THEORETICAL STUDIES ON THE ROLE OF FLOWS AND CURRENTS IN THE RFP

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Abstract

A number of issues associated with the interaction of plasma flows and currents with plasma fluctuations in RFP plasmas are addressed. Self-consistency arguments on the structure of turbulent mean field forces imply a relaxation behavior for both the current and plasma momentum. Nonlinear tearing mode interaction affect flow profile evolution through the production of localized electromagnetic torques. A model for shear flow generation is presented that drives spontaneous enhanced confinement regimes. A calculation of the tearing mode stability index is presented when a narrow flow shear layer is present in the exterior region. An extensive parameter survey yields an optimization for RFP profile control using lower hybrid waves. Alfven wave current drive prospects in RFPs are also addressed.

I. INTRODUCTION

The current profile is crucial to confinement in the reversed field pinch (RFP) through its role in plasma relaxation, the dynamo and anomalous transport. A growing body of recent experimental evidence suggests a similarly important role for flows. In this work we describe a number of theoretical models that calculate the interaction between plasma fluctuations and aspects of plasma flow and current profile evolution. In particular, calculations are presented that: 1) demonstrate a nontrivial coupling between parallel currents and momentum via turbulent mean field forces that suggests relaxation processes for both current and flow profiles; 2) provide a theory for the nonlocal interaction of tearing instabilities that affect flow profile evolution through localized electromagnetic torques; 3) introduce a model for shear flow generation that drives spontaneous enhanced confinement regimes; 4) calculate the presence of a narrow flow shear region on tearing mode stability; 5) show an optimization of lower hydrid current drive for current profile control; and 6) discuss the possibility of Alfven wave current drive in RFPs.

II. SELF-CONSISTENT MEAN FIELD FORCES IN TURBULENT PLASMAS

The properties of turbulent plasmas are described using the two-fluid equations [1]. Three global constraints for the fluctuation induced mean field forces that act on the ion and electron fluids are derived that generalize previous calculations using the MHD model [2]. The physical interpretation of the constraints is that plasma fluctuations dissipate energy while preserving the generalized helicity integrals for each species. These constraints imply functional forms for the parallel mean field forces in the Ohm's law and the momentum balance equations given by

$$\mathbf{F}_o\cdot\mathbf{B}_o = \nabla\cdot[\kappa_e^2\,\nabla(\frac{\mathbf{B}_o\cdot\mathbf{J}_o}{B_o^2}) + \Lambda_e\nabla(\frac{en_o\mathbf{u}_o\cdot\mathbf{B}_o}{B_o^2})] \quad , \qquad (1)$$

$$\mathbf{F}_M\cdot\mathbf{B}_o = \nabla\cdot[\kappa_i^2\,\nabla(\frac{en_o\mathbf{u}_o\cdot\mathbf{B}_o}{B_o^2}) + \Lambda_i\nabla(\frac{\mathbf{B}_o\cdot\mathbf{J}_o}{B_o^2})] \quad , \qquad (2)$$

where the anomalous transport coefficients satisfy $\kappa_e^2 > 0$, $\kappa_i^2 > 0$ and $(\Lambda_i + \Lambda_e)^2 < \kappa_e^2 k_i^2 /4$. The off-diagonal terms suggest a non-trivial relationship between current profile and parallel momentum evolution. The mean field forces attempt to relax the plasma to the state $\mathbf{J}_o = \lambda_o\mathbf{B}_o$, $n\mathbf{u}_o = \lambda_1\mathbf{B}_o$ to lowest order where λ_o and λ_1 are constants. This work suggests that the large flow profile changes observed following discrete dynamos in MST [3] may be described as a relaxation phenomenon in analogy with current profile relaxation.

III. NONLINEAR TEARING MODE INTERACTION

A novel aspect of nonlinear tearing mode interactions in RFP's is the production of localized electromagnetic torques which can influence the plasma flow properties [4]. The physics of this interaction is similar to the interaction of a resonant static field error and an isolated magnetic island. The nonlinear interaction of two tearing modes of incommensurate helicity with mode numbers $\mathbf{k}' = (m',n')$ and $\mathbf{k}-\mathbf{k}' = (m-m',n-n')$ can produce a magnetic signal resonant with a third mode with mode number $\mathbf{k}$. If the third mode has a rational surface in the plasma, a nonlinearly driven eddy current develops in the vicinity of the q = m/n surface. The eddy current in combination with a resonant magnetic perturbation produces a $\delta\mathbf{J}\times\delta\mathbf{B}$ torque at the rational surfaces of the form $T_{int} = \Sigma\ \ C(\mathbf{k},\mathbf{k}')\Psi_\mathbf{k}\Psi_{\mathbf{k}'}\Psi_{\mathbf{k}-\mathbf{k}'}\sin(\Delta\zeta)$ where $\Psi_\mathbf{k}$, $\Psi_{\mathbf{k}'}$ and $\Psi_{\mathbf{k}-\mathbf{k}'}$ are the mode amplitudes that satisfy a wave number resonance condition, $C(\mathbf{k},\mathbf{k}')$ is a geometric coupling coefficient and $\Delta\zeta = \int dt\,(\omega_\mathbf{k} - \omega_{\mathbf{k}'} - \omega_{\mathbf{k}-\mathbf{k}'})$ is the time dependent phase difference accounting for differential rotation, where the frequency $\omega_\mathbf{k} = \mathbf{k}\cdot\mathbf{V}(q = m/n)$ is taken to be due to bulk plasma rotation at the rational surface.

When T_{int} dominates the torque balance equation, a nonlocal momentum transport property ensues which forces the dominant three mode interaction to satisfy a three wave phase velocity resonance condition with $\Delta\zeta = 0$. For situations typical of the RFP, the dominant nonlinear interaction involves two core resonant m =1 modes and the m = 0 mode resonant at the reversal surface. While localized torques involving static field errors attempt to cause the modes to cease rotating and lock to the phase of the field error, the interior torques attempt to cause the plasma to rotate as a rigid rotor at the phase velocity of the m = 0 mode. In the MST RFP, the internal torques are generally larger than those associated with field errors, which suggests that the m = 0 mode plays an important role in the observed flow deceleration associated with a discrete dynamo event.

IV. MODELING OF FLOW SHEAR GENERATION

In Spontaneous Enhanced Confinement (SEC) discharges in MST [5], there is a transition in which $\mathbf{E}\times\mathbf{B}$ flow shear is generated after a sawtooth crash in a narrow layer outside the reversal radius. The transition is favored by wall conditioning, deep reversal, high plasma current, and low density. At the transition the pressure gradient contribution to the radial force balance is weak relative to that of toroidal flow. Initial measurements indicate that the magnetic Reynolds stress $\langle B_r B_\phi\rangle/\mu_o$ can become larger during a sawtooth crash relative to its value between sawtooth crashes. Preliminary measurements indicate the fluid Reynolds stress $\rho_o\langle u_r u_\phi\rangle$ is considerably smaller. The SEC transition differs from the transitions that create other flow shear induced transport barriers in tokamaks and stellarators [6] in at least three ways: 1) The magnetic Reynolds stress drives $\mathbf{E}\times\mathbf{B}$ flow generation initially; 2) a sawtooth crash is required to create Reynolds stress, not just provide heat to the edge; and 3) flow (toroidal) is damped by internal nonlinear couplings and external coupling to field errors at the wall, not by the neoclassical poloidal flow damping important in tokamak transitions. The observed change of the Reynolds stress cross-phase from approximately $\pi/2$ between sawteeth to near 0 during a sawtooth oscillation is modeled theoretically from the transient spectrum of edge-resonant tearing modes excited by the sawtooth crash on rational surfaces near the reversal radius. The modes make a nonzero contribution to the Reynolds stress due to the diamagnetic frequency, which is large in the edge region where the pressure gradient is large. Between sawtooth crashes, these modes have finite amplitude and their contribution to the Reynolds stress is small. During a crash, the nonlinear coupling to global modes excite the edge modes and their contribution to the Reynolds stress becomes significant. This drives a toroidal flow, which in turn drives the radial electric field. As fluctuation amplitudes decrease, the pressure gradient steepens. Once the edge resonant spectrum has relaxed after the sawtooth crash, the steepened pressure gradient maintains the radial electric field. The Reynolds stress of edge modes is balanced by internal torques associated with the coupling to the core-resonant tearing modes and the exterior torques due to field errors [4].

V. EFFECT OF LOCALIZED EXTERNAL FLOW SHEAR ON TEARING MODES

In MST, tearing mode fluctuation amplitudes decrease over all frequencies when a narrow layer of sheared **E**×**B** flow is generated in SEC discharges [5]. To determine if global tearing mode growth is affected by a narrow shear layer in the external kink part of the eigenmode, Δ', the tearing mode discontinuity parameter, is calculated where $\Delta' > 0$ is the instability criterion for linear and nonlinear tearing mode growth. The **E**×**B** flow profile is modeled to be $V = 0$ for $x < r_E - L_E/2$, $V = [x - r_E + L_E/2]V_E/L_E$ for $r_E - L_E/2 < x < r_E + L_E/2$, and $V = V_E$ for $x > r_E + L_E/2$, where $x = 0$ is the position of the rational surface, r_E is the position of the **E**×**B** shear layer, L_E is the layer width, and V_E is the flow shear strength. The flow shear is entirely outside the resistive layer, i.e., $r_E - L_E/2 > \delta$, where δ is the resistive layer width. For simplicity, a sheared slab geometry is chosen and the current gradient scale length is treated as constant over the tearing mode, i.e., J_o^{-1} dJ_o/dx = constant, where J_o is the equilibrium current. Matching external kink eigenmodes modified by flow shear with eigenmodes on either side of the shear layer, it is found that shear flow affects Δ' if $S \equiv V_E L_S/V_A L_E > 1$, where V_A is the Alfvén velocity, and L_S is the magnetic shear scale length. This criterion typifies prior work in which shear flow extended over the entire eigemode [7]. The expression for Δ' is given by

$$\Delta' = -2kC_0\pi\lambda \cot(\pi\lambda) + 4\chi(S)\,\Gamma(1 - \lambda), \qquad (3)$$

where $\lambda = \mu_o|J_0'|L_s/2B_o k$, k is the perpendicular wave number, C_o is a constant of order unity, Γ is the gamma function, and $\chi(S)$ is a negative function of flow shear strength that depends on the details of the matched eigenfunctions. The first term of (3) is the tearing criterion in the absence of shear flow, indicating the onset of tearing instability at $\lambda = 0.5$. The second term is the flow shear stabilization. When $ka \cong 1$ and $r_E - \lambda \cong a/3$, where a is the minor radius, $\chi(S)$ decreases monotonically from –0.05 at $S = \sqrt{3}$ to –0.4 for S = 10. For a current gradient scale length of order half the minor radius, the tearing mode is completely stabilized for $S \cong 10$. For MST, estimates of S place its value as only slightly larger than unity, in which case shear has a weak effect on Δ'. This suggests that the observed decrease of tearing mode activity is due in part to indirect effects wherein flow shear modifies edge conditions and lowers the resistivity. However, further increases in flow shear strength are predicted to have a marked direct effect on tearing mode activity.

VI. OPTIMIZATION OF LOWER HYBRID CURRENT DRIVE

Lower hybrid current drive (LHCD) has been shown theoretically to be an effective means of modifying the current profile in the outer region of the RFP and thereby suppressing large amplitude tearing modes [8]. Experimentally, pulsed poloidal current drive (PPCD) has led to a five-fold increase in the energy confinement time of the MST RFP [9]. These results strongly motivate us to experimentally implement the

LHCD scheme, which is not transient and potentially more controllable than PPCD. To refine our earlier theoretical results and to optimize the design of the LHCD experiment in MST, we have conducted an extensive parameter survey using a newly developed suite of codes [10] including MSTEQ (Grad-Shafarnov solver), GENRAY (generalized ray-tracing code), and CQL3D (3-D, relativistic, bounce-averaged, quasilinear, Fokker-Planck code). These codes are written for or adapted to toroidal RFP geometry. The parameter survey has confirmed our earlier theoretical results that in a typical MST discharge a judicious choice of wave frequency, parallel refractive index, wave spectrum and antenna location will produce an auxiliary parallel (~ poloidal) current which is well localized to the targeted radius (r ~ 0.65a) and sufficiently high in efficiency (~ 0.5 MA/1 MW). The parameter survey also reveals that wave penetration to the RFP core can be facilitated by high power RF-induced transparency, the process in which quasilinear velocity space diffusion flattens the electron distribution and reduces the Landau damping. This suggests a possible use of slow waves for core current drive and heating in the RFP. To avoid ion Landau damping, mode conversions and excessive wave scattering, it is desirable to use higher frequency slow waves. By using a slow wave at 800 MHz (~ 8 f_{LH} at r = a) with a parallel refractive index of 7.5, an inboard launch in MST is found to be optimal for efficiency and positioning over a range of RF power. For core current drive and heating, an outboard launch is shown to be more effective assuming sufficient RF-induced transparency.

VII. ALFVEN WAVE CURRENT DRIVE IN RFPs

A potentially attractive method of current generation in RFP's is Alfvén wave current drive. Driving current by resonantly pushing low parallel velocity electrons is highly efficient although the effect of electron trapping on efficiency remains unresolved. Nonresonant current drive, via the α-dynamo effect of Alfvén waves is inefficient in weakly nonuniform magnetic fields (e. g. in tokamaks) but its efficiency is greatly enhanced by magnetic stochasticity [11] (e. g. in RFP's) due to the increased dissipation of Alfvén waves [12]. The latter is reduced by FLR effects which are important for RFP parameters. Nevertheless, the stochasticity-enhanced α-effect of Alfvén waves is significant; for parameters of present-day RFPs the resulting current drive efficiency exceeds the Ohmic one while for RFP-based fusion reactors it is merely an order of magnitude smaller than the Ohmic efficiency [11].

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