Possibility of Stabilization of Resistive Wall Modes in RFPs by RF Ponderomotive Forces

V. A. Svidzinski, S. C. Prager

Physics Department, University of Wisconsin-Madison, WI 53706

Abstract. Ponderomotive forces from applied rf fields are generally too weak to stabilize MHD instabilities in toroidal plasmas for practical values of rf fields. We are investigating whether rf waves in a vacuum region between a plasma and wall can stabilize resistive wall instabilities. In our model the rf waves are generated in an RFP by an oscillating poloidal loop voltage. Because the resistive wall acts as a perfect conductor to high frequency rf fields, the rf field in the vacuum region is strongly compressed by the surface motion, thereby generating a strong restoring force. We consider a simple model in which the dielectric properties of the plasma, used to determine the rf penetration, are assumed to be that of an unmagnetized perfectly conducting plasma. Within this model we find stabilization at moderate rf power. However, the dielectric properties of a magnetized plasma, presently under study, weaken the effect.

The resistive wall modes are potentially dangerous instabilities in long-pulse Reversed Field Pinch (RFP) experiments. We investigate the possibility of stabilizing these modes by an rf voltage applied to a toroidal gap of an RFP. This stabilization results from the work done by the plasma against the ponderomotive force exerted by the rf field.

When there is a small vacuum region between plasma and resistive wall, the plasma surface perturbation strongly perturbs the rf fields in this region, generating a strong restoring force. The effect is significant for two reasons. First, the resistive wall acts essentially as a perfect conductor to high frequency rf fields. Second, the smaller the width of the vacuum region, the stronger the restoring force.

An approach of this kind was suggested in Ref. [1] for the ponderomotive stabilization of external kink modes in tokamaks. In this paper it was suggested to launch rf waves with ion Bernstein wave type antenna systems to create the wave structure with a strongly evanescent $E_{||}$ field. In this analysis the effects due to the perturbation of the applied rf wave intensity where neglected.

Our analysis relates to RFPs in which the magnetic field at the plasma surface is nearly poloidal. In this case the appropriate evanescent rf wave structure can be created by applying the rf voltage to the toroidal gap of an RFP. In our analysis the perturbation of rf intensity due to the perturbation of the plasma boundary plays crucial role.

Simplified consideration of rf pressure. We consider a cylindrical geometry shown in Fig. 1 with plasma radius a and resistive wall radius b. An rf voltage of frequency ω is applied to the toroidal gap. We assume that in this model the rf waves are excited by

CP595, *Radio Frequency Power in Plasmas: 14th Topical Conf.,* edited by T. K. Mau and J. deGrassie

a uniformly distributed electric field on the surface of the vessel:

$$\mathbf{E}(b) = \frac{1}{2}\left(E_A e^{-i\omega t} + c.c.\right)\mathbf{e}_\theta .$$

In this simplified consideration we assume the dielectric properties of the plasma to be that of an unmagnetized perfectly conducting plasma. In the frequency ranges of interest $\omega \ll \omega_{ci}$ or $\omega \gtrsim \omega_{ci}$ the condition $\lambda = \omega a/c \ll 1$ is satisfied.

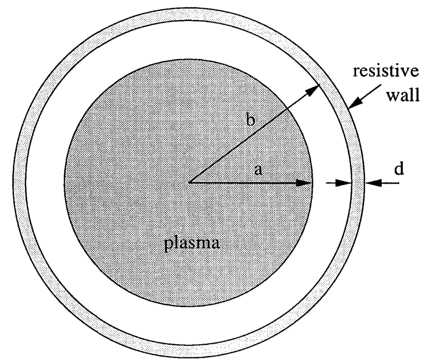

FIGURE 1.

From Maxwell's equations we find the nonzero components of unperturbed fields in the vacuum region satisfying the boundary conditions at $r = a$ and $r = b$:

$$E_\theta = \frac{\delta E_A}{\delta^2 - 1}\frac{(r-a)(r+a)}{ar}, \; B_z = \frac{2}{i\lambda}\frac{\delta E_A}{\delta^2 - 1}, \text{ where } \delta = b/a \tag{1}$$

The time averaged rf pressure on the plasma surface is

$$P_0 = \frac{|\mathbf{B}|^2}{16\pi} = \frac{1}{16\pi}\frac{4}{\lambda^2}\left(\frac{\delta}{\delta^2 - 1}\right)^2 |E_A|^2 .$$

Using a perturbative approach we find the rf pressure on a perturbed plasma surface. We consider the surface perturbation of the form

$$r_s = a + \frac{1}{2}\left(Ae^{im\theta + ikz} + c.c.\right) .$$

For the perturbed plasma surface, the rf fields in the vacuum layer are a superposition of the unperturbed fields of Eqs. (1) and a perturbation. The general form of the amplitudes of the perturbed rf fields satisfying Maxwell's equations is

$$E_z = \left[B^+ I_m(|k|r) + C^+ K_m(|k|r)\right] e^{im\theta + ikz} + \left[B^- I_m(|k|r) + C^- K_m(|k|r)\right] e^{-im\theta - ikz},$$
$$B_z = \left[D^+ I_m(|k|r) + F^+ K_m(|k|r)\right] e^{im\theta + ikz} + \left[D^- I_m(|k|r) + F^- K_m(|k|r)\right] e^{-im\theta - ikz}.$$

The boundary conditions at $r = b$, $E_\theta(b) = 0$ and $E_z(b) = 0$, and similar conditions on the plasma-vacuum boundary are used to find the coefficients in the above equations. Then we find the total time averaged rf pressure on the plasma surface:

$$P = P_0 + \tilde{p}_{\mathrm{rf}}\frac{1}{2}\left(Ae^{im\theta + ikz} + c.c.\right), \text{ with } \tilde{p}_{\mathrm{rf}} = -P_0\frac{g_{\mathrm{rf}}}{g'_{\mathrm{rf}}}2|k| > 0, \tag{2}$$

where $g_{\mathrm{rf}} = I_m(|k|a) - \frac{I'_m(|k|b)}{K'_m(|k|b)}K_m(|k|a)$, $g'_{\mathrm{rf}} = I'_m(|k|a) - \frac{I'_m(|k|b)}{K'_m(|k|b)}K'_m(|k|a)$.

Because in Eq. (2) $\tilde{p}_{\rm rf} > 0$, the perturbed rf pressure profile has a stabilizing effect on the plasma surface. This profile is such that the local force acting on the displaced plasma surface tends to restore the plasma to its equilibrium state.

In spite of its smallness, the rf pressure can influence the stability properties of the plasma. From Eq. (2) it follows that the perturbed rf pressure is proportional to $A/(b-a)$ ($g'_{\rm rf} \propto b-a$), while the MHD force acting on the perturbed plasma column is proportional to A/a (for the low frequencies of resistive wall modes). This means that for a sufficiently thin vacuum layer the perturbed MHD and rf forces can be comparable even when the unperturbed rf pressure is significantly smaller than the magnetic pressure.

Calculation of the growth rates. We derive the equation for the growth rates of the resistive wall modes in a similar way as done in Chap. 9 of Ref. [2] for the general screw pinch. We modify the pressure balance equation on the plasma-vacuum boundary by including the rf pressure calculated in the previous section. This pressure balance becomes:

$$\mathbf{B}_a\mathbf{B}^p_{1a} = \mathbf{B}_a\mathbf{B}^v_{1a} + 4\pi\tilde{p}_{\rm rf}\xi_a\,.$$

This leads to the equation for the growth rate ω_i:

$$\omega_i\tau_D = \frac{k^2b^2+m^2}{k^2b^2K'_m(|k|b)I'_m(|k|b)\left[1-\frac{I'_m(|k|a)K'_m(|k|b)}{I'_m(|k|b)K'_m(|k|a)}\right]}\frac{\delta W_\infty+\delta W_{\rm rf}}{\delta W_b+\delta W_{\rm rf}}\,, \tag{3}$$

where

$$\tau_D = 4\pi bd/\eta c^2\,,\ \delta W_{\rm rf} = 2\pi^2 R_0 a\tilde{p}_{\rm rf}\xi_a^2\,,$$

δW_∞ and δW_b relate to δW with a perfectly conducting wall located at $r=\infty$ and $r=b$, they are defined in Ref. [2]. $\delta W_{\rm rf}$ is proportional to the work done by the forces of rf pressure exerted on the plasma-vacuum surface when this surface is continuously perturbed by increasing ξ_a from 0 to its amplitude value.

For a mode, unstable without the rf pressure, $\delta W_\infty < 0$ and $\delta W_b > 0$. In Eq. (3) one can assume that $\delta W_{\rm rf} \ll \delta W_b$. Because $\delta W_{\rm rf} > 0$, the rf pressure either reduces the growth rate of an unstable mode or stabilizes it.

To estimate changes to the growth rates, we consider an equilibrium cylindrical state given by the Taylor's theory in the limit of zero plasma pressure. In this state $B_z/B_0 = J_0(\mu r)$, $B_\theta/B_0 = J_1(\mu r)$, where B_0 is the toroidal field on axis. B_z reverses on the outside if $\mu a > 2.4$.

To estimate δW_∞ we take the trial function $\xi(r) = \xi_a$, $0 \le r \le a$. With these assumptions the growth rate $\omega_i\tau_D$ is defined by the dimensionless parameters P_0/P_B ($P_B = B_0^2/8\pi$), μa, b/a, m, ka. The modes unstable without rf power are with $m=\pm1$. Because the growth rate is symmetric with respect to simultaneous change $m\to -m$, $k\to -k$, we consider the modes with $m=1$.

Figures 2(a)-(c) show the dependence of $\omega_i\tau_D$ versus ka for $\mu a = 2.4; 2.5; 2.6$. The dashed lines are $\omega_i\tau_D$ without rf pressure and the solid lines are $\omega_i\tau_D$ with $P_0/P_B = 0.05$. When $\omega_i > 0$ the mode is unstable, and when $\omega_i < 0$ the mode is stable.

From this figure one can see that without the rf pressure the mode is unstable for some range of wave numbers ka. With the rf pressure the mode is either stabilized

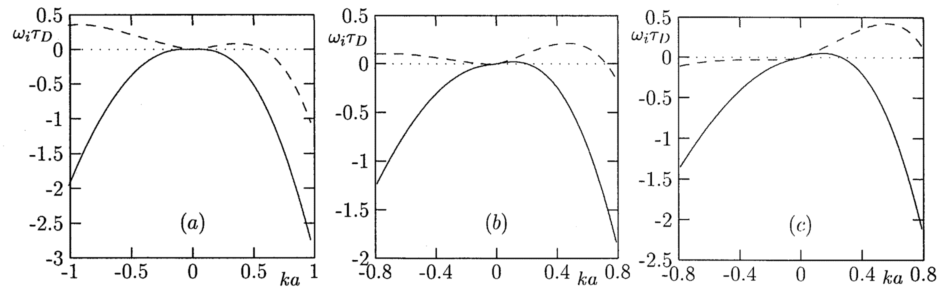

FIGURE 2. $\omega_i\tau_D$ vs. ka. (a) $\mu a = 2.4$, (b) $\mu a = 2.5$, (c) $\mu a = 2.6$. In all cases: (- - -) $P_0/P_B = 0$, (—) $P_0/P_B = 0.05$; $b/a = 1.1$, $m = 1$

or its maximum growth rate is reduced by approximately an order of magnitude. If one identifies an equivalent torus of length $2\pi R_0$, then the wave number k becomes quantized: $ka = na/R_0$. If we take the aspect ratio $R_0/a = 3$, then from Fig. 2 one can see that the mode is stabilized by the rf pressure for these wave numbers.

This analysis shows that the stabilization of the resistive wall modes in RFPs is possible with application of moderate rf power. From this point of view this approach seems to be attractive. Further analysis with a more realistic description of the dielectric properties of the plasma is necessary, however, for a more substantiated conclusion about the applicability of this approach.

Discussion. We performed a study of a more realistic model of a magnetized plasma in plane geometry in three different regimes. In the first case we considered rf frequency range $\omega \ll \omega_{ci}$. In this model there is a skin layer due to finite plasma resistivity and the propagation of rf waves is described by a resistive MHD model. The second case relates to $\omega \gtrsim \omega_{ci}$ and the skin layer is defined by electron inertia. In this model the propagation of em waves is described by a cold plasma dielectric tensor. In the third case we considered the second case with the assumption of zero skin depth.

All of these regimes reproduce the result of the ideal case of an unmagnetized plasma only for perturbations with $m = 0$, which is not the case of interest for external modes. Within the accuracy of calculations the rf pressure does not influence the perturbations with $m \neq 0$, i. e. it neither stabilizes nor destabilizes them. This suppression of the effect is because of the presence of a penetrating branch in the magnetized plasma with the electric field component perpendicular to the equilibrium magnetic field.

Effects due to perturbation of rf intensity considered with the proper description of the plasma dielectric properties should be taken into account in analysis of the methods of ponderomotive stabilization of external modes in fusion devices.

REFERENCES

1. D. A. D'ippolito Phys. Fluids **31**, 340 (1988).
2. J. P. Freidberg, *Ideal Magnetohydrodynamics*, (Plenum Press, New York, 1987).