Dielectric response of plasma in reversed field pinches near the ion cyclotron frequency

V. A. Svidzinski
University of Wisconsin–Madison, Madison, Wisconsin 53706

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The dielectric tensor of reversed field pinch (RFP) plasma is calculated near the magnetic axis for cylindrical azimuthally symmetric equilibria in the limit $k_\perp \rho_L \rightarrow 0$. Particle orbits are calculated to first order in the drift approximation near the axis such that geometrical corrections to gyrophase are included. It is demonstrated that these corrections are important for evaluation of the plasma heating rate near ion cyclotron resonance in RFPs while they can be neglected in tokamak modeling. Geometrical effects of this kind become important in RFPs because the ratio of poloidal and toroidal magnetic field components is substantially larger in RFPs relative to that in tokamaks. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1505639]

Recent study of the possibility of ion majority heating at the fundamental ion cyclotron frequency in reversed field pinches (RFPs)[1] showed that such heating can be effective in these low magnetic field devices. In this study, plasma heating near the magnetic axis was considered. It was found that the power absorption near the axis unexpectedly strongly depends on the model used. A brief description of the model follows. The RFP is approximated by a cylinder with plasma equilibrium of the Taylor state[2,3] which is uniform in axial and azimuthal directions. The plasma dielectric response is found from the linearized Vlasov equation. The unperturbed particle orbits are considered in a local reference frame connected with the direction of the static magnetic field (Stix frame). The local triad of this frame is defined before Eq. (3) in the present paper. The orbits are considered in the drift approximation to zero order in drift parameter $\epsilon_D = \rho_L / L$, where ρ_L is a typical Larmor radius and L is a characteristic length of variation of the equilibrium magnetic field. Because the higher harmonics are not important for the fundamental heating, the plasma dielectric tensor is derived in the limit $k_\perp \rho_L \rightarrow 0$. When rf fields are Fourier expanded in θ and z coordinates, the plasma dielectric tensor in the local frame is found to be the standard hot plasma dielectric tensor in a uniform magnetic field (see, e.g., Ref. 4), considered in the limit $k_\perp \rho_L \rightarrow 0$, but with the wave number

$$k_\parallel = \mathbf{k} \cdot \frac{\mathbf{B}}{B} = \frac{m \sin \theta_0}{r} + k_z \cos \theta_0 \tag{1}$$

used in place of k_z. In Eq. (1) θ_0 is the angle between $\mathbf{B}$ and z direction, and m and k_z are the azimuthal and axial wave numbers.

Such modeling of the plasma dielectric properties encounters a difficulty relating to the accuracy of calculation of the absorbed power. The question is which wave number $k_\parallel$ one should use in the argument of the plasma dispersion function $(\omega - \omega_{ci})/|k_\parallel| v_{Ti}$ for the calculation of the absorbed power near the axis. The above-noted model predicts $k_\parallel$ of Eq. (1). This wave number is often used in the rf modeling of tokamak plasmas (see later discussion). On the other hand since $\mathbf{B} \| \mathbf{z}$ at $r=0$, one should expect that $k_\parallel = k_z$ in the above-mentioned argument of the plasma dispersion function near the axis. Indeed, one can derive the dielectric tensor at $r=0$ in Cartesian coordinates xyz in the limit $k_\perp \rho_L \rightarrow 0$ for orbits of the zero order in ϵ_D. The result contains $k_\parallel = k_z$ in the argument of the plasma dispersion function related to the ion cyclotron heating by a left-handed wave.

For an equilibrium with a toroidal current density on axis the ratio $\sin \theta_0 / r \neq 0$ when $r \rightarrow 0$, therefore $k_\parallel$ of Eq. (1) is not equal to k_z at $r=0$ for $m \neq 0$. The two wave numbers $k_\parallel$: $k_\parallel^1$ of Eq. (1) and $k_\parallel^2 = k_z$ can give completely different results for the absorbed power in the cases of interest for central heating. In the example used in Ref. 1, $m = -1$, $k_z a = 1.5$ (a is the minor radius) the two wave numbers are $k_\parallel^1 a = 0.3$ and $k_\parallel^2 a = 1.5$. The use of the first wave number results in no heating while the second one results in a strong ion heating on axis.

Analysis in the present paper shows that the introduction of corrections of first order in ϵ_D in the equation for the gyrophase leads to corrections to wave numbers $k_\parallel$ in the argument of the plasma dispersion function in the perpendicular plasma response of the order $\sin \theta_0 / r$. In the above-noted models particle orbits are considered to zero order in ϵ_D. Since the difference between the wave numbers $k_\parallel^1$ and $k_\parallel^2$ is also of the order of $\sin \theta_0 / r$, these wave numbers are equally justified within the accuracy of the models. The above-noted example shows that the consideration of a particle's trajectories of the zero order in the drift parameter results in a poor accuracy of modeling of the plasma dielectric response in RFP when heating near $\omega = \omega_{ci}$ is examined. In Ref. 1 it was noted that the Stix frame is not inertial—it rotates when a particle moves along a magnetic field line. $k_\parallel = k_z$ from the model in inertial Cartesian coordinates xyz at $r=0$ was chosen as a more appropriate wave number for the estimates of heating rates near the axis. The present study shows that such a choice of an inertial reference frame is a step in the right direction toward the accurate result based on particle trajectories calculated to the first order in drift parameter.

In the present study we accurately calculate the dielectric properties of the plasma near the axis by considering particle orbits to first order in drift parameter. The first-order corrections account for magnetic field geometry and compensate the rotation of the reference frame. We derive the proper dielectric tensor analytically in the limit $k_\perp \rho_L \to 0$. Because the effects due to trapped particles are minimal near the magnetic axis, such a model accurately approximates the dielectric properties of RFP in this region.

The Stix frame, connected with the local direction of the equilibrium magnetic field, is not inertial for a moving particle. The difference between the Stix frame and an inertial frame leads to a correction in the equation for gyrophase when the particle's motion is considered in the drift approximation (see the discussion by Littlejohn[5] and Ref. 6). Another correction to the gyrophase is more directly related to the magnetic field geometry. It is due to nutation (or tilting back and forth) of the value of $\mathbf{b}$ ($\mathbf{b}=\mathbf{B}/B$) as it is seen by the gyrating particle.[5] We refer to both these corrections as geometrical corrections to the gyrophase. Such corrections belong to first-order terms in ϵ_D. In the rf modeling of tokamak plasmas with the poloidal magnetic field, emphasis is placed on accounting for the large parallel gradient in the equilibrium magnetic field. The parallel gradient of the magnetic field introduces significant numerical complication even in the case when the unperturbed particle orbits are calculated to zero order in ϵ_D. For this reason, the higher accuracy particle orbits are not considered in practical full wave simulations, see Refs. 7 and 8 (FISIK, TORIC codes) and Refs. 9 and 10 (ALCYON code). On the other hand, the rotational transform in tokamaks is relatively small. As will be seen from the analysis in this paper, the correction to $k_\parallel$ of Eq. (1) in the perpendicular plasma response due to the geometrical corrections to the gyrophase is $\sim \sin\theta_0/r$ with $\sin\theta_0 = B_\theta/B$. The ratio B_θ/B is small in tokamaks, which probably justifies the neglect of the effects due to geometrical corrections to the gyrophase in rf modeling of tokamak plasmas.

In a recent formulation of the constitutive relation of axisymmetric toroidal plasmas by Brambilla[11] the particle's gyrophase is calculated in the Stix frame to zero order in ϵ_D. Thus the geometrical corrections to particle's gyrophase are not included in this formulation. In a detailed analysis of the rf response of tokamak plasmas made by Lamalle[12] the drift approximation is used to first order in ϵ_D inclusively and the geometrical corrections to the gyrophase are included in a general description. In the following comparison of different theoretical models made by Lamalle[13] it was identified that the correction to the gyrophase due to the curvature and twist of the magnetic field lines is not included in the full wave simulations in tokamaks. The emphasis in this comparison is made on the proper modeling of the parallel gradient of magnetic field, while the geometrical corrections to the gyrophase are not included in the main points of comparison. Thus we may conclude that such geometrical effects are considered as not important in present day rf modeling of tokamaks, which seems to be appropriate in view of our remark about the smallness of the corresponding correction for tokamak magnetic field.

In RFPs, however, the ratio B_θ/B is substantially larger than in tokamaks (poloidal and toroidal components of magnetic field in RFPs are comparable[14]). As follows from our earlier discussion the geometrical corrections to the gyrophase in RFPs are important and they have to be included in RFP modeling for a proper evaluation of power absorption near ion cyclotron resonance. Our calculation is different from the analysis in Refs. 12 and 13 in the following. We consider the region near magnetic axis where particle orbits are simplified such that an explicit analytical result is possible. In these references a general formulation in tokamak geometry is made such that analytical results are possible only in limiting cases. This formulation is not valid near the axis because for the basis used in Ref. 12, the expansion over the drift parameter ϵ_D diverges as $r \to 0$ (see the remark in the following). Thus our result is not the limiting case of expressions obtained in the above-mentioned references. We now briefly outline the derivation of the corrected plasma dielectric response.

We derive the dielectric tensor by integration of the linearized Vlasov equation considering simplified particle orbits and taking the limit $k_\perp \rho_L \to 0$. We modify the standard calculations performed in the case of a uniform magnetic field (see, e.g., Ref. 15) to our case of twisted field lines in the RFP. The plasma current is obtained in terms of an integral along the unperturbed particle orbits,[15]

$$\mathbf{j}(\mathbf{r},t) = -\sum_\alpha \frac{n_\alpha e^2}{m_\alpha} \int \mathbf{v}\, d\mathbf{v} \int_{-\infty}^{t} dt'\, \mathbf{E}(\mathbf{r}',t') \frac{\partial F_\alpha}{\partial \mathbf{v}'}, \qquad (2)$$

where α denotes ions (with $Z_\alpha = 1$) or electrons; $\mathbf{r}'$, $\mathbf{v}'$ are the solution of the equations of motion in equilibrium magnetic field satisfying the "final" conditions $\mathbf{v}'(t'=t)=\mathbf{v}$, $\mathbf{r}'(t'=t)=\mathbf{r}$. The positioning of the density n_α outside the integral is justified in the limit of zero Larmor radius. In our derivation the unperturbed distribution function F_α is a normalized Maxwellian. Similar calculations can be carried out for non-Maxwellian distribution functions as well.

The equilibrium magnetic field has components $B_\theta(r)$, $B_z(r)$ ($B_r=0$). The local triad of unit vectors of the Stix frame is $\mathbf{e}_r$, $\mathbf{e}_{\tilde\theta}$, $\mathbf{e}_{\tilde z}$, where $\mathbf{e}_{\tilde z} \| \mathbf{B}$ and $\mathbf{e}_{\tilde\theta} = \mathbf{e}_{\tilde z} \times \mathbf{e}_r$. We consider vectors $\mathbf{j}$, $\mathbf{E}$, $\mathbf{v}$, $\mathbf{v}'$ in the Stix frame. Assuming time dependence $\propto \exp(-i\omega t)$ for $\mathbf{j}$ and $\mathbf{E}$, we perform a Fourier transform of Eq. (2) in θ and z coordinates. We obtain

$$\mathbf{j}(r) = -\sum_\alpha \frac{n_\alpha e^2}{m_\alpha} \int \mathbf{v}\, d\mathbf{v} \times \int_{-\infty}^{t} dt'\, \mathbf{E}(r) \frac{\partial F_\alpha}{\partial \mathbf{v}'} \cdot e^{-i[m(\theta-\theta') + k_z(z-z') - \omega(t-t')]}. \qquad (3)$$

Now the components of $\mathbf{j}$ and $\mathbf{E}$ are the Fourier harmonics of corresponding components of rf current and electric field with the azimuthal and axial wave numbers m and k_z. In Eq. (3) we assumed that $r'=r$ due to the limit $k_\perp \rho_L \to 0$.

To find the orbits with improved accuracy one should consider particle's equations of motion in drift approximation to first order in the parameter ϵ_D, see, e.g., Ref. 6. Accurate treatment of these equations leads to a relatively complicated calculation of plasma dielectric response. Such calculation for the helical magnetic field is made by

Mikhailovskii.[16] In this calculation the geometrical corrections to the gyrophase are neglected while other effects of the helical magnetic field are retained. We accurately calculate the plasma dielectric response near the axis; in this region the drift contributions are small and the equations of motion are significantly simplified.

Near the axis the velocity components in the Stix frame are considered in the form

$$v_{\tilde{z}}=v_{\parallel}, \quad v_r=v_{\perp}\cos(\varphi-\theta), \quad v_{\tilde{\theta}}=v_{\perp}\sin(\varphi-\theta) \tag{4}$$

and the time evolution is described by

$$\dot{v}_{\parallel}=0, \quad \dot{v}_{\perp}=0, \tag{5}$$

$$\dot{\varphi}=-\omega_{c\alpha}-\frac{v_{\parallel}}{2}\mathbf{e}_{\tilde{z}}\cdot(\nabla\times\mathbf{e}_{\tilde{z}}) = -\omega_{c\alpha}-\frac{v_{\parallel}}{2}\left(\theta_0'+\frac{\sin\theta_0\cos\theta_0}{r}\right). \tag{6}$$

In Eq. (4) θ is the particle's azimuthal position in cylindrical coordinates (not the position of the guiding center) while the right-hand side of Eq. (6) is evaluated at the location of the guiding center. The prime in Eq. (6) denotes the derivative with respect to r. φ in Eqs. (4) and (6) can be considered as a gyrophase defined in a quasi-inertial frame (i.e., free from the singularity of the Stix frame). The geometrical correction to the gyrophase in Eq. (6) is due to the nutation of $\mathbf{b}$ as seen by the gyrating particle. The correction due to twisting of the Stix frame is incorporated in Eq. (4). A direct substitution of particle velocity of Eqs. (4)–(6) into the equations of motion shows that the inaccuracy in these equations is of the order of $\epsilon_D\sin\theta_0$ when $\sin\theta_0\gtrsim\epsilon_D$ or ϵ_D^2 when $\sin\theta_0\lesssim\epsilon_D$. This inaccuracy is reduced by the factor $\sin\theta_0$ relative to the case when geometrical corrections to the gyrophase are not included. Thus the orbits of Eqs. (4)–(6) provide an adequate accuracy for the calculation of resonant rf plasma response near the axis. It should be noted that the orbits of Eqs. (4)–(6) are not obtained from the standard form of the drift approximation (Ref. 6). Because of the divergence of $\nabla\times\mathbf{e}_{\tilde{\theta}}$ when $r\to 0$ the Stix reference frame is not appropriate for formulating the drift approximation near the axis.

The integral over the particle velocity $\mathbf{v}$ in Eq. (3) is performed in cylindrical coordinates connected with the Stix frame. Particle velocity at a moment t' is calculated in the Stix frame connected with the point at which particle is located. We introduce left- and right-handed perpendicular field components E_+ and E_- such that $E_{\pm}=(E_r\pm iE_{\tilde{\theta}})/\sqrt{2}$. Taking into account Eq. (4) the time integral in Eq. (3) becomes

$$\int_{-\infty}^{t}dt'\left\{\frac{v_{\perp}}{v}\frac{E_+}{\sqrt{2}}e^{-i[(m+1)(\theta-\theta')-(\varphi-\varphi')]-i\tilde{\varphi}} + \frac{v_{\perp}}{v}\frac{E_-}{\sqrt{2}}e^{-i[(m-1)(\theta-\theta')+(\varphi-\varphi')]+i\tilde{\varphi}} + \frac{v_{\parallel}}{v}E_{\tilde{z}}e^{-im(\theta-\theta')}\right\}\frac{\partial F_{\alpha}}{\partial v}e^{-i[k_z(z-z')-\omega(t-t')]}, \tag{7}$$

where $v=\sqrt{v_{\perp}^2+v_{\parallel}^2}$. In Eq. (7) $\tilde{\varphi}$ is a dummy variable of integration in velocity space (we changed $\tilde{\varphi}=\varphi-\theta$). Within the considered accuracy one can substitute the coordinates of the guiding center in place of θ' and z' in Eq. (7), such that

$$\theta'=\theta-\frac{v_{\parallel}\sin\theta_0}{r}(t-t'), \tag{8}$$

$$z'=z-v_{\parallel}\cos\theta_0(t-t'). \tag{9}$$

The use of guiding center coordinates in Eq. (7) instead of the exact particle coordinates is justified due to the following. The correction to the z coordinate due to finite Larmor radius is a small parameter which can be neglected in the limit $k_{\perp}\rho_L\to 0$. The same condition holds for the θ coordinate when $r\gg\rho_L$, but it is not valid when $r\sim\rho_L$. From relations between E_r, E_θ and E_x, E_y we find that as $r\to 0$,

$$E_+=\frac{E_r+iE_\theta}{\sqrt{2}}=\frac{E_x+iE_y}{\sqrt{2}}e^{-i\theta}, \tag{10}$$

$$E_-=\frac{E_r-iE_\theta}{\sqrt{2}}=\frac{E_x-iE_y}{\sqrt{2}}e^{i\theta}, \tag{11}$$

which means that $E_+(r=0)\neq 0$ only for $m=-1$ and $E_-(r=0)\neq 0$ only for $m=1$. Similarly $E_{\tilde{z}}(r=0)\neq 0$ only for $m=0$. Thus we can deduce that for small radii $r\lesssim\rho_L$ the $\theta-\theta'$ dependence vanishes in Eq. (7) for the realistic electric fields which validates Eq. (8) in this region as well.

We substitute orbits of Eqs. (8) and (9) and $\varphi-\varphi'$ resulting from Eq. (6) into Eq. (7). We compare the resulting integral with the corresponding integral in a uniform plasma with magnetic field in z direction. The terms in the resulting integral are obtained from the corresponding terms for the uniform magnetic field by substitution $k_z\to k_{\parallel}^+$ in the term with E_+, $k_z\to k_{\parallel}^-$ in the term with E_- and $k_z\to k_{\parallel}$ in the term with $E_{\tilde{z}}$, where

$$k_{\parallel}=\frac{m\sin\theta_0}{r}+k_z\cos\theta_0,$$
$$k_{\parallel}^+=k_{\parallel}+k_s+k_c, \quad k_{\parallel}^-=k_{\parallel}-k_s-k_c, \tag{12}$$
$$k_s=\frac{1}{2}\left(\theta_0'+\frac{\sin\theta_0\cos\theta_0}{r}\right), \quad k_c=\frac{\sin\theta_0}{r}.$$

Therefore the dielectric tensor in our geometry can be found from the dielectric tensor in the uniform magnetic field by the appropriate substitution of wave numbers $k_{\parallel}^+$, $k_{\parallel}^-$, $k_{\parallel}$ in place of k_z. The correction k_s is due to the nutation of $\mathbf{b}$ within gyro-orbit (or due to the shear of magnetic field), the correction k_c is due to the twisting of the local coordinate frame.

The resulting conductivity tensor in our geometry is

$$j_+=\frac{j_r+ij_{\tilde{\theta}}}{\sqrt{2}}=\frac{\omega}{4\pi i}\sum_{\alpha}\frac{\omega_{p\alpha}^2}{\omega^2}\frac{\omega}{|k_{\parallel}^+|v_{T\alpha}}Z\left(\frac{\omega-\omega_{c\alpha}}{|k_{\parallel}^+|v_{T\alpha}}\right)E_+, \tag{13}$$

$$j_{-}=\frac{j_r-ij_{\tilde{\theta}}}{\sqrt{2}}=\frac{\omega}{4\pi i}\sum_{\alpha}\frac{\omega_{p\alpha}^2}{\omega^2}\frac{\omega}{|k_{\parallel}^{-}|v_{T\alpha}}Z\left(\frac{\omega+\omega_{c\alpha}}{|k_{\parallel}^{-}|v_{T\alpha}}\right)E_{-}, \tag{14}$$

$$j_{\tilde{z}}=-\frac{\omega}{4\pi i}\sum_{\alpha}\frac{\omega_{p\alpha}^2}{\omega^2}\left(\frac{\omega}{|k_{\parallel}|v_{T\alpha}}\right)^2 Z'\left(\frac{\omega}{|k_{\parallel}|v_{T\alpha}}\right)E_{\tilde{z}}, \tag{15}$$

where $v_{T\alpha}=\sqrt{2T_{\alpha}/m_{\alpha}}$, $\omega_{p\alpha}$, $\omega_{c\alpha}$ are the r dependent plasma and cyclotron frequency, Z is the plasma dispersion function, $k_{\parallel}$, $k_{\parallel}^{+}$, and $k_{\parallel}^{-}$ are defined by Eqs. (12). The conductivity tensor defined by these equations contains different wave numbers for different wave polarizations when $r=0$. From Eqs. (10) and (11) it follows that the $m=1$ component is purely right-handed and the $m=-1$ component is purely left-handed on axis. This is a general property of fields in cylindrical coordinates. Therefore the conductivity tensor on axis is calculated with wave number $k_{\parallel}^{+}=k_z+k_s=k_z+\theta_0'$ for left-handed circular polarization, $k_{\parallel}^{-}=k_z-k_s=k_z-\theta_0'$ for right-handed polarization, and $k_{\parallel}=k_z$ for parallel electric field. Such difference between parallel wave numbers is due to the helical structure of magnetic field. The corrections to parallel wave number further separate the plasma dielectric response for different polarizations. Note that the term k_c in equations for $k_{\parallel}^{+}$ and $k_{\parallel}^{-}$ compensates the term $m \sin\theta_0/r$ for these polarizations. Thus the transition from the Stix frame to an inertial frame removes the term $m\sin\theta_0/r$ in Eq. (1) in the region near the axis. The correction due to shear of magnetic field k_s acts in the same direction as k_c and further shifts wave numbers in perpendicular plasma response from that defined by Eq. (1).

From the conductivity tensor defined by Eqs. (13)–(15) in the Stix frame one can find the plasma dielectric tensor T in cylindrical coordinates. Such dielectric tensor properly describes the plasma dielectric response near the axis in our cylindrical model and provides better modeling for heating near the axis. The tensor includes geometrical corrections to the gyrophase which are found to be important in rf modeling of RFP plasmas near ion cyclotron resonance. The power absorbed on the magnetic surface can be found by averaging the quantity

$$P_{\text{abs}}=\frac{\omega}{8\pi}\mathbf{E}^{*}\cdot\mathsf{T}^{\mathrm{A}}\cdot\mathbf{E}$$

over a magnetic surface (see Ref. 15). Regarding the estimates of the absorbed power near axis in Ref. 1 (in which left-handed $m=-1$ component is considered) we can note that one should use $k_{\parallel}=k_z+\theta_0'$ in the argument of the plasma dispersion function instead of $k_{\parallel}^1=k_z-\theta_0'$ of Eq. (1) or $k_{\parallel}^2=k_z$ (used in Ref. 1). The correction θ_0' to $k_{\parallel}^2$ would somewhat improve wave absorption near the axis in the examples in Ref. 1. Because of the relatively low value of the safety factor in RFPs, θ_0' can be a noticeable correction to k_z of realistic antennas in the ion cyclotron frequency range. We can conclude that the geometrical corrections to the gyrophase and probably other corrections to particle orbits of first order in ϵ_D (which appear starting from midradius where $\sin\theta_0\sim 1$) have to be included in accurate modeling of plasma dielectric properties in RFPs near ion cyclotron resonance. Such corrections are probably less important in rf modeling of tokamak plasmas in this frequency range.

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