

LIMITATIONS OF THE HAMILTONIAN TREATMENT FOR COLLISIONLESS ASTROPHYSICAL ACCRETION FLOWS

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Abstract. In the Hamiltonian treatment of purely mechanical systems, the canonical and actual momentum of a particle are the same. In contrast, for a plasma of charged particles and electromagnetic fields, those two momenta are different. We show how this distinction is fundamental in identifying the limitations of a recent attempt by Binney (2003) to rule out two-temperature collisionless astrophysical accretion flows from Hamiltonian theory. This illustrates the Hamiltonian method for astrophysical plasmas, its relation to the equations of motion, and its role in practical calculations. We also discuss how the complete Hamiltonian treatment of a plasma should couple the particle motion to a fully dynamical treatment of the electromagnetic fields. Our results stand independent from the discussion of Quataert (2003) who argued that time scale calculated in Binney (2003) is not the equipartition time as claimed.

Key words: accretion: accretion discs – black hole physics – galaxies: active – X-ray: stars – binaries

1. INTRODUCTION

An accretion disc of hot plasma orbiting a massive black hole and slowly spiralling in from the action of viscosity is widely considered to be the main energy source for very luminous extragalactic objects such as quasars and other anomalously bright active galactic nuclei (AGN) (Shapiro & Teukolsky 1983; Krolik 1999). Recently, the determination of the density and temperature of the gas in the accretion disc surrounding the central black hole of our own Milky Way Galaxy as well as discs around other nearby galactic centers has become possible from X-ray data taken by orbiting X-ray telescopes such as XMM Newton and the Chandra X-ray Observatory. From these measurements, the accretion rate, \dot{M} , can be determined. The standard theory of geometrically thin accretion discs around black holes predicts that about 0.1 of $\dot{M}c^2$ is converted into escaping radiation. This estimate is consistent with the radiative efficiency of the integrated luminosity of quasars with the observed space density of supermassive black holes. However, a puzzling discrepancy has emerged: the measured luminosities of the central sources in some nearby galaxies are smaller than this standard estimate by

3 to 5 orders of magnitude.

This discrepancy led to the development of a newer, but also popular, type of geometrically thick accretion disc models called “advection dominated accretion flows” (ADAFs) (e.g., Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1995; Narayan, Mahadevan & Quataert 1998) in which the gravitational binding energy of the accreting material is retained as internal energy within the hot plasma and ultimately crosses the event horizon of a black hole as the plasma falls in without radiating significantly. In the vicinity of the black hole horizon, the gravitational binding energy of the plasma is a fraction of $\dot{M}c^2$ and, therefore, the associated internal energy per particle in an ADAF is of order $m_p c^2 \sim 1 \text{ GeV}$. Because electrons are more mobile, they are the primary radiating particles. Thus, in order for such a weakly radiating accretion flow to exist, three main assumptions of the ADAF model must be satisfied: (1) the internal energy dissipated in the accretion process via viscosity must go almost entirely into ions. (2) the heat transfer from ions to electrons must be slow enough, so that only a tiny fraction of the dissipated thermal energy received by the ions is transferred to electrons during the time it takes the gas to lose its angular momentum and fall onto the black hole. (3) the effective viscosity must be very high in order that the gas loses its angular momentum quickly and can indeed accrete faster than the ion-electron thermal coupling times.

If the assumptions were true, an accretion flow with a given \dot{M} could have a low enough number density and high enough temperature that Coulomb collisions are inefficient in establishing equipartition of energy between ions and electrons during the accretion time. In the absence of any other plasma process that could speed up the ion-electron energy transfer, the electrons could remain at temperatures which are orders of magnitude lower than the ions in the black hole engine environments. Since electrons produce practically all of the radiation in a given disc, the luminosity rises steeply with increasing electron temperature. Thus keeping electrons at a much lower temperature achieves the main goal of ADAFs: substantially lowering the luminosity for a given accretion rate (e.g., Ichimaru 1977; Narayan & Yi 1995; Quataert & Gruzinov 2000; Narayan, Igumenshchev & Abramowicz 2000; Narayan 2002) compared to standard thin discs. ADAFs are thick discs because the heat dissipated by the accretion is stored as thermal energy of the ions, which puffs up the disc.

Despite the important implications of the above assumptions if they were true, these assumptions have not been proven or disproven. Doing so requires understanding the subtle plasma physics of the interactions between ions and electrons with magnetic and electric fields. The assumptions have therefore been the subject of much deserved attention (Begelman & Chiueh 1988; Bitsnovaty-Kogan & Lovelace 1997; Quataert 1998; Gruzinov 1998; Blackman 1999; Quataert & Gruzinov 2000). One central issue is whether or not collective long range interactions could be important for momentum transfer, energy dissipation and thermal equilibration processes occurring not just between pairs of particles (like Coulomb collisions) but in the whole volume of plasma, shortening the electron ion equilibration time and ruling out the ADAFs.

Since the basic microphysical processes of a plasma must involve known electromagnetic interactions between particles and fields, it is tempting to address the above ADAF assumptions starting from a very basic treatment of electromagnetic theory. This was recently pursued by Binney (2003), who presented a

general argument that assumption (2) above is invalid. He used a Hamiltonian formalism to calculate the change of energy and angular momentum of a particle moving in a general time-dependent electromagnetic field and averaged the result obtained over many particles to estimate the corresponding rates for the plasma. He concluded that the ion-electron equipartition time t_{equi} is smaller than the characteristic angular momentum loss time, t_{res} (or residence time according to Binney's (2003) terminology). If this result were true, and the approach correct, it would rule out two-temperature accretion flows and close the whole ADAF chapter in astrophysics.

Scientists working in plasma astrophysics usually start with writing down Lorentz forces acting on particles and Maxwell equations to analyze interactions in astrophysical plasmas. The idea of Binney (2003) to invoke general Hamiltonian analysis is novel and original. However, in this paper we follow along with the calculation of Binney (2003) and discover two fatal problems therein: (1) The calculation of Binney assumed that the particle angular momentum and the canonical angular momentum are the same, which leads to an incorrect and non-gauge invariant angular momentum equation. (2) The electric and magnetic fields are not included dynamically, as they must be for a plasma. We show that when the Hamiltonian analysis is performed correctly, no new conclusions can be made out of it that cannot be made by writing down usual Lorentz forces and Maxwell equations. Unfortunately, this exclude Hamiltonian analysis as a tool to resolve the problem of the existence of ADAF. We hope that a broader consequence of our consideration will help to elucidate the role and limitations of the Hamiltonian formalism in the context of such plasma dynamics problems.

2. ANNOTATED DISCUSSION OF THE HAMILTONIAN APPROACH OF BINNEY (2003)

2.1. The rate of change of the Hamiltonian

Binney (2003) uses a Hamiltonian formalism to calculate the change of energy and angular momentum of a particle under the action of electromagnetic and gravitational fields. For a given time-dependent electromagnetic field characterized by the potentials \vec{A} and ψ , and gravitational potential Φ , the Hamiltonian for the non-relativistic motion of a particle of mass m and charge q in Cartesian coordinates is (e.g., Landau & Lifshitz 1988b)

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} + q\psi + m\Phi, \quad (1)$$

where Φ is assumed to be time-independent and axially symmetrical, and the canonical momentum of the particle, \vec{p} , is related to its velocity \vec{v} by

$$\vec{p} = m\vec{v} + q\vec{A}, \quad (2)$$

where we set the speed of light $c = 1$ to match the notation of Binney (2003). Binney (2003) then calculates the time derivative of H . From general Hamiltonian theory it is known that the total time derivative of any quantity F along the

trajectory of a particle is given by

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \{H, F\}, \quad (3)$$

where the Poisson bracket for position and momenta canonical variables s_i, p_i is $\{H, F\} \equiv \frac{\partial F}{\partial s_i} \frac{\partial H}{\partial p_i} - \frac{\partial F}{\partial p_i} \frac{\partial H}{\partial s_i}$ with repeated indices summed, and $\partial F/\partial t$ is the time derivative when s_i, p_i are held fixed (Landau & Lifshitz 1988a). Since $\{H, H\} = 0$, the rate of change of the Hamiltonian is

$$\begin{aligned} \frac{dH}{dt} &= \frac{\partial H}{\partial t} = -q \frac{\vec{p} - q\vec{A}}{m} \cdot \frac{\partial \vec{A}}{\partial t} + q \frac{\partial \psi}{\partial t} \\ &= -q\vec{v} \cdot \frac{\partial \vec{A}}{\partial t} + q \frac{\partial \psi}{\partial t}. \end{aligned} \quad (4)$$

2.2. Hamiltonian is not the particle energy

Binney (2003) then proceeds to identify dH/dt from Eq. (4) as a rate of change of the energy of a particle. He considers only the first term in the right-hand side of expression (4) to obtain the lower limit on the rate of energy transfer, and the upper limit on the equipartition time

$$t_{\text{equi}} \sim \frac{H}{|dH/dt|} < \frac{H}{|q\vec{v} \cdot \partial \vec{A}/\partial t|}. \quad (5)$$

These expressions need to be summed over the particles in some small volume to obtain the corresponding rates for the plasma as a whole. Assuming that individual positive and negative charges have the same charge magnitude $|q|$, we introduce the current density $\vec{j} = |q| \sum_{l,m} (\vec{v}_l^+ - \vec{v}_m^-)$, where the sum is over ions and electrons in some local volume of plasma where \vec{v}_l^+ is the velocity of the l th positive charge, and \vec{v}_m^- is the velocity of the m th negative charge. The result of separately summing up H and dH/dt over the charges then leads to:

$$t_{\text{equi}} < \frac{H^{\text{tot}}}{|\int d^3\mathbf{x} \vec{j} \cdot \partial \vec{A}/\partial t|}. \quad (6)$$

We must pause at this juncture. Binney's (2003) presentation of (6) as a measure of the time rate of change of the particle energy is flawed because H is not in general a measure of the particle energy. Specifically, note that the Hamiltonian of Eq. (1) explicitly depends on time via $\vec{A} = \vec{A}(\vec{r}, t)$ and $\psi = \psi(\vec{r}, t)$ and is not gauge invariant: the addition of a time derivative of some scalar function to ψ changes H . Physical quantities such as the actual energy of a particle must be gauge invariant. It is not H that represents the kinetic energy of a particle, but rather it is the first term in H that represents the particle kinetic energy. This term is explicitly gauge invariant (note that a gauge transformation also changes \vec{p}) due to its dependence on \vec{A} . It is because of these points, that the identification of the Hamiltonian with the energy of a particle and dH/dt as the work done on a particle by the electric field cannot be correct. Rather, it is the kinetic energy of

a particle, $K_e = mv^2/2$, that must be estimated to determine equilibration times – the time derivative of the first term in Eq. (1).

The equipartition time is not correctly obtained by (5) but rather by $t_{\text{equi}} = \frac{1}{2}m_e v^2/|dK_e/dt|$, where dK_e/dt is simply the time rate of change of the electron energy subject to acceleration by interaction with the electromagnetic fields. It is clear without detailed derivation that dK_e/dt is simply given by $q\vec{v} \cdot \vec{E}$, the work done per unit time by the Lorentz force. Unlike in (5) the correct expression for dK_e/dt will have $\vec{\nabla}\psi$ terms, which are particularly dominant when close interactions of electrons and ions occur (Coulomb collisions). Therefore, the correct estimate of t_{equi} is rather different from (5) but is nothing new compared to the estimate of the conventional work done by the electric field.

Binney (2003) further argues that in obtaining (5) the $q\partial\psi/\partial t$ terms of (4) can be ignored because they cancel when averaging over particles with opposite charges in a quasi-neutral plasma. Actually, this cancellation is not guaranteed because a particle with a given charge in a plasma always attracts particles of the opposite charge within its Debye sphere. As a result, the change in the potential at the location of a particle correlates with its charge and the cancellation of $q\partial\psi/\partial t$ between species is not obvious.

We have identified problems with (6) as presented by Binney (2003), but let us for the moment, continue to follow and dissect the arguments that he presents in comparing the equilibration and infall (angular momentum loss) times. We now discuss the latter and the pitfalls therein.

2.3. Canonical angular momentum is not the particle angular momentum

To compare with the energy equilibration time (6), Binney (2003) proceeds to derive a similar time scale for the loss of angular momentum of the plasma. If the angular momentum loss time were the longer of the two, he would rule out ADAFs because the electrons and protons would equilibrate before the plasma falls onto the black hole. The fundamental problem with his subsequent calculation is that it relies on incorrectly identifying the canonical and particle angular momenta, as we now show.

Specifically, Binney (2003) identifies the angular momentum of a particle L_z with the ϕ -component of the canonical momentum p_ϕ . He uses Hamiltonian theory (e.g., Landau & Lifshitz 1988a) to write

$$\frac{dL_z}{dt} = \{H, p_\phi\} = -\frac{\partial H}{\partial \phi} = q\vec{v} \cdot \frac{\partial \vec{A}}{\partial \phi} - q\frac{\partial \psi}{\partial \phi}. \quad (7)$$

He then notices that the right-hand side of expression (7) differs from the right-hand side of expression (4) by replacing ∂t by $\partial \phi$ and changing sign. Therefore, the analogous arguments and calculations leading to the estimate (6) are repeated to obtain an estimate of the “residence time” or the maximum time before the plasmas loses its angular momentum and falls into the black hole:

$$t_{\text{res}} \sim \frac{L_z^{\text{tot}}}{|dL_z^{\text{tot}}/dt|} \sim \frac{L_z^{\text{tot}}}{|\int d^3\vec{x} j \cdot \partial \vec{A} / \partial \phi|}. \quad (8)$$

Dividing both sides of (6) by both sides of (8), approximating $H^{\text{tot}}/L_z^{\text{tot}}$ by Ω_ϕ (acceptable when the plasma is substantially supported by Keplerian rotation as

in the case of ADAFs), and approximating the ratio of integrals by the ratio of the corresponding derivatives of \vec{A} one obtains

$$\frac{t_{\text{equi}}}{t_{\text{res}}} < \Omega_\phi \frac{|\partial\vec{A}/\partial\phi|}{|\partial\vec{A}/\partial t|}. \quad (9)$$

The ratio of derivatives of \vec{A} can be approximated by the inverse frequency of a pattern of \vec{A} propagating in ϕ direction. The lowest important frequency of the pattern in \vec{A} is the Keplerian frequency Ω_ϕ . Then, the final estimate follows:

$$\frac{t_{\text{equi}}}{t_{\text{res}}} < 1. \quad (10)$$

Taken at face value, this relation would imply that electrons receive a significant fraction of the thermal energy of ions before they have time to fall through the black hole event horizon, and ADAFs are impossible. We now pinpoint the key problem with the approach that led to this conclusion.

3. HAMILTONIAN EQUATIONS OF PLASMAS FROM FIRST PRINCIPLES

The key problem with the result and calculation that leads to (10) above is the mis-identification of the angular momentum of a particle L_z with the ϕ -component of the canonical momentum p_ϕ above Eq. (7). To show that these are not the same and the important consequences, we now rigorously derive dL_z/dt starting from first principles.

The forms of the Hamiltonian and canonical momenta depend on the choice of the coordinate system used to describe the particle motion. In cylindrical coordinates, expressions (1) and (2) need to be modified. The general procedure to derive Hamiltonian equations is to start with a Lagrangian (which can be obtained from the covariant action). The Lagrangian is invariant under the transformations of space coordinates s_i . In cylindrical coordinates $(s_1, s_2, s_3) = (r, \phi, z)$ and the Lagrangian \mathcal{L} in the non-relativistic limit is (e.g., Landau & Lifshitz 1988b):

$$\mathcal{L} = \frac{m}{2} \left(\dot{r}^2 + r^2 \dot{\phi}^2 + \dot{z}^2 \right) + q \left(A_r \dot{r} + r A_\phi \dot{\phi} + A_z \dot{z} \right) - q\psi - m\Phi, \quad (11)$$

where $\dot{r} = dr/dt$, $\dot{\phi} = d\phi/dt$, and $\dot{z} = dz/dt$ along the trajectory of a particle. The conjugate canonical momenta are $p_i = \partial\mathcal{L}/\partial\dot{s}_i$. In cylindrical coordinates

$$p_r = m\dot{r} + qA_r, \quad p_\phi = mr^2\dot{\phi} + qrA_\phi, \quad p_z = m\dot{z} + qA_z. \quad (12)$$

The actual particle angular momentum relative to the z -axis is $L_z = mr^2\dot{\phi}$. Eq. (12) allows us to relate L_z to ϕ -component of conjugate momentum as

$$L_z = p_\phi - qrA_\phi. \quad (13)$$

Therefore, in electromagnetism $L_z \neq p_\phi$ contrary to the statement of Binney (2003). As a result, his Eq. (7) for dL_z/dt is incorrect.

To derive the correct expression for dL_z/dt , we use the fact that the Hamiltonian $H = \sum_i \dot{s}_i \partial \mathcal{L} / \partial \dot{s}_i - \mathcal{L}$. Then using (11) and substituting for \dot{s}_i from Eqs. (12) gives

$$H = \frac{1}{2m} [(p_r - qA_r)^2 + (p_\phi/r - qA_\phi)^2 + (p_z - qA_z)^2] + q\psi + m\Phi. \quad (14)$$

This expression is different from the expression (1) for H in Cartesian coordinates by the factor $1/r$ multiplied with p_ϕ . Let us take d/dt along the particle trajectory of all terms in relation (13). One has

$$\begin{aligned} \frac{dp_\phi}{dt} &= \{H, p_\phi\} = -\frac{\partial H}{\partial \phi} \\ &= q \left(\dot{r} \frac{\partial A_r}{\partial \phi} + r\dot{\phi} \frac{\partial A_\phi}{\partial \phi} + \dot{z} \frac{\partial A_z}{\partial \phi} \right) - q \frac{\partial \psi}{\partial \phi}, \end{aligned} \quad (15)$$

where the curly bracket is again the Poisson bracket and we have used Eqs. (12). This result exactly reproduces the right-hand side of equation (7) but the left-hand side should be dp_ϕ/dt , **not** dL_z/dt . There are extra terms to dL_z/dt according to relation (13):

$$\begin{aligned} \frac{dL_z}{dt} &= \frac{dp_\phi}{dt} - q\dot{r}A_\phi - qr \frac{\partial A_\phi}{\partial t} - qr \frac{\partial A_\phi}{\partial r} \dot{r} - \\ &qr \frac{\partial A_\phi}{\partial \phi} \dot{\phi} - qr \frac{\partial A_\phi}{\partial z} \dot{z}. \end{aligned} \quad (16)$$

Substituting $\frac{dp_\phi}{dt}$ from Eq. (15) into Eq. (16) and collecting together terms with \dot{r} and \dot{z} we obtain

$$\begin{aligned} \frac{dL_z}{dt} &= q \left[r\dot{r} \left(\frac{1}{r} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (rA_\phi) \right) + \right. \\ &r\dot{z} \left. \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \right] - rq \left(\frac{1}{r} \frac{\partial \psi}{\partial \phi} + \frac{\partial A_\phi}{\partial t} \right). \end{aligned} \quad (17)$$

When all components of potentials are combined in Eq. (17), we are left with only components of magnetic and electric fields:

$$\begin{aligned} \left(\frac{1}{r} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (rA_\phi) \right) &= -B_z, \\ \left(\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) &= B_r, \\ -\frac{1}{r} \frac{\partial \psi}{\partial \phi} - \frac{\partial A_\phi}{\partial t} &= E_\phi. \end{aligned}$$

Therefore, Eq. (17) becomes

$$\frac{dL_z}{dt} = qrE_\phi + qr(v_z B_r - v_r B_z). \quad (18)$$

This is nothing but the torque produced by the ϕ component of the Lorentz force $q\vec{E} + q(\vec{v} \times \vec{B})$ acting on a particle. Indeed such a result could be anticipated from the very beginning, without using the Hamiltonian or Lagrangian formalism for the derivation. In fact, Eq. (7) after Binney (2003) would already raise initial concerns from the fact that its left-hand side dL_z/dt , is a gauge invariant quantity, while the right-hand side is not gauge invariant.

The conclusion of Binney (2003) that we reproduced in the previous section culminating in Eq. (10), crucially relies on the incorrect presence of the actual rather than canonical momentum on the left side of Eq. (7). We have shown that when this Eq. (7) is corrected, no new results come from the Hamiltonian formalism that do not already come from simply writing down the Lorentz force acting on a particle. The latter leads to classical estimates of equilibration times used in standard two-temperature accretion flow calculations (e.g., Narayan et al. 1998), and by contrast, the relation expressed in Eq. (10) is simply invalid.

4. INCLUSION OF THE ELECTROMAGNETIC FIELDS AS DYNAMICAL VARIABLES AND IMPLICATIONS FOR CALCULATING LOSS TIMES

The energy transfer from protons to electrons in the turbulent accretion plasma is mediated by electromagnetic fields (waves) excited by plasma instabilities. Particles and waves can exchange energy, (see for example Begelman & Chiueh (1988) in this accretion flow context). In order to quantitatively assess the effectiveness of the energy transfer between particles and waves, one needs to couple the dynamical evolution of the electromagnetic fields to dynamical evolution of the particle motion. This means that in general, the electromagnetic fields \vec{E} and \vec{B} (or \vec{A} and ψ) need to be treated as dynamical variables, not merely as background fields. This contrasts Binney (2003), who assumes that the electromagnetic field is fixed: \vec{A} and ψ are considered as externally imposed background fields on the dynamical system of particles, not as dynamical variables themselves.

One might be tempted to extend the intended Hamiltonian approach of Binney (2003) to include dynamical electromagnetic fields. In such an approach, the electromagnetic fields can be decomposed into the sum of normal modes and each such mode could be treated as a dynamical degree of freedom. One could then write a Hamiltonian containing both particles and electromagnetic fields and use it to calculate the time derivatives of the total energy and angular momentum of the system. But the result of such calculation is predetermined: the total energy and angular momentum of all particles and electromagnetic fields (including radiation) must be conserved in time. Hamiltonian equations of motion written down for such a Hamiltonian will simply be Maxwell equations for the electromagnetic fields combined with the equations of particles motion under the action of Lorentz forces. Such a procedure would not lead to new results but simply lead us back to the conventional methods of analyzing the equations of plasma dynamics which start with the equations of motion in the first place.

5. CONCLUSIONS

We have elucidated the role of the Hamiltonian formalism in the study of plasmas and its relation to the equations of motion. The latter is the usual starting point for the practical study of plasma dynamics in applications to laboratory and

astrophysical plasmas. We have illustrated how the Hamiltonian formalism does not provide any more information than that which is contained in the equations of motion when it comes to the practical calculation of dynamical time scales of a system.

We have illustrated the importance of understanding these basic derivations by means of an application to a very current topic in astrophysics, namely, two-temperature, low luminosity accretion flows commonly used as an explanation for the otherwise mysterious quiescent accretion engines at the centers of galaxies. In particular, we have discussed Binney's (2003) attempt to use general Hamiltonian methods to obtain a constraint on the ratio of ion-electron equipartition time to the angular momentum loss time of particles in these flows. If the former time scale were shorter than the latter, these accretion flows would be ruled out.

We have shown that the mathematically correct Hamiltonian formalism does not provide any new information for estimating the ion-electron equipartition time beyond conventional non-Hamiltonian approaches. This is revealed when one corrects the Hamiltonian approach of Binney (2003) by not equating the canonical angular momentum to the actual particle angular momentum. The revised calculation shows that the expression for angular momentum change of a particle used by Binney (2003) is incorrect (evidenced also by the fact that it is not gauge-invariant) and thus the subsequent conclusion that the equilibration time between electrons and ions in accretion plasma is always shorter than the accretion time is unsupported. Instead, if performed correctly, Hamiltonian expressions for the rate of change of angular momentum of a particle in electromagnetic fields lead to the usual torque provided by the Lorentz force.

We have also pointed out that to incorporate particle-wave interactions occurring in turbulent plasmas, one must treat the electromagnetic fields as dynamical variables in the Hamiltonian formalism. Including these excitations will simply lead to a conserved Hamiltonian, once again providing no new information beyond conventional plasma physics approaches.

Finally, we note that Quataert (2003) also argued that the conclusion of Binney (2003) is incorrect. Quataert (2003) argued that the time scale on which the energy of a particle changes due to the work by the electric field is not the time scale on which the true heating or change in entropy occurs. He mentions two examples where this difference is evident. First is the motion of a particle in a slowly varying magnetic field, with characteristic variation time much longer than Ω^{-1} , where $\Omega = eB/mc$ is the cyclotron frequency. After some time the magnetic field returns to its initial value everywhere. In this case, t_{equi} calculated by the method of Binney (2003) [expression (6) above] would be the characteristic variation time of the magnetic field. At the same time, in the absence of collisions, the energies of particles remain the same because of the conservation of the adiabatic invariant. His second example is an undamped Alfvén wave. In this wave the energy is transferred periodically between fields and particles, but there is no net heating. As an extension of the argument about different time scales for adiabatic and dissipative energy changes, Quataert (2003) mentions that particles are heated at discrete wave-particle resonances, not explicitly accounted for in Binney (2003).

Although these two examples of non-dissipative energy changes of particles in time variable magnetic fields are clear and correct, the statistical nature of the turbulence (presumably existing in any accretion flow due to the non-linear development of the magneto-rotational instability (MRI, e.g. Balbus & Hawley

1998)) does not allow one to conclude that *all* particle-turbulence energy exchange processes will occur as in Quataert's two examples. Therefore, by themselves, these arguments of Quataert (2003) do not disprove the derivation of Binney (2003). In particular, Binney argued that the rate of heating may be estimated from equation (2) in his paper [repeated as equation (4) above in this paper] as follows: "Thus this equation describes the mechanism by which equipartition is established between ions and electrons; the net direction of the energy flow is mandated by the general principles of statistical physics, and the rate of flow may be estimated from Equation (2)." This statement does not contradict the specific energy transfer examples of Quataert described in our previous paragraph above. The reason is that it is not clear how statistically important the examples of Quataert are for a realistic accretion flow. We have found different and more fundamental reasons that rigorously disprove the results of Binney (2003).

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