Isorotation and differential rotation in a magnetic mirror with imposed E×B rotation

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Doppler spectroscopy of helium impurities in the Maryland Centrifugal Experiment reveals the simultaneous existence of isorotating and differentially rotating magnetic surfaces. Differential rotation occurs at the innermost surfaces and is conjectured to cause plasma voltage oscillations of hundreds of kilohertz by periodically changing the current path inductance. High-speed images show the periodic expulsion of plasma near the mirror ends at the same frequencies. In spite of this, the critical ionization velocity limit is exceeded, with respect to the vacuum field definition, for at least 0.5 ms. © 2012 American Institute of Physics.

I. INTRODUCTION

Isorotation in magnetized plasmas refers to plasma rotation at the same angular velocity for a given magnetic field line. It was first proposed by Ferraro1 to explain the non-uniform rotation of the Sun. It is now common to assume isorotation when using magnetohydrodynamic (MHD) models to describe astrophysical as well as laboratory magnetized plasmas with rotation.2-5 This assumption, however, relies on what is sometimes yet another assumption: high electrical conductivity along the magnetic field (infinite, if invoking ideal MHD), such that plasma remains “frozen-in” to the field.6 The Maryland Centrifugal Experiment (MCX)7 was designed assuming isorotation. The main purpose of MCX is to study centrifugal confinement and heating of plasmas under imposed E×B rotation, where E is the electric field in the radial direction produced by a center electrode in a cylindrical chamber, and B is the axial magnetic field produced by external coils. A schematic diagram of MCX is shown in Fig. 1. Under ideal conditions, plasma isorotates at different angular speeds across the mirror radius, creating shear flow that helps suppress instabilities found in traditional magnetic mirrors (i.e., those without imposed rotation).8

Here, we present high-speed images as well as spectroscopic measurements of MCX plasma rotation profiles, \( \Omega(r) \), and ion temperature profiles, \( T_i(r) \). These measurements were motivated by the appearance of a new instability during MCX discharges; the plasma load voltage, \( V_p \), exhibits oscillations of hundreds of kilohertz (hereafter called high frequency), not seen in the plasma load current, \( I_p \). Figure 2 shows typical \( V_p \) and \( I_p \) traces for the plasma discharges with the high-frequency oscillation. Note that both traces include oscillations on the order of a few kilohertz (low frequency), which have been explained in previous work by numerical simulations and magnetic measurements.9,10 In the low frequency case, azimuthal modes arise from high plasma pressure that overwhelms the shear flow stabilization effect and from the nonlinear interplay between these modes. In the present case, the high frequency component of \( V_p \) appeared during plasma discharges after recent efforts to reduce impurities by implementing cleaner vacuum practices and, therefore, reduce energy losses which occur partly through atomic line radiation. Plasma discharges have since resulted in higher sustained \( V_p \) and lower \( I_p \) (time-averaged over spans of milliseconds). The significance of high \( V_p \) comes from its relation to rotational velocity, \( u_\phi \), which may be approximated by \( u_\phi = V_p/aB \), where \( a = R_{MCX} \) is the MCX vessel radius at \( z = 0 \), and \( B \) is the magnetic field. Rotational velocities have been shown in previous work to be closely correlated to \( V_p/aB \) using Doppler spectroscopy of impurity lines.7

This paper is organized as follows: a phenomenological description for the cause of the high-frequency voltage oscillations is presented in Sec. II; the spectroscopic measurements and high-speed images that support the model are presented in Sec. III. A brief discussion of the results is presented in Sec. IV.

II. PHENOMENOLOGY

MCX must have insulators to allow E to be non-zero across the B field. Ideally, the only components of these fields effecting a rotation are the radial electric field, \( E_r \), and the axial magnetic field, \( B_z \), such that the rotational velocity is

\[
    u_\phi = - \frac{E_r B_z}{|B|}. \tag{1}
\]

The hypothesis proposed here to explain the high frequency \( V_p \) oscillations is that these are caused by inductance changes in the plasma, which in turn may be caused by the twisting of magnetic flux surfaces due to differential rotation along the mirror axis. A temperature drop at the mirror ends increases plasma resistivity and, consequently, allows a differential rotation along flux surfaces since the MHD frozen-in-flux condition is no longer met. Note that temperature drops discussed in Refs. 8 and 11 are estimated to be only

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centimeters away from the insulator, where the magnetic field is highest and plasma density the lowest, and no significant deformation of the magnetic field is expected there. Away from the insulators, as the field decreases and density increases, differential rotation causes deformation of the magnetic field, creating an azimuthal component of the magnetic field, $B_\phi$, and consequently an axial component of the $E \times B$ drift. A resistive drop along the magnetic field also implies the possibility of an axial component of the electric field, $E_z$. The resulting components of the plasma drift velocity are (in cylindrical coordinates)

$$u_r = \frac{E_z B_\phi}{|B|^2}$$  \hspace{1cm} (2)

$$u_\phi = -\frac{E_r B_z + E_z B_r}{|B|^2}$$ \hspace{1cm} (3)

$$u_z = \frac{E_r B_\phi}{|B|^2}.$$ \hspace{1cm} (4)

The effect of the axial $E \times B$ drift component is to pile up helicity towards one region. This is shown schematically in Fig. 3; the top panel shows one turn difference between the two regions (represented by disks), while the bottom panel shows two turns. Eventually, field stretching becomes untenable and the magnetic configuration will tend to reset itself by merging or reconnecting the field at the affected region. Note that $B_\phi$ changes sign for $\pm z$ (where the z-axis origin is defined in Fig. 1), since it is assumed that a resistive drop occurs on both sides of the mirror and the maximum rotation occurs at $z = 0$. The following observations can also be made for Eqs. (2)–(4): $E_r$ has the same sign on both sides of the mirror, and therefore, $u_r$ is convergent or divergent on both sides; $E_z$ alternates signs with $B_\phi$, and thus $u_\phi$ is either positive or negative on both sides; and the $E_z B_\phi$ component in $u_\phi$ always subtracts to $E_z B_z$ in Eq. (3), and is thus detrimental to centrifugal confinement.

With no differential rotation, MCX plasma may be modeled as a resistor in parallel with a capacitor. It is thus reasonable to question whether capacitive changes, and not inductive changes, are the leading cause of the voltage spikes. The available diagnostics at MCX, or those used in previous experiments, could not collect enough information to answer this question quantitatively. We may, however, obtain some insight at the phenomenological level by noting that the stored charge in the MCX plasma is directly proportional to the density. Recently reported measurements of the density at two distinct axial locations in MCX shows two turns. Eventually, field stretching becomes untenable and the magnetic configuration will tend to reset itself through reconnection with a frequency of order $\Omega_2 - \Omega_1$.  

FIG. 1. Schematic diagram of MCX. The magnetic field is azimuthally symmetric about the centerline. The dotted vertical lines ($z = 0$ cm and $z = -78$ cm) correspond to the Doppler measurement sites, while the rectangles at $z = \pm 1.3$ m represent the MCX insulators that limit the plasma. The capacitor bank, $C_B = 1.8$ mF, is attached to the center electrode (negative voltage) and vacuum vessel (ground). Plasma voltage and current measurement locations are indicated on the discharge circuit, where $R_L = 2$ Ω. The lower circuit is the simplified model used for the plasma.

FIG. 2. Typical current (top) and voltage (bottom) traces for plasma discharges analyzed in this work.

FIG. 3. Schematic representation of differential rotation ($\Omega_2 > \Omega_1$) along magnetic field lines, and the resulting axial flow component ($E \times B$)$_z$. Helicity buildup is untenable in this configuration, and the magnetic field will tend to reset itself through reconnection with a frequency of order $\Omega_2 - \Omega_1$.  

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show oscillations in its magnitude which occur in the low frequency range and never in the high frequency range considered here (the density histories for the present discharges have similar characteristics to those shown in Fig. 2 of Ref. 14). Therefore, we expect capacitive changes to be in the low frequency range. The MCX electrical model is thus modified to include a plasma inductance, \( L_p \), as shown in Fig. 1. The contribution from this inductance to \( V_p \) is \( L_p I_p + I_p L_p \), and the circuit impedance is

\[
|Z_p| = \sqrt{\frac{(L_p \omega)^2 + R_p^2 (C_p L_p \omega^2 - 1)^2}{1 + (C_p R_p \omega)^2}}. \tag{5}
\]

Note that if \( L_p \to 0 \), \( |Z_p| \to 0 \) for \( \omega \to \infty \). That is, at high frequencies, a capacitive-only plasma would produce a low \( V_p \), which is not observed here.

III. MEASUREMENTS

High-speed images of the insulator region, obtained from plasmas with high frequency \( V_p \) oscillations, also provide insight about the effects of differential rotation. Fig. 4 shows images from plasma discharges with identical conditions. Figs. 4(a) and 4(b) were taken with 200 ns exposures and show the typical shot-to-shot variation from a large set of images; Fig. 4(c) was taken with a 1 ms exposure. The corresponding line plots on the right column show light intensity values along vertical dotted lines. The inner MCX electrode is the dark bar in the images, corresponding approximately to pixels between 180 and 220. The confined, rotating plasma is expected between pixels 130 and 270. However, Fig. 4(a) shows plasma beyond the confining flux surfaces, indicated by red arrows for line \( A-A' \). Fig. 4(b) and its corresponding \( B-B' \) line, in contrast, show no plasma outside these flux surfaces. When camera exposure times were increased to \( 1 - 10 \) \( \mu \)s, or more, the images would look more like Fig. 4(c). (For 1 ms exposures, the shot-to-shot image reproducibility was excellent.) In the context of the paradigm proposed here, the ejection of plasma from the flux surfaces is expected to occur given the \( \langle E \times B \rangle_z \) component induced by differential rotation, and with the same high voltage frequencies mentioned above. Fig. 4(c), therefore, shows an integration of light over many ejection cycles. In contrast to the insulator region, high-speed images of plasmas in the midplane region show no structure or plasma ejecta at high frequencies, indicating that plasma ejection from the mechanism proposed here is particular to the insulator region.

Doppler spectroscopy was used to radially resolve velocity and temperature profiles in plasma discharges with high frequency oscillations. An array of 10 optical fibers with collimation lenses (beam width \( \sim 1 \) cm) was first setup at \( z = 0 \) cm (midplane, Fig. 1) in order to measure the radial profile of Doppler shift from impurities in a hydrogen plasma. The same 10 fibers were then moved to \( z = -78 \) cm. The experimental setup and the velocity unfolding technique are similar to that used by Ghosh et al. in Ref. 17 to resolve the radial velocity profile in MCX, except that this time 10 fibers were used (instead of 5) to obtain higher radial resolution. First ionized state atomic emission lines from helium (\( \text{He II, 468.54 nm} \)) were measured using a 1 m Turner-Czerny spectrometer with a 1200 line/mm grating and a signal integration time of 500 \( \mu \)s. The chamber was pre-filled with neutral helium, which made up less than 5% of the neutral particles inventory. Hydrogen was injected separately using a gas-puff valve to achieve a total pressure of 5 mTorr shortly before breakdown. In past experiments, first and second ionized states of carbon had been used in MCX to measure Doppler velocities. However, cleaner conditions have reduced carbon signals significantly. Particle mass-to-charge ratio, \( q/m \), was recently realized to be an issue in rotating plasmas. 18 The relatively high \( E \) fields required to sustain

![FIG. 4. High-speed images of plasma near the insulator region, with image pixel values along vertical dotted lines shown on the corresponding plots on the right column. Axes correspond to pixel number in the images, taken during experimental run MCX20100624.](image)
discharges in MCX would yield particle trajectories that significantly differ from ideally circular trajectories, depending on \( q/m \). \( \text{He II} \) was chosen because \( q/m \) is the most similar to that of hydrogen, while still emitting visible radiation that can be captured by our spectrometer.

The processed spectroscopic data from plasma discharges with identical setup parameters (midplane field 0.2 T, mirror field 1.4 T, and an initial voltage between MCX electrodes of 10 kV) is shown in Fig. 5. The radial parameter, \( \rho \), for midplane and transition regions, is the radius normalized to the magnetic flux surface values, where \( \rho = 0 \) is the innermost flux surface with plasma rotating due to the \( E \times B \) effect, and \( \rho = 1 \) is the outermost one. The \( \rho = 0 \) value corresponds to the vacuum flux surface definition and is not expected to change (due to diamagnetism) during plasma rotation more than the width of the measuring chords. Diamagnetism is expected to be significant for these measurements for outermost flux surfaces. For this reason, the \( \rho = 1 \) value was chosen by analyzing signals at \( z = -78 \) cm with plasma rotation, in order to take diamagnetism changes into account. Note that there are only eight chords displayed in Fig. 5 for the \( z = -78 \) cm plane; the remaining two chords had signals that were too low and with large error bars, indicating that these chords were beyond the outer rotating flux surfaces.

The measured velocities are highest during the time interval 4.5–5.0 ms, which also correspond to the highest external heating power (\( \sim 10 \) MW) through the plasma. Also, during this interval, angular rotation profiles coincide the most, except for the flux surfaces near \( \rho = 0.2 \). The radial extent in differential rotation increases at the later times, but so do the velocity measurement uncertainties. Temperature measurement uncertainties increase concomitantly. Care was taken to distinguish between turbulence broadening and Doppler broadening by also measuring oxygen impurity temperatures,\(^\text{19}\) which come from water molecules desorbed from the walls. Nevertheless, for the first time interval, the broadening condition on the inner surfaces (\( \rho < 0.5 \)) could not be assessed since the oxygen signal was too weak. Note that previously reported ion temperatures were inferred to be radially constant.\(^\text{20}\) The present measurements agree with past ones at the outer edge, but show a radial profile with an order of magnitude higher temperature at the inner surfaces.

**IV. DISCUSSION**

The angular velocity difference between the midplane and transition regions in these inner surfaces is \( \sim 3 – 4 \times 10^6 \) rad/s. In the context of the model discussed above, this frequency corresponds to the magnetic field

![Graph showing spectroscopic measurements at three different time intervals](image)

**FIG. 5.** Spectroscopic measurements at three different time intervals during plasma discharges, at \( z = 0 \) cm (red circles) and at \( z = -78 \) cm (blue squares). The dotted line represents the hydrogen critical ionization velocity at midplane, which is exceeded for the first interval (\( u_{ci} = 51.4 \) km/s, mapped at \( z = 0 \) cm with \( R_m = 7 \) results in \( u_{ci} = 136 \) km/s). The error bars correspond to one standard deviation of the measured values. Measurements at \( z = +78 \) cm (not shown) indicate the same behaviour as those at \( z = -78 \) cm.
wrapping and resetting itself once around the axis, creating the inductive voltage with a frequency within the spectrum shown in the inset of Fig. 2. This, of course, is a phenomenological interpretation of the inductive changes that have to occur to sustain the measured voltages. The data presented here show that differential rotation occurs only on the inner surfaces for all time intervals. Pressure is the lowest on these inner surfaces, and thus, drift instabilities are unlikely to play a role in the axial differential rotation. Magnetic diagnostics at the external magnetic flux surfaces do not show a correlation with the voltage spikes, indicating that the effects of differential rotation are confined to the surfaces where it occurs.

The rotation measurements presented here also show, for the first time in MCX, the apparent surpassing of the critical ionization velocity, $u_{\text{crit}}$, at midplane, if only for a fraction of the discharge time. This velocity, which is dependent on the ion's mass and ionization potential, has been determined to be caused by plasma surface interactions and has turned out to be a velocity limit in most previously reported experiments on rotating plasmas. Mapping this velocity from the insulator to the midplane by magnetic flux conservation and requiring isorotation, we obtain $u_{\text{midplane}} = u_{\text{insulator}}\sqrt{R_m}$, where $R_m$ is the mirror ratio. As shown in Fig. 5, the isorotation condition is satisfied for the radial interval in which $u_{\text{crit}}$ is apparently exceeded. However, the centrifugal effect creates an outward shift in the magnetic field at midplane, effectively increasing $R_m$, which in turn increases $u_{\text{crit}}$ at midplane. For the data shown here, $R_m$ would have to increase by 56% with respect to the initial value for the maximum velocity to remain at $u_{\text{crit}}$. This displacement is being investigated with the aid of numerical simulations. Results from these simulations indicate that $R_m$ increases by at most 10%, making it likely that the velocities reported here are indeed above $u_{\text{crit}}$. Research is ongoing to extend the duration of the super-$u_{\text{crit}}$ phase beyond the 4.5–5.0 ms interval and will be reported elsewhere.

The theoretical upper bound for MCX rotation has been found to be the Alfvén velocity, $u_A = B/\sqrt{n_i m_i}$, where $n_i$ and $m_i$ are the ion number density and mass, respectively. Previously reported experiments in MCX have confirmed this limit for a wide range of parameters. However, all velocities presented here satisfy $u_d/u_A < 1$. The Alfvén velocity limit could in principle be increased by increasing $B$ or decreasing $n_i$. Since one of the goals of the MCX configuration is to maximize the confined plasma density, decreasing $n_i$ is undesirable. Increasing $B$ is, therefore, the best option to increase $u_A$, and it is limited only by the practical engineering constraints found in fabricating appropriately sized coils required to produce high magnetic fields.

It is reasonable to expect that the high frequency component in the voltage would have a detrimental effect on temperature and confinement. However, temperature and density at midplane remain as high or higher than in previous experiments. The perturbations created by the differential rotation may be increasing the heating efficiency of the capacitor bank, by launching contained Alfvén waves that propagate from the mirror side and get absorbed in the midplane. The theory of this heating effect for rotating plasma configurations has been studied by Fetterman and Fisch, and is the subject of ongoing research.

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