Anomalous ion heating from ambipolar-constrained magnetic fluctuation-induced transport

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A kinetic theory for the anomalous heating of ions from energy stored in magnetic turbulence is presented. Imposing self-consistency through the constitutive relations between particle distributions and fields, a turbulent Kirchhoff's Law is derived that expresses a direct connection between rates of ion heating and electron thermal transport. This connection arises from the kinematics of electron motion along turbulent fields, which results in granular structures in the electron distribution. The drag exerted on these structures through emission into collective modes mediates an effective ambipolar constraint on transport. Resonant damping of the collective modes by ions produces the heating. In collisionless plasmas the rate of ion damping controls the rate of emission, and hence the ambipolar-constrained electron heat flux. The heating rate is calculated for both a resonant and nonresonant magnetic fluctuation spectrum and compared with observations. The theoretical heating rate is sufficient to account for the observed twofold rise in ion temperature during sawtooth events in experimental discharges.
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I. INTRODUCTION

Anomalous ion heating is a feature of many laboratory plasma experiments with high levels of magnetic fluctuations. The reversed field pinch (RFP),[1–3] spheromak,[4] and the magnetic reconnection experiment (MRX) (Ref. 5) all exhibit an anomalously high ion temperature. In these cases the temperature cannot be explained by the standard Ohmic heating process in which ions are heated by collisions with electrons. In the Madison Symmetric Torus (MST),[6–8] the average core ion temperature during both the initial temperature rise and the subsequent flat top is inconsistent with the collisional equilibration process. For example, in recent measurements the ion temperature is equal to the electron temperature between sawtooth events, even though the heating is Ohmic.[8] During sawtooth events the intensity of a background of magnetohydrodynamic dynamo fluctuations undergoes a significant enhancement. Simultaneously, the ion temperature increases by as much as 100%, while the electron temperature drops (see Fig. 10 in Ref. 7). The ion temperature increase indicates a correlation between ion heating and magnetic fluctuation level. The decrease of electron temperature is noteworthy. Because it falls below the ion temperature it can only be explained by an enhanced electron energy loss, or a transient decrease in the Ohmic heating of electrons. The latter is ruled out by the increase in loop voltage that accompanies sawtooth events. During startup in the MST, the ion temperature also rises above the electron temperature.[7] The spheromak and MRX plasmas differ from the RFP in many ways, yet share in common the presence of anomalous ion heating and significant levels of magnetic turbulence. It thus seems plausible that anomalous ion heating is a natural by-product of magnetic turbulence.

Despite its importance in experiments, theoretical work on anomalous ion heating has been limited. Ion cyclotron resonance damping[9] and ion viscous damping[10,11] have been invoked for ion heating in the RFP. Within the viscous heating scenario it has also been argued[3] that the same fluctuations that heat the ions lower the electron temperature through an enhancement of the electron energy flux. However, for viscous dissipation to be effective, a cascade to small scales seems likely. In a cascade of magnetic turbulence, electrons are also heated through Landau resonances, and the rates of electron and ion heating are comparable.[9] In such a case it is not clear how a rise in ion temperature might be connected to a drop in electron temperature. Ion heating has also been considered in the two-fluid theory that resolves ion and electron structures in forced magnetic reconnection.[12] In that theory, the heating of ions occurs through a dc electric field, and is therefore quite different from the manifestly turbulent process considered herein.

In this paper we show that ion heating is a natural by-product of magnetic turbulence through a constraint associated with Ampére's Law and quasineutrality that allows ions to slow electron loss rates. This leads to a turbulent Kirchhoff's Law expressing a direct connection between anomalous electron transport, which removes energy from electrons, and ion heating. The process originates with the granular structure produced by the magnetic turbulence in the electron distribution function and its shielding by the plasma dielectric.[13] The role of turbulent granularity in magnetic fluctuation-induced electron transport has been documented in a series of papers examining successively higher transport moments.[14–16] For the particle and field-aligned momentum fluxes, it was found that the transport from mag-

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netic turbulence nearly vanishes relative to the electrostatic fluxes and the flux of the non self-consistent quasilinear approximation.[14,15] The residual transport is smaller than the quasilinear or electrostatic fluxes by the ratio of ion to electron current contributions in Ampére's Law, and therefore resembles an ambipolar-constrained transport flux. However, it is not the conventional constraint of an ambipolar electric field, because it arises from an electromagnetic dielectric response to granular structure. This paper does not deal with conventional ambipolarity, hence this effective ambipolar constraint of the dielectric response to granular structure will hereafter be referred to as simply an ambipolar constraint. In prior calculations it was also found that for electron thermal transport, the flux of perpendicular energy breaks into ambipolar- and nonambipolar-constrained components.[16] The former dominates when the magnetic fluctuations responsible for transport are resonant at distant rational surfaces. This is the situation in the MST edge, where the fluctuations driving electron thermal transport are the core-resonant unstable tearing modes. The measured flux reflects an ambipolar constraint because it can be expressed as a Rechester–Rosenbluth diffusivity, but with the ion thermal velocity in place of the electron thermal velocity.[16] The flux is also convective, despite a robust temperature gradient. Near the rational surfaces of transport-causing fluctuations, located in the core region in the MST, the nonambipolar-constrained component dominates. The flux is larger than that of remote locales by the ratio of electron to ion thermal velocities. Subsequent to the predictions of Ref. 16, core transport in MST was inferred through profile measurements and power balance analysis. The thermal diffusivity increases by a factor of 40 in going from the edge to the core.[17] The rise in diffusivity occurs near the reversal surface, the region that separates the transport-causing core fluctuations from smaller scale edge turbulence.

We extend here the prior calculations of ambipolar constraints in magnetic fluctuation-induced transport[14–16] to calculate the heating of ions intrinsic to these constraints. Although the origin of these constraints and the underlying physics that introduces them has been detailed in the literature,[13,18–22] we present here a brief introduction. The granular or clumpy component of the distribution function arises simply from the inhomogeneity of turbulent mixing. Particles that are barely separated experience nearly the same scattering force, and hence remain correlated longer than those with large separation. In magnetic turbulence this is closely related to the magnetic field topology, with neighboring field lines remaining bundled for a greater distance along the field line, than field lines that are far apart. This inhomogeneity of turbulent mixing is lost if the distributions and fields are treated solely as normal modes, because the normal mode ansatz makes the mixing rate identical for all separations.[13] Therefore, the clumpy part of the distribution, which arises in a complete description from the weak relative scattering at short separation, is distinct from the normal mode component. By definition, a normal mode entails a coherent relationship between the distribution function and the potentials, with the distribution function $h(\mathbf{k})$, electrostatic potential $\phi(\mathbf{k})$, and the parallel component of the magnetic vector potential $A_\parallel(\mathbf{k})$ all proportional to $\exp(i\mathbf{k}\cdot\mathbf{x})$. This coherent relationship implies that the normal mode component of the distribution $h^c(\mathbf{k})$ (where the superscript c signifies coherent) can be written in terms of dielectric susceptibilities R_ϕ and R_A as

$$h^c(\mathbf{k}) = R_\phi(\mathbf{k})\phi(\mathbf{k}) + R_A(\mathbf{k})A_\parallel(\mathbf{k}). \tag{1}$$

Because the clump is not captured in the normal mode response its contribution $\tilde{h}(\mathbf{k})$ at wave number $\mathbf{k}$ is incoherent, i.e., it is not proportional to either potential at the wave number $\mathbf{k}$. When the full distribution $h^c(\mathbf{k})+\tilde{h}(\mathbf{k})$ is substituted into the quasineutrality condition or Ampére's Law, the normal-mode component can be collected into dielectric tensor elements. The clump component is incoherent and cannot. As a simple illustration, consider Poisson's equation for purely electrostatic fluctuations having coherent responses in the electrons and ions and an incoherent component in the electron distribution,

$$k^2\phi(k) = 4\pi e\int d^3v[h_i^c(\mathbf{k}) - h_e^c(\mathbf{k}) - \tilde{h}_e(\mathbf{k})].$$

The coherent densities $h_i^c(\mathbf{k}) = R_\phi^i\phi(\mathbf{k})$, $h_e^c(k) = R_\phi^e\phi(\mathbf{k})$ can be collected into a dielectric $k^2\varepsilon(\mathbf{k}) = k^2 - 4\pi e\int d^3v(R_\phi^i - R_\phi^e)$, yielding

$$k^2\varepsilon(\mathbf{k})\phi(\mathbf{k}) = -4\pi e\int d^3v\,\tilde{h}_e(\mathbf{k}). \tag{2}$$

This expression is clearly analogous to the shielding of a test charge, where the incoherent charge density $q\int d^3v\,\tilde{h}$ appears in place of the test charge density $q_{\text{test}}\delta(\mathbf{x})$.

The shielding physics constrains transport because the quasineutrality condition [Poisson's equation for $k \ll (\text{Debye length})^{-1} = \lambda_D^{-1}$] and Ampére's Law are the constitutive relations required to close the drift-kinetic description and make it self-consistent. Electron transport moments necessarily involve the incoherent distribution, i.e., $\langle U_e(\mathbf{v})\rangle = \int d^3v\,U_e(\mathbf{v})h_e = \int d^3v\,U_e(\mathbf{v})(h_e^c + \tilde{h}_e)$, where $U_e(\mathbf{v})$ is any function of velocity. The description is only complete when the fields that force $h_e^c + \tilde{h}_e$ are determined self-consistently from the constitutive relations. Imposing an expression like Eq. (2) constrains the transport in two crucial ways. First it introduces ion physics in an electron transport moment through $\varepsilon(\mathbf{k})$, yielding ambipolar constrained flux components. Second, because $\varepsilon(\mathbf{k})$ contains h_e^c, it modifies the role played by h_e^c in transport from that predicted by quasilinear theory, or any other non self-consistent description of transport. In the self-consistent magnetic fluctuation-induced transport of particles, field-aligned momentum, and parallel energy, h_e^c cancels out altogether, leaving only the ambipolar-constrained components. These cancellations and the appearance of ion dynamics reflect the energy and momentum constraints in wave–particle interaction physics intrinsic to the system of the drift-kinetic and Maxwell's equations.

The interaction of clumps with the shielding dielectric is in fact a finite-amplitude analog to discreteness effects in a nonturbulent plasma with finite plasma parameter.[23] In the latter case, moving discrete particles (scales $<\lambda_D$) drag the dielectric cloud (scales $>\lambda_D$), inducing emission into the dielectric at its zeros or collective resonances, whatever they may be. The rate of emission is equal to the dielectric absorption rate, as expressed by Kirchhoff's Law. Transport is described by the Lenard–Balescu equation, and relaxation rates, which require dissipation, are proportional to the absorption rate. For transport moments that do not relax under like-particle collisions only (e.g., the particle flux), the emission rate, which controls relaxation, can only be proportional to absorption by the other, unlike-particle species. An analogous process occurs for clumps. The above set of statements describing discreteness physics holds for clumps, if the words ''discrete particles'' are replaced by ''clumps,'' ''λ_D'' by ''turbulent correlation length,'' ''Kirchhoff's Law'' by ''turbulent Kirchhoff's Law,'' and ''Lenard–Balescu equation'' by ''turbulent kinetic equation.''

Ion heating, the subject of this paper, originates in the above process as the energy absorbed by the ions from emission by the clump component of the electron distribution. Ion heating is generic in the sense that the process is not specific to any particular type of mode or ion absorption mechanism. We will also keep the calculation fairly generic by stipulating that there is an unstable collective resonance in the turbulent spectrum, i.e., an instability that drives the turbulence, but not specifying its details. We therefore calculate the electron heat flux and the ion heating rate associated with some specified level of turbulence consistent with some instability and its saturation. To compare with experiment, we can use measured fluctuation levels, but must determine if approximations made in the calculation are compatible with the experimental conditions. For the simple expressions used in this calculation (the forms are given in the next section) the collective mode of the dielectric into which the granular fluctuations emit is a drift-Alfvén mode. For the ion absorption rate, a variety of dissipation mechanisms could be invoked, but we will use Landau damping as the one that seems most consistent with conditions attendant to the absorption of parallel energy in the MST.

These calculations yield an ion heating rate that is sufficient to explain the observed ion temperature rise in the MST core during sawtooth events. The temperature evolution is obtained from the heating rate using a simple zero-dimensional transport model. The model incorporates the anomalous ion heating rate from the ambipolar-constrained flux of parallel electron energy, and the electron heat loss rate from the nonambipolar-constrained flux of perpendicular energy. (As explained above, a spectrum of locally resonant magnetic fluctuations yields a transport of perpendicular energy that is dominated by the nonambipolar-constrained component.) We use the model to predict the electron and ion temperature transients associated with sawtooth events in the MST. Prior to the sawtooth, Ohmic heating and anomalous transport in the electrons are balanced to yield a stationary electron temperature. A similar balance between electron-ion collisional equilibration, anomalous ion heating, and ion transport losses yields a stationary ion temperature. The sawtooth event is imposed in the model as a transient rise in magnetic fluctuation level and a subsequent return to the original level over a time scale of the order of a millisecond. This heuristic modeling exercise leads to ion and electron temperature transients that are qualitatively like those of experiment. The ion temperature rises by 100% and the electron temperature drops below the ion temperature. After the sawtooth events the signals return to their pre-sawtooth values. The decay rate of the ion temperature to its pre-sawtooth value is considerably slower in the model than in experiment because the confinement time inferred from the simple steady state balance is larger than the experimental value. The modeling exercise is crude but significant because it shows that the anomalous ion heating rate of the turbulent Kirchhoff's Law is sufficient to account for the net ion temperature rise and the dip in electron temperature. It is too crude to offer accurate modeling of relaxation rates.

This paper is organized as follows. In Sec. II we introduce the general theoretical ideas behind our calculation. This includes the basic kinetic equations on which the theory is based and the self-consistency constraints which govern the kinematics and energetics of the electron and ion distribution functions. In Sec. III we detail the calculation of the anomalous flux of electron parallel energy due to magnetic turbulence, and present the energy balance that ties this flux to ion heating. An explicit expression for the latter is presented for the cases of resonant and nonresonant magnetic fluctuation spectra. In Sec. IV, we address the flux of the perpendicular component of electron energy, and construct the simple zero-dimensional transport model. Representative temperature time histories are presented for the sawtooth modeling exercise. A conclusion section ends the paper.

II. THEORETICAL FRAMEWORK

The basis for the heating and transport calculations described in this paper is the drift-kinetic equation (DKE). The DKE is obtained by averaging the Vlasov equation over the rapidly oscillating component of motion under the assumption that all terms in the equation are of order $\delta=\rho/L\ll 1$ compared to gyromotion (ρ is the particle's gyroradius and L is a scale length characterizing the plasma). As usual in low-β magnetic turbulence, we assume $k_\parallel/k_\perp\ll 1$. In the Coulomb's gauge ($\nabla\cdot\mathbf{A}=0$), this implies that the perpendicular components of the magnetic potential, $|A_\perp|=(k_\parallel/k_\perp)A_\parallel$, can be neglected. Choosing as velocity variables the parallel velocity $v_\parallel$, the magnetic moment $\mu=mv_\perp/(2B_0)$, and the gyrophase φ (where $\parallel$ and $\perp$ refer to the parallel and perpendicular directions of the unperturbed magnetic field), and neglecting effects due to equilibrium magnetic field inhomogeneity and curvature, the DKE for the electron species is given by

$$\frac{\partial \overline{f_e}}{\partial t}+(v_\parallel\mathbf{b}+\mathbf{v}_{E\times B})\cdot\frac{\partial \overline{f_e}}{\partial \mathbf{x}}+\frac{q_e}{m_e}\mathbf{E}\cdot\mathbf{b}\frac{\partial \overline{f_e}}{\partial v_\parallel}=0. \qquad (3)$$

Here $\overline{f_e}$ is the gyroaveraged electron distribution function, $\mathbf{b}=\mathbf{B}_{\rm tot}/B_0$ is the unit vector in the direction of the magnetic field, $\mathbf{v}_{E\times B}=(c/B_0)\mathbf{E}\times\mathbf{b}_0$ is the $E\times B$ drift velocity, and $q_e=-e$. In the case under consideration the electric field has only a perturbed component, $\mathbf{E}$, while the magnetic field has both equilibrium ($\mathbf{B}_0$) and perturbed ($\mathbf{B}$) parts. The latter is related to the vector potential by $\mathbf{B}=\nabla A_\parallel\times\mathbf{b}_0$. Then $\mathbf{b}\simeq\mathbf{b}_0$

$+\tilde{\mathbf{b}}$, with $\tilde{\mathbf{b}}=\nabla A_\parallel\times\mathbf{b}_0/B_0$, and the electric field has both electrostatic and magnetic components, $\mathbf{E}=-\nabla\phi-(1/c)\times(\partial\mathbf{A}/\partial t)$. The streaming term contains a magnetic flutter operator, $v_\parallel\tilde{\mathbf{b}}\cdot\nabla$, and the $E\times B$ drift has a magnetic component $-(1/B_0)(\partial\mathbf{A}/\partial t)\times\mathbf{b}_0$. Since the fields ϕ and $A_\parallel$ are also assumed to vary slowly (in space and time) with respect to the space-time scales associated with the motion of gyration, the gyroaveraged electron distribution function $\overline{f_e}$ can be divided into an averaged part $\langle\overline{f_e}\rangle$ and a perturbed part defined as $\delta\overline{f_e}\equiv\overline{f_e}-\langle\overline{f_e}\rangle$. The averaging operator $\langle\cdots\rangle$ is now an ensemble average which removes any fast time-space variation associated with the perturbing fields. The averaged part can in turn be decomposed into an equilibrium piece, approximately a local Maxwellian distribution $f_{e,M}(v)=n_0/(\pi^{3/2}v_e^3)\exp(-v^2/v_e^2)$ [where $v_e\equiv(2T_e/m_e)^{1/2}$], and a slowly time-dependent part which describes its evolution on a transport time scale. Using this decomposition for $\overline{f_e}$ in Eq. (3), ensemble averaging, and taking the energy moment, we arrive at the following energy balance describing the time evolution of the kinetic energy associated with $\langle\overline{f_e}\rangle$:

$$\frac{\partial}{\partial t}\int d^3v\frac{m_e v^2}{2}\langle\overline{f_e}\rangle=H_e-\nabla\cdot\mathbf{Q}_e. \tag{4}$$

The first term on the right-hand side is the electron Joule heating, while an explicit form for the energy flux present in the remaining term is

$$\mathbf{Q}_e=\int d^3v\frac{m_e(v_\perp^2+v_\parallel^2)}{2}\frac{c}{B_0}\left\langle\mathbf{b}_0\times\nabla\left(\phi-\frac{v_\parallel}{c}A_\parallel\right)\delta\overline{f_e}\right\rangle, \tag{5}$$

where we have written the total energy in terms of parallel and perpendicular components and note that the flux of energy can similarly be broken into components representing the flux of parallel and perpendicular energy. In this expression for the flux, the perturbed distribution function $\delta\overline{f_e}$ can be replaced by its nonadiabatic part $h_e\equiv\delta\overline{f_e}+q_e/T_e[\phi-(v_\parallel/c)A_\parallel]$, since the adiabatic contribution (the second term in the definition of h_e) does not produce transport. The equation for h_e is derived from Eq. (3). We obtain

$$\frac{\partial h_e(\mathbf{x},\mathbf{v},t)}{\partial t}+\left(v_\parallel\mathbf{b}_0-\frac{c}{B_0}\nabla\phi\times\mathbf{b}_0+v_\parallel\frac{\nabla A_\parallel\times\mathbf{b}_0}{B_0}\right)\cdot\nabla h_e$$
$$=\sum_{\mathbf{k},\omega}\frac{iq_e}{T_e}f_{e,M}(\omega-\omega_{*,e}^T)\left(\phi-\frac{v_\parallel}{c}A_\parallel\right)_{\mathbf{k},\omega}$$
$$\times\exp(-i\mathbf{k}\cdot\mathbf{x}+i\omega t), \tag{6}$$

where

$$\omega_{*,e}^T\equiv\omega_{*,e}\left[1+\eta_e\left(\frac{v^2}{v_e^2}-\frac{3}{2}\right)\right],$$

$$\omega_{*,e}\equiv\frac{cT_e}{q_eB_0}(\mathbf{k}\times\mathbf{b}_0\cdot\mathbf{r})\frac{1}{L_{n_e}}$$

are the diamagnetic frequencies, $L_{n_e}^{-1}\equiv n_e^{-1}\partial n_e/\partial r$, $L_{T_e}^{-1}\equiv T_e^{-1}\partial T_e/\partial r$, $\eta_e\equiv L_{n_e}/L_{T_e}$, and we have dropped the velocity space nonlinearities, which play a negligible role in the dynamics under consideration. The right-hand side is a source term due to inhomogeneities in the equilibrium distribution function. This term is responsible for transport. On the left-hand side, the perturbing fields ϕ and $A_\parallel$ produce nonlinearities which advect the perturbed distribution function. Electrostatic fluctuations give rise to the only retained perpendicular drift, the $E\times B_0$ velocity. The flutter term, associated with the fluctuating magnetic potential, introduces the ballistic streaming of particles along the perturbed magnetic field.

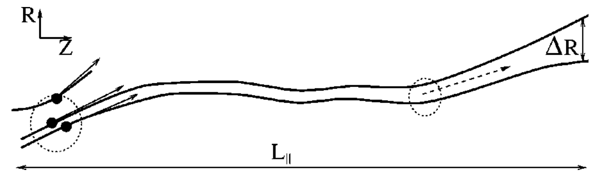

FIG. 1. Schematic representation of the propagation and decay of a clump in steady-state magnetic turbulence.

As described in the Introduction, the nonadiabatic electron distribution function can be broken into a coherent (h_e^c) and an incoherent component ($\tilde{h}_e$),

$$h_e=h_e^c+\tilde{h}_e. \tag{7}$$

The coherent component supports a normal mode response and can therefore be written as

$$h_e^c(\mathbf{k},\omega;\mathbf{v})=R_\phi^e(\mathbf{k},\omega;\mathbf{v})\phi(\mathbf{k},\omega)+R_A^e(\mathbf{k},\omega;\mathbf{v})A_\parallel(\mathbf{k},\omega), \tag{8}$$

where R_ϕ^e and R_A^e are the electron contributions to the susceptibilities introduced in Eq. (1). These functions are obtained from the solution of Eq. (6). In the absence of the nonlinear terms, whose convolutions in Fourier space produce driving of $h_e(\mathbf{k})$ by potentials at wave numbers other than $\mathbf{k}$, the solution of Eq. (6) is obviously coherent, with

$$R_\phi^e=\frac{q_e}{T_e}f_{eM}\frac{\omega-\omega_{*,e}^T}{\omega-k_\parallel v_\parallel},\quad R_A^e=-\frac{v_\parallel}{c}R_\phi^e. \tag{9}$$

Under standard renormalized treatments of the nonlinearities, the solution of Eq. (6) remains coherent, yielding responses with a broadened propagator. In weak to moderate turbulence this broadening can be neglected. The nonlinear driving of $h_e(\mathbf{k})$ by the Fourier potential amplitudes at other wave numbers, which produces the incoherent distribution $\tilde{h}_e(\mathbf{k})$, is only recovered in renormalized treatments of the two-point correlation $\langle h_e(\mathbf{x}_1,\mathbf{v}_1,t)h_e(\mathbf{x}_2,\mathbf{v}_2,t)\rangle$. In that type of calculation, which is not repeated here, the vanishing of relative scattering at small separations in phase space $(\mathbf{x}_1-\mathbf{x}_2)\to 0$, $(\mathbf{v}_1-\mathbf{v}_2)\to 0$, and the short range correlation, or clump, it produces is identified directly with $\tilde{h}_e(\mathbf{k})$.

A simplified picture of the propagation and decay of localized incoherent fluctuations in a region of steady-state perturbed magnetic field lines is presented in Fig. 1. A group of perturbed magnetic field lines locally bunched together spatially decorrelate (i.e., separate from one another) by spreading radially a distance ΔR (decorrelation length) for a longitudinal displacement $Z=L_\parallel$. A group of electrons which are initially approximately at the same Z location, are separated by a distance less than ΔR (within the encircled region to the left of the picture) and have equal velocities, ballistically propagate along the field lines. During the streaming, the electrons at the edge of the region decorrelate faster from an electron in the center than do electrons in an interior region close by the central electron. Consequently, electrons within a radial correlation length form a clump that decays as the electrons stream along the field (as indicated by the smaller encircled region to the right). In the more general context of time-dependent magnetic turbulence, the clump lifetime depends on the Lyapunov time, i.e., the time that two neighboring field lines will remain correlated (diffuse together), as well as on the relative streaming velocities of the individual electrons. Moreover, there are self-consistency effects: when electrons follow a magnetic field line that changes direction, the change in electron momentum acts back on the field line through the constitutive relations, changing the field configuration. The proper accounting of these self-consistency effects lies at the heart of this paper. As electron clumps move through the plasma, they drag the shielding cloud, and are forced by conservation constraints to emit into the dielectric. The dielectric absorbs this emission at a collective resonance.

The ions can also be described by a DKE similar to Eq. (6). Of particular importance is the evolution equation for the mean kinetic energy,

$$\frac{\partial}{\partial t}\int d^3v\frac{m_i v^2}{2}\langle\overline{f_i}\rangle=\langle E_\parallel J_{i\parallel}\rangle-\nabla\cdot\mathbf{Q}_i\,, \tag{10}$$

which contains the ion heating term. For fluctuations that are predominantly magnetic, the ion heating term is given by

$$H_i\equiv\langle E_\parallel J_{i\parallel}\rangle=-\frac{1}{c}\left\langle\frac{\partial A_\parallel}{\partial t}J_{i\parallel}\right\rangle, \tag{11}$$

where $J_{i\parallel}$ is supplied by the appropriate moment of the nonadiabatic ion distribution. As with the electron distribution, the nonadiabatic ion distribution contains a normal mode, or coherent, part. For magnetic turbulence it is readily apparent that the incoherent ion component is negligible. As explained, the incoherent part results from streaming-particle motion along the field, hence $\tilde{h}_i$ is smaller than $\tilde{h}_e$ by $(m_e/m_e)^{1/2}$. The ion distribution can therefore be written as

$$h_i(\mathbf{k},\omega;\mathbf{v})=R^i_\phi(\mathbf{k},\omega;\mathbf{v})\phi(\mathbf{k},\omega)+R^i_A(\mathbf{k},\omega;\mathbf{v})A_\parallel(\mathbf{k},\omega). \tag{12}$$

For weak to moderate turbulence under MST-like conditions, where the fluctuation frequency is low, the dissipative part of the ion susceptibilities is governed by the ion Landau damping. For strong turbulence the resonances are broadened and ion Compton scattering comes into play.

The shielding plasma potentials $\phi(\mathbf{k},\omega)$ and $A_\parallel(\mathbf{k},\omega)$ are governed by quasineutrality and Ampére's Law, which, in our framework, can be succinctly written as

$$\begin{bmatrix} d^{ei}_{\phi\phi}(\mathbf{k},\omega) & d^{ei}_{\phi A}(\mathbf{k},\omega)\\ d^{ei}_{A\phi}(\mathbf{k},\omega) & d^{ei}_{AA}(\mathbf{k},\omega)\end{bmatrix}\begin{bmatrix}\phi(\mathbf{k},\omega)\\ A_\parallel(\mathbf{k},\omega)\end{bmatrix}=\begin{bmatrix}-4\pi e\tilde{n}(\mathbf{k},\omega)\\ -\dfrac{4\pi}{c}\tilde{J}_\parallel(\mathbf{k},\omega)\end{bmatrix}. \tag{13}$$

In the above, $\tilde{n}=\int d^3v\tilde{h}(\mathbf{v})$ and $\tilde{J}_\parallel=e\int d^3v v_\parallel\tilde{h}(\mathbf{v})$ are the density and current density sources associated with the clump population. By definition of incoherent fluctuations $[\tilde{h}(\mathbf{k},\omega;\mathbf{v})\propto\phi(\mathbf{k}',\omega'),A_\parallel(\mathbf{k}',\omega')]$ these fluctuations cannot be included in the elements of the dispersion tensor, i.e., in the left-hand side of Eq. (13). The elements of the dispersion tensor are given by

$$\begin{bmatrix} d^{ei}_{\phi\phi}(\mathbf{k},\omega) & d^{ei}_{\phi A}(\mathbf{k},\omega)\\ d^{ei}_{A\phi}(\mathbf{k},\omega) & d^{ei}_{AA}(\mathbf{k},\omega)\end{bmatrix}=4\pi e\int d^3v\begin{bmatrix} R^e_\phi(\mathbf{k},\omega,\mathbf{v})-R^i_\phi(\mathbf{k},\omega,\mathbf{v}) & R^e_A(\mathbf{k},\omega,\mathbf{v})-R^i_A(\mathbf{k},\omega,\mathbf{v})\\ \dfrac{v_\parallel}{c}[R^e_\phi(\mathbf{k},\omega,\mathbf{v})-R^i_\phi(\mathbf{k},\omega,\mathbf{v})] & \dfrac{v_\parallel}{c}[R^e_A(\mathbf{k},\omega,\mathbf{v})-R^i_A(\mathbf{k},\omega,\mathbf{v})]\end{bmatrix}. \tag{14}$$

Inverting Eq. (13) we obtain the following self-consistent expressions for the potentials:

$$A_\parallel(\mathbf{k},\omega)=-4\pi L^{-1,ei}(\mathbf{k},\omega)\left[d^{ei}_{\phi\phi}(\mathbf{k},\omega)\frac{1}{c}\tilde{J}_\parallel(\mathbf{k},\omega)-d^{ei}_{A\phi}(\mathbf{k},\omega)e\tilde{n}(\mathbf{k},\omega)\right], \tag{15}$$

$$\phi(\mathbf{k},\omega)=-4\pi L^{-1,ei}(\mathbf{k},\omega)\left[d^{ei}_{AA}(\mathbf{k},\omega)e\tilde{n}(\mathbf{k},\omega)-d^{ei}_{\phi A}(\mathbf{k},\omega)\frac{1}{c}\tilde{J}_\parallel(\mathbf{k},\omega)\right]. \tag{16}$$

In these expressions, the coherent shielding response manifests itself through the dispersion elements (14), while the shielded incoherent fluctuations are represented by the clump density $\tilde{n}$ and current density $\tilde{J}_\parallel$. Later on, by using these potentials in the electron heat flux and the ion heating rate expressions we will ensure self-consistency. The plasma dielectric response is described by the elements of the dispersion tensor,

$$L^{ei}(\mathbf{k},\omega)=\det\begin{bmatrix} d^{ei}_{\phi\phi}(\mathbf{k},\omega) & d^{ei}_{\phi A}(\mathbf{k},\omega)\\ d^{ei}_{A\phi}(\mathbf{k},\omega) & d^{ei}_{AA}(\mathbf{k},\omega)\end{bmatrix}. \tag{17}$$

Collective modes correspond to zeros of the dielectric, each with a dispersion relation $\omega=\omega(\mathbf{k})$ obtained from $L^{ei}(\mathbf{k},\omega)=0$.

In the remainder of this paper we calculate the anomalous electron energy transport and associated ion heating assuming fluctuations that are predominantly magnetic. We will assume limited spectrum linewidth broadening, and hence neglect nonlinear broadening contributions to the electron and ion propagators R^e and R^i. In this approximation, the propagators are given by

$$R_\phi^s=\frac{q_s}{T_s}f_{sM}\frac{\omega-\omega_{*,s}^T}{\omega-k_\parallel v_\parallel},\quad R_A^s=-\frac{v_\parallel}{c}R_\phi^s \tag{18}$$

[$s=e(s=i)$ referring to electrons (ions)]. As a consequence the only nonlinear effect in the calculation follows from the inclusion of the incoherent part of the electron distribution, $\tilde{h}_e$. Moreover, the electron linear response will be approximated by its resonant part, consistent with collisionless dynamics. These assumptions lead to cancellations in the electron parallel energy flux, leaving only the drag term associated with the shielding of the incoherent fluctuation from the ion contribution to the dielectric. Because of this, the electron parallel energy flux will be proportional to the ion heating. If there is a principal part of the linear response from collisions and/or a significant nonlinear broadening contribution, the electron parallel energy flux retains a quasilinear electron–electron term and an electron–electron drag-like term, and the simple proportionality with the ion heating is lost. With regard to the ion response, the linear approximation is for simplicity. Resonance broadening contributions in the ions have no effect on the cancellation just mentioned and do not change the results qualitatively.

In the remaining sections, for simplicity, we will display only the results of the heat flux and ion heating associated with the magnetic potential $A_\parallel$. However, the calculations are carried out retaining the full electromagnetic response, as needed in order to properly impose self-consistency.

III. RADIAL FLUX OF PARALLEL ELECTRON KINETIC ENERGY AND ION HEATING RATE

In this section we will calculate the radial flux of electron parallel energy and the associated ion heating rate. To derive a self-consistent expression for the flux of parallel electron energy in the presence of incoherent fluctuations, we first assume steady-state, homogeneous turbulence and perform a space–time average on the Fourier transformed expression of Eq. (5). Then, dividing the nonadiabatic electron distribution h_e into its coherent and incoherent parts [Eq. (7)], we express the radial component of the parallel electron energy flux in terms of the Lenard–Balescu turbulent collision integral,[14,24]

$$\overline{Q_e^\parallel}=\Re\int d^3v\frac{m_e v_\parallel^2}{2}\sum_{\mathbf{k},\omega}\frac{ic}{B_0}(\mathbf{k}\times\mathbf{b}_0\cdot\mathbf{r})\frac{v_\parallel}{c}\times[R_A^e(\mathbf{k},\omega;v)\langle A_\parallel A_\parallel\rangle_{\mathbf{k},\omega}+\langle A_\parallel\tilde{h}_e(\mathbf{v})\rangle_{\mathbf{k},\omega}], \tag{19}$$

where we have adopted the notation $\langle aa\rangle_{\mathbf{k},\omega}=\langle a(-\mathbf{k},-\omega)a(\mathbf{k},\omega)\rangle$, and where the terms proportional to the electrostatic responses have been omitted as noted above. This expression describes the radial loss of parallel heat associated with electron streaming along the perturbed magnetic field. The first term inside the square brackets is the quasilinear diffusion contribution, while the second term, arising from the incoherent part of the distribution, is a drag-like term which represents the shielding of clumps by the plasma dielectric and the consequent emission process into collective modes. This shielding and drag process enters Eq. (19) when the calculation is made self-consistent by relating the magnetic potential present in the two correlations to the charge distribution through Ampére's Law and quasineutrality. Introducing Eq. (15) into Eq. (19) and making use of the definitions of d_{AA}^{ei} and $d_{\phi A}^{ei}$ [Eq. (14)] and of the identity $(1/L^{ei})^*=L^{ei}/|L^{ei}|^2$ [where L^{ei} is defined in Eq. (17), and the superscript $*$ indicates complex conjugation], we obtain the following expression for the parallel electron heat flux:

$$\overline{Q_e^\parallel}=\Re\sum_{\mathbf{k},\omega}\frac{ic}{B_0}(\mathbf{k}\times\mathbf{b}_0\cdot\mathbf{r})|4\pi L^{ei,-1}|^2\frac{e}{c}\int d^3v\int d^3v'\frac{m_e v_\parallel^2}{2}\frac{v_\parallel}{c}\left(\frac{v_\parallel'}{c}d_{\phi\phi}^{ei}-d_{A\phi}^{ei}\right)\{R_A^e(\mathbf{v})[d_{\phi\phi}^{ei*}\langle\tilde{J}_\parallel\tilde{h}_e(\mathbf{v}')\rangle_{\mathbf{k},\omega}-ecd_{A\phi}^{ei*}\langle\tilde{n}\tilde{h}_e(\mathbf{v}')\rangle_{\mathbf{k},\omega}]-R_A^e(\mathbf{v}')[d_{\phi\phi}^{ei*}\langle\tilde{J}_\parallel\tilde{h}_e(\mathbf{v})\rangle_{\mathbf{k},\omega}-ecd_{A\phi}^{ei*}\langle\tilde{n}\tilde{h}_e(\mathbf{v})\rangle_{\mathbf{k},\omega}]+R_A^i(\mathbf{v}')[d_{\phi\phi}^{ei*}\langle\tilde{J}_\parallel\tilde{h}_e(\mathbf{v})\rangle_{\mathbf{k},\omega}-ecd_{A\phi}^{ei*}\langle\tilde{n}\tilde{h}_e(\mathbf{v})\rangle_{\mathbf{k},\omega}]\}. \tag{20}$$

The first two contributions inside the curly brackets are proportional to R_A^e and represent quasilinear diffusion, a process involving only electrons, and the drag exerted on electron clumps by the electron component of the shielding dielectric, respectively. The last term, proportional to R_A^i, comes from the drag exerted on electron clumps by the ion component of the dielectric. These drag terms are called the electron–electron and electron–ion drag, respectively. As discussed in Sec. II, we approximate the electron response in Eq. (20) with its resonant linear part, $R_A^e\propto\delta(\omega-k_\parallel v_\parallel)$. Moreover, the incoherent fluctuations propagate ballistically at the phase velocity $u=\omega/k_\parallel$. This property can be demonstrated by a solution of the two-point equation which governs the dynamics of the correlation of incoherent density, current density and the incoherent distribution function.[13,25,26] In terms of the Fourier transformed correlation, the two-time, two-point correlation can be written in terms of a one-time, two-point correlation multiplied by a ballistic operator,

$$\langle \tilde{K}\tilde{h}_e\rangle_{\mathbf{k},\omega}=2\pi\delta(\omega-k_\parallel v_\parallel)\langle \tilde{K}\tilde{h}_e\rangle_{\mathbf{k}},$$

where the quantity $\tilde{K}$ is either $\tilde{n}$ or $\tilde{J}_\parallel$. Because of the $\delta(\omega-k_\parallel v_\parallel)$ factor in both the two-point correlation and the electron coherent response, the quasilinear diffusion and the electron–electron drag cancel each other, and the dissipative ion–electron interaction of the electron–ion drag remains as the only parallel energy transport mechanism. This is the cancellation of h_e^c alluded to in the Introduction. Expressing the remaining correlations of either $\tilde{n}$ or $\tilde{J}_\parallel$ with $\tilde{h}_e$ in terms of $A_\parallel$, again using Eqs. (15) and (16), we obtain

$$\overline{Q_e^\parallel}=\Re\, i\sum_{\mathbf{k},\omega}(\mathbf{k}\times\mathbf{b}_0\cdot\mathbf{r})\frac{m_e u^3}{2B_0}\langle A_\parallel A_\parallel\rangle_{\mathbf{k},\omega}\left[\int d^3v R_A^i(\mathbf{k},\omega;v)\right]. \tag{21}$$

The flux of parallel electron energy is ambipolar, regardless of the nature of the fluctuation spectrum or ion response. Anticipating here a result presented in the following section, the intrinsic ambipolarity of the electron parallel energy flux will not be found in the perpendicular energy flux. In the latter, which is given by Eq. (20) but with $m_e v_\parallel^2/2$ replaced by $m_e v_\perp^2/2$, the quasilinear conductive flux survives the cancellation of drag and diffusion. There are thus nonambipolar-constrained and ambipolar-constrained components. Which dominate depends on the form of the magnetic fluctuation spectrum.[16]

We now consider the velocity integral over the ion response and the spectral sums present in Eq. (21). Again following the discussion of Sec. II, we will perform the integration using an adiabatic ion approximation, i.e., $R_A^i\propto(\omega-k_\parallel v_\parallel)^{-1}$. After the integration is performed, the flux of parallel electron energy can be cast as

$$\overline{Q_e^\parallel}=-v_i\left(\mathcal{D}_n\frac{1}{L_{n_i}}+\mathcal{D}_T\frac{1}{L_{T_i}}\right)n_0T_e, \tag{22}$$

with transport coefficients $\mathcal{D}_n$ and $\mathcal{D}_T$ that still include the $\mathbf{k}$ summation. To perform the latter, we first note that because magnetic fluctuation-induced transport is produced by particles streaming along turbulent fields, the flux is critically sensitive to spectral variations in $k_\parallel$. Based on the previous discussion on the observed features of the fluctuation spectrum in MST,[16] we have obtained explicit expressions for the transport coefficients in Eq. (22) for the two distinct cases of a resonant and nonresonant spectrum of width $\Delta k_\parallel$. In the first case, the spectrum is peaked about $k_\parallel=0$, as it is in the core of MST. The transport coefficients are given by

$$\mathcal{D}_n=\sum_{\mathbf{k}_\perp,\omega}D_M\left(\frac{v_i}{v_e}\right)^2\left(1-\frac{\omega}{\omega_{*i}}\right)\left(1+\frac{\Delta u^2}{v_i^2}\right)\exp[-(\Delta u/v_i)^2],$$

$$\mathcal{D}_T=\sum_{\mathbf{k}_\perp,\omega}D_M\left(\frac{v_i}{v_e}\right)^2\frac{3}{2}\left(1+\frac{\Delta u^2}{v_i^2}+\frac{2}{3}\frac{\Delta u^4}{v_i^4}\right)\times\exp[-(\Delta u/v_i)^2].$$

Here we have simplified the notation by introducing the magnetic diffusivity $D_M\equiv(\sqrt{\pi}/\Delta k_\parallel)(\tilde{B}/B_0)^2$, and by defining the quantity $\Delta u\equiv\omega/(\Delta k_\parallel/2)$. Moreover ω is understood to be the real part of the frequency. Assuming a spectrum width that encompasses the core resonant tearing modes ($m=1$, $n=6-8$), and a frequency on the order of the diamagnetic frequency, the parameter Δu is small in the core of the MST. Expanding to zero order in this quantity we obtain

$$\mathcal{D}_n=\sum_{\mathbf{k}_\perp,\omega}D_M\left(\frac{v_i}{v_e}\right)^2\left(1-\frac{\omega}{\omega_{*i}}\right), \tag{23}$$

$$\mathcal{D}_T=\sum_{\mathbf{k}_\perp,\omega}\frac{3}{2}D_M\left(\frac{v_i}{v_e}\right)^2. \tag{24}$$

In the nonresonant case, we assume a peaked spectrum that is shifted off $k_\parallel=0$ by an amount $k_{\parallel 0}>0$. The latter case represents the spectrum typical of the edge region of the MST, having no power in the locally resonant modes for which $k_\parallel\simeq 0$ and being dominated by modes resonant at remote rational surfaces. In this case, the transport coefficients are given by ($u_0\equiv\omega/k_{\parallel 0}$)

$$\mathcal{D}_n=\sum_{\mathbf{k}_\perp,\omega}D_M\left(\frac{v_i}{v_e}\right)^2\left(1-\frac{\omega}{\omega_{*i}}\right)\frac{\Delta k_\parallel}{k_{\parallel 0}}\left(\frac{u_0}{v_i}\right)^4\times\exp[-(u_0/v_i)^2] \tag{25}$$

and

$$\mathcal{D}_T=\sum_{\mathbf{k}_\perp,\omega}D_M\left(\frac{v_i}{v_e}\right)^2\frac{\Delta k_\parallel}{k_{\parallel 0}}\left(\frac{u_0}{v_i}\right)^4\left(\frac{u_0^2}{v_i^2}-\frac{1}{2}\right)\times\exp[-(u_0/v_i)^2]. \tag{26}$$

These expressions have been simplified with an expansion to lowest order in the parameter $(\Delta k_\parallel/2)/k_{\parallel 0}$, which is small in virtue of the narrow width of the spectrum.

We now consider the heating term (11) for the ions. As we have found, the electron parallel energy flux, Eq. (22), consists only of the electron–ion drag term, which comes from the incoherent part of the distribution function. Physically this term reflects the shielding of the incoherent fluctuations by the plasma dielectric and the consequent emission into the plasma normal mode. The damping of these modes on the ions produces anomalous ion heating. Fourier transforming Eq. (11) we obtain

$$\overline{H_i}=-\Re\, i\sum_{\mathbf{k},\omega}k_\parallel\frac{u}{c}\langle A_\parallel J_{\parallel,i}\rangle_{\mathbf{k},\omega},$$

where we assume steady-state, homogeneous turbulence. The ion current can be related to the electron response using Ampére's Law,

$$J_{\parallel,i}=\int d^3v\, ev_\parallel[\delta f_e^{ad}+R_A^eA_\parallel]+\int d^3v\, ev_\parallel\tilde{h}_e+\frac{ck^2}{4\pi}A_\parallel,$$

where again the electrostatic contribution has been omitted. Introducing this expression in the ion heating expression (disregarding the adiabatic part δf_e^{ad} and the term proportional to k^2 since they will give no contribution) we obtain

$$\overline{H_i}=\Re\, i\sum_{\mathbf{k},\omega}k_\parallel\frac{u}{c}\int d^3v\, ev_\parallel[R_A^e(\mathbf{k},\omega;v)\langle A_\parallel A_\parallel\rangle_{\mathbf{k},\omega}+\langle A_\parallel\tilde{h}_e(\mathbf{v})\rangle_{\mathbf{k},\omega}].$$

Using the self-consistency constraints Eq. (13) and performing the velocity space integration over the resonant electron response, we find that the ion heating can be expressed in terms of the electron parallel energy flux associated with magnetic turbulence,

$$\overline{H_i}=\sum_{\mathbf{k},\omega} 2\frac{|\Omega_e|}{u}\frac{k_\parallel}{\mathbf{k}\times\mathbf{b}_0\cdot\mathbf{r}}\overline{Q_e^\parallel}(\mathbf{k},\omega). \tag{27}$$

Here $\overline{Q_e^\parallel}(\mathbf{k},\omega)$ is the right-hand side of Eq. (21), excluding the summation sign, $\Omega_e\equiv -eB/(m_e c)$, and $\mathbf{k}\times\mathbf{b}_0\cdot\mathbf{r}$ is essentially $k_\perp$. This equation represents a turbulent energy balance which ties the ion heating to the ambipolar-constrained electron parallel energy transport. Since the underlying physical process is the absorption by the ions of waves emitted by electron clumps, we refer to it as a turbulent Kirchhoff's Law. Note that this relation is general, i.e., it applies to any plasma with magnetic fluctuations, independent of any specific mode, provided the assumptions of a collisionless plasma and moderate line broadening are satisfied. The first proportionality factor in Eq. (27), $|\Omega_e|/u$, is the inverse distance traveled by an electron in a gyroperiod. The inverse dependence on u reflects the fact that a slowing of the electron loss rate to u enhances the energy transfer to ions. In MST-like plasmas, this factor is large. Even though the factor $k_\parallel/k_\perp$ is small, the overall proportionality factor in Eq. (27) is large (of the order of 10^3-10^4 cm^{-1} for MST parameters). Physically, the large factor applied to ion heating relative to the radial electron energy flux is explained by the fact that the magnetic field lines wrap many times around the torus before diffusing radially by a small amount. As a consequence, electrons following the field lines have the opportunity to deposit, through the ambipolarity constraint, a significant amount of energy to the ions before undergoing appreciable radial diffusion. We also note that the direct proportionality between the ion heating and the electron flux implies a decrease of the electron temperature due to parallel heat losses whenever ion heating is present. However, this loss is small compared to nonambipolar constrained heat losses, if such occur.

In the adiabatic ion limit ($u/v_i<1$) we first perform the required integration over the ion response, and then the $k_\parallel$ summation. For a resonant (superscript r) and nonresonant (superscript nr) spectrum we obtain, respectively,

$$\overline{H_i^r}=\sum_{\mathbf{k}_\perp,\omega} D_M v_i n_0 T_i \frac{1}{L_{n_i}^2}\frac{\omega}{\omega_{*i}}\left[\left(\frac{\omega}{\omega_{*i}}-1\right)-\frac{1}{2}\eta_i\right] \tag{28}$$

and

$$\overline{H_i^{nr}}=\sum_{\mathbf{k}_\perp,\omega} D_M v_i n_0 T_i \frac{1}{L_{n_i}^2}\frac{\omega}{\omega_{*i}}\left[\left(\frac{\omega}{\omega_{*i}}-1\right)+\eta_i\left(\frac{1}{2}-\frac{u_0^2}{v_i^2}\right)\right]\frac{\Delta k_\parallel}{k_{\parallel 0}}\left(\frac{u_0}{v_i}\right)^2\exp[-(u_0/v_i)^2]. \tag{29}$$

Here we have again simplified the results by expanding in the small parameters $\Delta u/v_i$ (resonant case) and $(\Delta k_\parallel/2)/k_{\parallel 0}$ (nonresonant case). The presence of an ion thermal velocity in the ion heating expressions reflects the ambipolar physics which leads to and ultimately governs the heating process.

Having performed the required summations, we are now in a position to quantify better the relation between the ion heating and the electron parallel energy flux presented in Eq. (27). We choose parameters typical of the core of the MST. Loosely speaking, the core is where the $m=1$, $n=6-8$ tearing modes are resonant. In the MST this occurs approximately at $r\simeq 0.5a$ ($a=52$ cm being the minor radius). These parameters, which include the width $\Delta k_\parallel$ of the fluctuation spectrum, the profile parameters L_{n_i} and L_{T_i}, and the frequency of the relevant fluctuations, are not well established, either theoretically or experimentally. To obtain $\Delta k_\parallel$ we observe that the magnetic fluctuation spectrum in the MST (Ref. 27) is broad and its power is concentrated in the $m=1$, $n=6-8$ core-resonant tearing modes. Using the polynomial function model[28] to fit typical MST equilibrium profiles for the magnetic field, we obtain $\Delta k_\parallel=0.86$ m^{-1}. Temperature and density profile diagnostics in the MST have limited resolution. There is, however, some evidence that at $r/a\simeq 0.5$ both profiles tend to be broad, with the density usually being more peaked. We assume $L_{n,i}=-70$ cm and $\eta_i=0.1$. The flatness of these profiles is also responsible for the small values of the diamagnetic frequencies in the core region. We assume $\omega_{*,i}=-5000$ Hz and $\omega_{*,e}=10\,000$ Hz. Finally we turn to an assessment of the mode frequency ω. The theoretical expressions we have derived are sensitive to the value of the fluctuation frequency. For example, consider the electron parallel energy flux driven by a resonant fluctuation spectrum [Eq. (22)] with transport coefficients given by Eqs. (23) and (24). With $\eta_i\ll 1$, it is easily seen that the total flux is outward when $\omega>0$ [i.e., for mode rotation in the direction of the electron diamagnetic frequency, as defined after Eq. (6)]. On the other hand, for mode rotation in the ion direction ($\omega<0$) the flux is outward if $|\omega|<|\omega_{*i}|$. Similarly, the ion heating [Eq. (28)] is positive when $\omega>0$, but becomes negative for negative values of the frequency such that $|\omega|<|\omega_{*i}|$. Unfortunately, fluctuation frequencies in the plasma frame are not well known from measurement. Likewise, there are no theoretical predictions available for fluctuation frequencies at the present time. The frequency depends not just on pressure gradients, but on the nonlinear torques that govern rotation. Deriving the frequency lies outside the scope of this work. Therefore, in our calculations we will assume that the modes rotate (in the plasma frame) in the electron diamagnetic direction, and take $\omega=0.8\omega_{*,e}=8000$ Hz.

Using these parameters as representative of the core of the MST, we quantify the relation between the ion heating rate and electron parallel energy transport by considering the following temperature evolution equations for ions and electrons:

$$\frac{d}{dt}\left(\frac{3}{2}n_i T_i\right)=H_i, \tag{30}$$

$$\frac{d}{dt}\left(\frac{3}{2}n_e T_e\right)=-\boldsymbol{\nabla}\cdot\mathbf{Q}_e^\parallel, \tag{31}$$

where $Q_e^{\parallel}$ and H_i are given, respectively, by Eqs. (22)–(24) and (28). Note that Eq. (31) includes only the flux of parallel energy, not the potentially larger perpendicular energy flux discussed in the next section. Hence T_e evolution in the exercise described here only tracks part of the electron energy loss. Assuming initial values of 350 and 150 eV for the electron and ion temperatures respectively, and integrating Eqs. (30) and (31) with $\tilde{B}/B=3\%$ for a time interval of 0.5 ms, we find that the ion temperature increases to 540 eV while the electron temperature decreases slightly to 349 eV. Even if the flux of electron parallel energy associated with these levels of magnetic fluctuations is small and leads to a very small decrease in T_e, the corresponding ion heating rate is large and induces a significant increase in T_i during a sawtooth crash.

To end this section, we note that a comparison of the fluxes of electron parallel energy and ion heating rate indicates that these are significantly smaller for a nonresonant spectrum [Eqs. (25) and (29)] than for a resonant spectrum [Eqs. (23) and (28)]. The reduction factor is proportional to the nondimensional parameters $\Delta k_\parallel/k_{\parallel 0}$ and u_0/v_i. We recall that $\Delta k_\parallel$ is the width of the spectrum, $k_{\parallel 0}>0$ is its center-point, and $u_0\equiv\omega/k_{\parallel 0}$. The wave vector $k_{\parallel 0}$ is equal to the average value of the wave vectors associated with the modes $m=1$, $n=6$–8 which drive the edge spectrum in the MST. We find $k_{\parallel 0}=4.5\ \mathrm{m}^{-1}$. Using this value and other data from the MST edge, we find that the parallel heat transport coefficients are reduced by a factor $\simeq 1.5\times 10^{-5}$, while the ion heating rate is reduced by $\simeq 1.8\times 10^{-3}$.

IV. RADIAL FLUX OF ELECTRON PERPENDICULAR KINETIC ENERGY

The anomalous ion heating described in the previous section is tied to the flux of parallel electron energy. This flux is ambipolar, independent of the fluctuation spectrum, and therefore small. As already briefly mentioned, the structure of the flux of perpendicular energy differs substantially from that of parallel energy because the quasilinear conductive flux survives the cancellation of drag and diffusion. For a resonant spectrum this implies $Q_e^{\perp}\gg Q_e^{\parallel}$. For a nonresonant spectrum, however, the major contribution comes from the ion drag, or ambipolar-constrained part of the flux, so that $Q_e^{\perp}\simeq Q_e^{\parallel}$. Consequently, it is very important to consider the flux of perpendicular energy whenever the dominant contribution to the fluctuation spectrum comes from resonant modes (as in the core of the MST). In this case, the flux of perpendicular electron heat dominates the energy losses, and is responsible for the decrease of the electron temperature observed during bursts of magnetic activity in the MST.[7]

In this section we present expressions for the radial component of the electron perpendicular heat flux for both resonant and nonresonant spectra. As in the parallel case, we expect ion heating to be associated with the ambipolar part of the perpendicular electron heat flux. Calculation of this heating lies outside the drift-kinetic formalism employed herein, and is left for future work. The radial component of the anomalous perpendicular electron heat flux can be calculated following a similar procedure as that employed in Sec. III to calculate the parallel heat flux.[16] We have

$$\overline{Q_e^{\perp}}=-\left[v_e\left(\mathcal{D}_T^{ee}\frac{1}{L_{T_e}}\right)+v_i\left(\mathcal{D}_n^{ei}\frac{1}{L_{n_i}}+\mathcal{D}_T^{ei}\frac{1}{L_{T_i}}\right)\right]n_0T_e, \tag{32}$$

where in the resonant case the transport coefficients are given, to zero order in $[\omega/(\Delta k_\parallel/2)]/v_i<1$, by

$$\mathcal{D}_T^{ee}=\sum_{\mathbf{k}_\perp,\omega}D_M,\quad \mathcal{D}_n^{ei}=\sum_{\mathbf{k}_\perp,\omega}D_M\left(1-\frac{\omega}{\omega_{*i}}\right),$$
$$\mathcal{D}_T^{ei}=\sum_{\mathbf{k}_\perp,\omega}\frac{1}{2}D_M. \tag{33}$$

For a nonresonant spectrum, we expand in $(\Delta k_\parallel/2)/k_{\parallel 0}<1$ and obtain

$$\mathcal{D}_T^{ee}=\sum_{\mathbf{k}_\perp,\omega}D_M\frac{\Delta k_\parallel}{k_{\parallel 0}}\left(\frac{u_0}{v_e}\right)^2,$$
$$\mathcal{D}_n^{ei}=\sum_{\mathbf{k}_\perp,\omega}D_M\left(1-\frac{\omega}{\omega_{*i}}\right)\frac{\Delta k_\parallel}{k_{\parallel 0}}\left(\frac{u_0}{v_i}\right)^2\exp[-(u_0/v_i)^2], \tag{34}$$

$$\mathcal{D}_T^{ei}=\sum_{\mathbf{k}_\perp,\omega}D_M\left(\frac{u_0^2}{v_i^2}-\frac{1}{2}\right)\frac{\Delta k_\parallel}{k_{\parallel 0}}\left(\frac{u_0}{v_i}\right)^2\exp[-(u_0/v_i)^2]. \tag{35}$$

Note that in deriving these expressions we have assumed that the perpendicular energy of the electrons in the incoherent distribution, $\tilde{v}^2\equiv\int d^2v_\perp v_\perp^2\tilde{h}/\int d^2v_\perp\tilde{h}$, is equal to the thermal energy v_e^2.

From these expressions we see that, in the core region (resonant spectrum), the major contribution to the flux comes from its quasilinear part because of its v_e-rate (compared to the v_i-rate of the drag part). In the edge region (nonresonant spectrum) the transport coefficients present in the drag part of the flux are reduced by a factor $\propto(\Delta k_\parallel/k_{\parallel 0})(u_0/v_i)^2$. This is much larger than the reduction of the quasilinear part of the flux $\propto(\Delta k_\parallel/k_{\parallel 0})(u_0/v_e)^2$. Consequently, the major contribution in the edge region comes from the drag part.

We end this section by making a more quantitative comparison between the fluxes of parallel and perpendicular energy in the core. We obtain

$$\frac{\overline{Q_e^{\perp}}}{\overline{Q_e^{\parallel}}}\propto\frac{v_eD_ML_{T_e}^{-1}n_0T_e}{v_iD_M(v_i/v_e)^2L_{T_i}^{-1}n_0T_e}$$
$$=\left(\frac{v_e}{v_i}\right)\left(\frac{v_e}{v_i}\right)^2\frac{L_{T_i}}{L_{T_e}}\simeq\left(\frac{m_i}{m_e}\right)^{3/2}\gg 1,$$

where in the last step we have assumed equal temperatures.

V. A HEURISTIC TRANSPORT MODEL

To determine if the expressions presented in the previous sections are able to qualitatively reproduce the time evolution of the electron and ion temperatures during a sawtooth crash in the core region of the MST, we consider the following heuristic transport model for coupled evolution of T_i and T_e:

$$\frac{d}{dt}\left(\frac{3}{2}n_i T_i\right)=H_i^{\mathrm{an}}+H_i^{\mathrm{cl}}-\frac{\frac{3}{2}n_i T_i}{\tau_{Ei}}, \tag{36}$$

$$\frac{d}{dt}\left(\frac{3}{2}n_e T_e\right)=H_e^{\mathrm{Ohm}}+H_e^{\mathrm{cl}}-\frac{S}{V}(Q_e^{\mathrm{an},\parallel}+Q_e^{\mathrm{an},\perp})-\frac{\frac{3}{2}n_e T_e}{\tau_{Ee}}. \tag{37}$$

Here there is a term depending on the ion energy confinement times τ_{Ei}, which reflects losses associated with classical and electromagnetic anomalous ion transport. The corresponding term in the electron equation reflects classical and electrostatic anomalous electron transport losses.

We use this model to simulate the core region of the MST, taking the anomalous ion heating rate H_i^{an} to be given by Eq. (28), $Q_e^{\parallel}$ by Eq. (22) with coefficients given by Eqs. (23) and (24), and $Q_e^{\perp}$ by Eqs. (32)–(33). The quantities S and V are a surface in the core region $(0\leqslant r/a\leqslant 0.5)$, and the volume it encloses. The remaining terms are the classical heating associated with equipartition, H^{cl}, which in cgs units can be expressed as

$$H_i^{\mathrm{cl}}=-H_e^{\mathrm{cl}}=5\times 10^{-37}\frac{(m_e m_i)^{1/2}n_0\ln\Lambda_{ie}}{(m_e T_i+m_i T_e)^{3/2}}(T_e-T_i)$$

(where $\ln\Lambda_{ie}\simeq 17$), and the Ohmic heating deposited to the electrons, $H_e^{0\mathrm{hm}}$, which we assume is equal to 2.5×10^7 (erg/s)/cm^3 in the core region of the MST. For the remaining parameters we use the same values as in the integration of Sec. III. We first find that, in steady-state, $\tau_{Ee}=1.5$ ms and $\tau_{Ei}=3.1$ ms. Experimental data on transport in the MST indicate that these values are too large by a factor of 2 or 3. However, since our goal is to assess qualitatively the effects of our expressions for Q_e and H_i, we neglect this discrepancy and proceed with the simulation, keeping in mind that transport rates will not be realistic. We integrate the two equations in time, following the evolution of T_e and T_i in response to a change in the magnetic fluctuation level that simulates the occurrence of a sawtooth crash. The result of the integration is presented in Fig. 2. This figure shows how our model is able to capture the main trends of the ion and electron temperature responses to a sawtooth cycle (compare with Fig. 10 of Ref. 7).

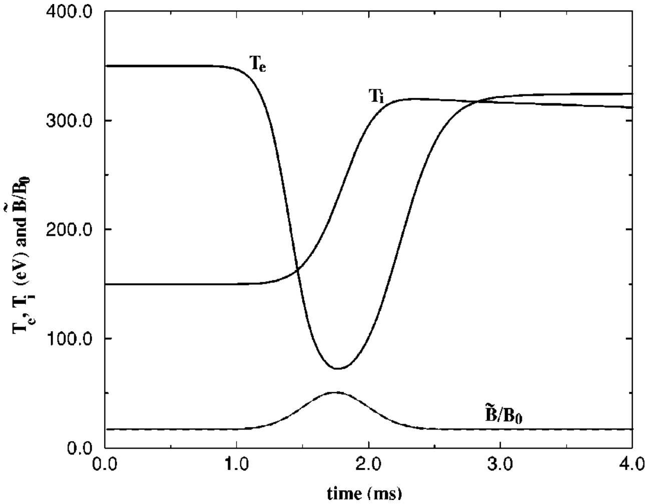

FIG. 2. Time evolution of $\tilde{B}/B_0$, T_e, and T_i.

VI. SUMMARY AND CONCLUSIONS

We have considered the problem of anomalous ion heating in magnetic turbulence, showing that ion heating is a natural by-product of magnetic turbulence. The heating is due to the absorption by ions of waves emitted by granulations in the electron distribution function. Since these are created and regulated by the magnetic turbulence itself, their presence represents a continuous saturation process that converts the energy stored in magnetic turbulence into ion kinetic energy. Physically the granulations, or incoherent fluctuations, consist of groups of electrons which stream together along the perturbed magnetic field lines, remaining correlated for long times. As a consequence, these bunched electrons effectively act as macroparticles. During their ballistic propagation, they are shielded by the plasma dielectric, which consists of coherent electron and ion responses. The shielding process links the dynamics of clumped electrons and the dielectric, allowing a continuous exchange of momentum and energy between these two plasma constituents.

To describe this process we use a drift-kinetic formalism, retaining the $E\times B$ and magnetic flutter nonlinearities. The latter describes streaming of electrons along perturbed magnetic field lines. The conservative energetics of the shielding physics is imposed through quasineutrality and Ampére's Law. Within this mathematical framework, we have evaluated the magnetic fluctuation-induced electron energy flux, and the rate of conversion of magnetic energy into ion kinetic energy. The principal findings are as follows:

(1) Under the assumptions of moderate resonance broadening and collisionless dynamics, the magnetic fluctuation-induced flux of electron energy parallel to the magnetic field is ambipolar. The source of the ambipolar constraint originates with the wave-particle resonance factor $\delta(\omega-k_\parallel v_\parallel)$, which applies to both the incoherent electron fluctuations and the coherent electron response. This leads to a cancellation of the quasilinear diffusion with the electron–electron drag, leaving the dissipative interaction of the electron–ion drag as the only parallel energy transport mechanism. This result does not require any particular form of the magnetic fluctuation spectrum or any particular choice of the ion response.

(2) The reduction of the electron loss rate to one controlled by the ion thermal velocity implies an anomalous transfer of energy to the ions. Consequently, the ion heating rate is proportional to the radial flux of electron parallel energy. The heating is accomplished through the absorption by ions of waves emitted by electron clumps. Hence, the proportionality relationship can be represented as a turbulent generalization of Kirchhoff's Law usually associated with discreteness effects. There is a significant amount of heating associated with the flux

of parallel electron energy because electrons stream over large parallel distances, all the time heating ions, before undergoing appreciable radial diffusion. The flux of electron parallel energy in the turbulent Kirchhoff's Law implies a decrease of the electron temperature due to parallel heat losses whenever ion heating is present. This decrease is small because the ambipolar-constrained parallel energy flux is small.

(3) In the MST, the fluctuations that drive transport are resonant in the core region and nonresonant at the plasma edge. While the turbulent Kirchhoff's Law holds everywhere by virtue of the ubiquitous ambipolar character of the parallel energy flux, the total electron energy loss (including both parallel and perpendicular energy) varies considerably, depending on whether the fluctuation spectrum is resonant or nonresonant. For a resonant spectrum, the flux of perpendicular electron energy is nonambipolar, and hence much greater than the corresponding parallel energy flux. This suggests that the decrease in electron temperature observed in the MST core during sawtooth events is likely due to electron perpendicular energy losses. In the core of the MST, where the spectrum is resonant, we have found that the flux of perpendicular energy is of the order of 10^4 larger than that of parallel energy.

(4) We have solved a simple zero-dimensional transport model for the nonlinearly coupled electron and ion temperatures. The model incorporates the anomalous ion heating rate from the ambipolar-constrained parallel energy flux, and the electron heat loss rate from the nonambipolar-constrained flux of perpendicular energy (as just observed, $Q_e^{\perp} \gg Q_e^{\parallel}$ in the core). Simulating a sawtooth event as a transient burst of magnetic energy during one millisecond, we have obtained ion and electron temperature transients that are qualitatively like those observed in the MST.

The ion heating calculation presented in this paper is generic, i.e., it is not specific to any particular type of mode. We stipulate only that there is a collective resonance in the turbulent spectrum that drives instability. Therefore, in principle our results could apply to other situations with anomalous ion heating and high levels of magnetic fluctuations. These include magnetic fusion experiments with relaxed magnetic fields, such as the spheromak, and experiments with reconnection, such as the MRX. Astrophysical situations are also included. Whether, in fact, the ion heating observed in these other cases is due to the mechanism studied herein remains speculative and must await further study.

A variety of other issues requires further study and clarification. Our results do not directly apply to collisional regimes or those with strongly broadened resonances. In these cases the simple proportionality between electron parallel energy flux and ion heating is replaced by a more complicated relationship. We have also assumed stationary turbulence. The existence of growing modes, for example, alter the simple resonant energy exchange mechanism between electron clumps and normal modes. The turbulent Kirchhoff's Law ties ion heating to the radial flux of electron parallel energy. We also expect ion heating associated with the ambipolar-constrained part of the electron perpendicular heat flux. We have presented expressions for the electron perpendicular energy flux, but have not attempted to generalize the turbulent Kirchhoff's Law by including ion heating from the flux of perpendicular energy. This requires gyrokinetic formalism and is left for future work. Future work must also address the issue of the fluctuation frequency. We anticipate that diamagnetic effects and rotation play a role, but also the incoherent emission process, which is known to broaden the power spectrum at fixed wave number.

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