be expressed as \( T_{EM} \)

\[ \rho(r) \frac{\partial \Omega_{\phi}}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu_{\perp}(r) \frac{\partial \Omega_{\phi}}{\partial r} \right) + \sum_{s} T_{EM}^{m,n}(t) + \delta(r - r_{s}^{m,n}), \]  

where the first term on the right hand side is the viscous torque density, \( \rho(r) \) is the plasma mass density, and \( \Delta \Omega_{\phi}(r,t) \) is the plasma toroidal angular velocity reduction profile. The perpendicular dynamic viscosity \( \mu_{\perp}(r) \) is related to the kinematic viscosity \( \nu_{kin}(r) \) through \( \mu_{\perp}(r) = \rho(r) \nu_{kin}(r) \). In the second term, the delta function implies that the EM torque acts only at the resonant radius, where \( T_{EM} \) is given by (A12).

5. TM island angular evolution

The fluid and TM angular velocities are related via the no-slip condition \( T_{EM} \) that ensures the co-rotation between the TM island and the plasma flow at the resonances

\[ \frac{dx_{m,n}(t)}{dr} = n \left[ \Omega_{\phi}(r_{s}^{m,n}, t_{0}) + \Delta \Omega_{\phi}(r_{s}^{m,n}, t) \right], \]  

FIG. 8. Typical shape of the three tearing mode eigenfunctions \( \psi_{s,m,n}^{s}, \psi_{b,m,n}^{s}, \) and \( \psi_{c,m,n}^{s} \), which are representing the interaction at the resonant surface (s), the wall (b), and coils (c), respectively.
where $\Omega_\phi$ is the unperturbed toroidal angular velocity profile, as defined in Section II. The above expression (A18) assumes that poloidal rotation is negligible, $d\alpha_m n/dt \equiv -\alpha_m n(t) \approx \eta (\alpha_m n(t))$, see Section II for details.

6. TM amplitude evolution

The TM amplitude evolution is modeled by the Rutherford equation.\(^{34,47}\) The present version includes the RMP–TM interaction and the interaction between TM and eddy currents induced in the shell. Using the approach in Ref. 34 together with Equations (A9)–(A11) and the wall dispersion relation,\(^{53}\) Rutherford’s equation takes the form

$$
d\sqrt{\Psi_m^n(t)} = G_{m,n}^{m,n} \left( E_{m,n} + D_{m,n}(t) \right) \sqrt{\Psi_m^n} + \frac{\sqrt{\Psi_m^n}}{\sqrt{\Psi_n}} E_{m,n} \cos(\Delta \alpha_{m,n}(t)) - \frac{\Lambda}{G_{m,n}^{m,n} (\lambda_{0,n}^m)^2} \ln \left( \frac{G_{m,n}^{m,n}}{\sqrt{\Psi_m^n}} \right), \tag{A19}
$$

where

$$
D_{m,n}(t) = \frac{E_{m,n} E_{n,n} \left( \sqrt{K_{m,n}^2} - E_{m,n} \right)}{K_{m,n} - \sqrt{2K_{m,n} + E_{m,n} E_{n,n}}}, \tag{A20}
$$

$$
G_{m,n} = \left( \frac{B_{m,m}}{n} \right) \left( \frac{1}{\sigma \left( m_n^2 + n_n^2 \right)} - 2 \eta \right), \tag{A21}
$$

$$
\lambda_{0,n}^m = \frac{r m^2}{\sigma - 2 \eta \left( m_n^2 + n_n^2 \right)} \left| r - e_{r,n} \right|, \tag{A22}
$$

$$
\alpha_{m,n}(t) = \frac{\partial B_{m,m}}{\partial x}, \tag{A23}
$$

$\Lambda \approx 1.6$ and $\eta$ is the parallel resistivity. The time scale is in the order of the resistive time scale $\tau_r$. In Equation (A19), the first term is the linear instability drive (or stability if $E_{m,n}^2 < 0$). The second term is due to the TM interaction with the shell. The third term is the interaction with the RMP, which produce oscillations in the amplitude according to $\cos[\Delta \alpha_{m,n}(t)]$. The last term is the non-linear saturation.
60See supplementary material at http://dx.doi.org/10.1063/1.4953438 for the digital format of the data shown in this paper.