

TUTORIAL PAPERS

Does flow shear suppress turbulence in nonionized flows?*

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The suppression of turbulence by mean flow shear is shown to apply to nonionized flows as well as plasmas. However, besides the criterion that the shearing rate exceed the turbulent decorrelation rate, there are three additional conditions. These stipulate that the shear flow must be stable, that turbulence must remain in the domain of flow shear for longer than an eddy turnover time, and that the dynamics should be two dimensional (2D). In nonionized flows, these conditions are not typically satisfied, explaining why shear suppression is not a familiar phenomenon in hydrodynamics. The three conditions are discussed in the context of nonionized and plasma flows. Two examples of suppression in nonionized flows are presented. One involves the formation of coherent structures in 2D Navier–Stokes turbulence and the other involves large-scale turbulence in the stratosphere. © 2000 American Institute of Physics. [S1070-664X(00)91805-6]

I. INTRODUCTION

The suppression of turbulence and turbulent transport by $E \times B$ flow shear in magnetically confined plasmas is so widespread that it has been called universal.^{1,2} This phenomenon is regarded as the central engine of the H-mode (high confinement mode), an edge transport barrier observed in all auxiliary heated tokamaks, in stellarators, and in mirrors. Transport barriers associated with $E \times B$ flow shear have also been detected in the reversed field pinch, and are believed to apply to the Z pinch. The phenomenon is invariant with respect to plasma location, operating in internal and edge transport barriers alike. Implicit in these observations, and explicitly demonstrated in numerous model calculations and simulations, suppression of turbulence by $E \times B$ flow shear occurs for many different types of turbulence and instabilities. Given this quasiuniversal character, it is striking that this phenomenon is not a familiar feature of nonionized flows. Scouring the literature of nonionized flows, it is possible to identify isolated instances where suppression of turbulence or fluctuations by flow shear appears to be operating;² however, the physical mechanism has not been identified in any case we have encountered.

Although the suppression of turbulence by flow shear in plasmas is a feature of the $E \times B$ velocity, a flow of charged particles in an ionized plasma, there is ample reason to expect that suppression of turbulence by flow shear ought to occur in nonionized fluids. The charge-independent $E \times B$ velocity is identical for ions and electrons. As the universal advector of fluctuations, it plays the same role in plasma continuum equations as the mean and turbulent flows of nonionized continuum descriptions.³ A demonstration of suppression of turbulence by flow shear in a nonionized flow is

of interest for plasma physics. It would help validate the basic mechanism of shear suppression in plasmas,⁴ enhancing confidence that plasma transport barriers indeed operate through the suppression of turbulence by flow shear and not the confluence of other complicated effects intrinsic to confined plasmas (e.g., confining fields and geometries, multiple fluctuations and instabilities, nonambipolar particle losses, multiple states, bifurcations, etc.). Lacking at present such an experiment, it is essential for the credibility of the shear suppression mechanism to determine why this mechanism is not familiar in hydrodynamics, what features of nonionized flows are responsible, and the implications for fusion plasmas.

We show that suppression of turbulence by flow shear is not a universal feature of all turbulent flows, but subject to several conditions (besides the condition that the shear straining rate exceed the turbulent decorrelation rate⁴). These conditions are often satisfied in fusion plasmas, but not in nonionized flows. The conditions are: (1) the sheared flow must be stable, (2) turbulence must remain in the region of strong flow shear for longer than a turbulent correlation time, and (3) two-dimensional (2D) flow is desirable for making the phenomenon identifiable. When these conditions are violated, turbulence is driven by shear instead of suppressed, it is advected through the region of shear before the nonlinearity can decorrelate fluctuations, and suppression of vorticity in certain directions is intermingled with amplification in others.

Table I compares plasma flows with nonionized flows in the context of the conditions listed above. While there are nonionized flows that satisfy all three conditions, they are exceptional. One case is large-scale flow in the atmosphere and oceans, known as quasigeostrophic flow. Suppression of quasigeostrophic turbulence by flow shear has been observed in simulations^{5,6} and is postulated to occur in the

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TABLE I. Situation in plasma and nonionized flows with regard to conditions for suppression.

	Stability (Shear flow must be stable)	Long time (Turbulence must remain in shear region longer than nonlinear time)	2D (To avoid 3D amplification of vorticity)
Magnetic fusion plasmas	<i>Usually stable:</i> Magnetic shear and rotation stabilize flow shear	<i>Yes:</i> Flow shear is present over entire path of circulating flow	<i>Yes:</i> Strong ambient magnetic field \Rightarrow $k_{\parallel} \ll k_{\perp}$
Non ionized flows	<i>Often unstable:</i> Wall flows, jets, wakes, boundary layer flows, flow past objects are unstable	<i>Frequently no:</i> In strongly sheared engineering flows, turbulence is often in shear region transiently (e.g., wind tunnel with local constriction)	<i>Almost never:</i> Exception: large-scale flows in atmosphere, ocean (flow time scale > Earth's rotational period)

stratosphere.⁶ The situation where turbulence is advected through a region of strong flow shear in a time shorter than the eddy turnover time is quite common in nonionized flows, particularly engineering flows.⁷ A widely used technique known as rapid distortion theory (RDT) can be employed to trace out the fluid motions. We show that RDT is the short time, linear counterpart of the long time, turbulent decorrelation theory of Biglari, Diamond, and Terry (BDT).⁴ In three-dimensional (3D) flows, the suppression mechanism applies only to vorticity perpendicular to both the directions of the flow and the shear. Vorticity in either the direction of the flow or the shear is amplified through the process of vortex tube tilting and stretching. Since the fluctuation spectrum is dominated by the most intense fluctuations, measured signals and transport are generally dominated by the amplified components of the turbulence and not the suppressed components. We show, however, that there is a 3D generalization of BDT that applies to all components of the vorticity under a Lagrangian construct known as the potential vorticity.⁸ Since the potential vorticity also incorporates the effects of compressibility, suppression is found to apply in a general way to compressible fluctuations in 3D.

This paper is organized as follows. In Sec. II we discuss each of the three conditions for suppression in the context of both fusion plasmas and nonionized fluids. This section includes analyses of shear flow stability, an examination of the relationship of RDT to BDT, a description of vortex tube stretching, and an analysis of potential vorticity suppression. Section III presents two cases in nonionized fluids where turbulence and transport are suppressed. One deals with the formation of coherent structures in decaying 2D Navier–Stokes turbulence, and the other with quasigeostrophic turbulence. The conclusions are presented in Sec. IV.

II. REQUIREMENTS FOR SHEAR SUPPRESSION OF TURBULENCE

A. Stable shear flow

Unstable shear flow routinely arises in nonionized fluids, leading to the common association of shear flow with the driving of turbulence and not its suppression. Nonetheless,

some shear flows are stable and thus capable of suppressing turbulence. Stability can be achieved under a variety of mechanisms, but general stability criteria are difficult to formulate. Necessary and sufficient conditions depend on the precise details of spatial variations of flow, density, rotation, etc., and are rarely available.⁹ Certain principles, however, can be enumerated. To this end, consider the Kelvin–Helmholtz (KH) instability in a fluid of uniform density for a continuous mean flow with a region of linear shear connected at opposite ends to regions of oppositely directed uniform flow. Specifying x and y as the directions of the mean flow and shear, $\bar{u}(y) = U_0$ for $y > L$; $\bar{u}(y) = -U_0$ for $y < -L$; and $\bar{u}(y) = U_0 y/L$ for $-L < y < L$. The instability is governed by the Navier–Stokes equation. Taking the curl and assuming inviscid 2D perturbation (no variation in the z direction),

$$\frac{d}{dt} [\nabla \times (\bar{\mathbf{u}} + \nabla \tilde{\phi} \times \mathbf{z})] = \left(\frac{\partial}{\partial t} + \bar{u}(y) \frac{\partial}{\partial x} + \nabla \tilde{\phi} \times \mathbf{z} \cdot \nabla \right) \nabla^2 \tilde{\phi} - \frac{d^2 \bar{u}}{dy^2} \frac{\partial \tilde{\phi}}{\partial x} = 0, \quad (1)$$

where the fluctuating flow $\mathbf{u} = \nabla \tilde{\phi} \times \mathbf{z}$ is expressed in terms of the stream function $\tilde{\phi}$, and $\mathbf{z} \nabla^2 \tilde{\phi} = \nabla \times \mathbf{u}$ is the vorticity. The last term on the left-hand side is required for instability, i.e., the flow must have a second derivative. For the present example this is provided by the discontinuities in slope at $x = \pm L$. For fluctuations centered at the vorticity maximum $y = 0$, the growth rate is

$$\gamma_k = U_0 k_x \left(\frac{1}{k_x L} - 1 - \frac{1}{2k_x^2 L^2} \frac{\sinh(2k_x L)}{\exp(2k_x L)} \right)^{1/2}, \quad (2)$$

where k_x is the Fourier wave number in the x direction. The growth rate is positive provided $k_x L < 0.64$, with the growth rate maximum near $k_x L = 0.4$. Very long wavelengths ($k_x L \ll 0$) are unstable, but the growth rate goes to zero as k_x goes to zero. The instability condition $k_x L < 0.64$ can be interpreted as allowing a perturbation with $u_y \lesssim U_0$ to sample both signs of the curvature $d^2 \bar{u}/dy^2$ occurring at $y = \pm L$. This permits the interchange of vortex filaments to relax the

unstable shear profile. The requirement that flow curvature change sign in the domain of instability, known as the Rayleigh inflection point instability criterion, applies to arbitrary profiles as a *necessary* condition for instability.¹⁰ The flow curvature is the gradient of vorticity for the above case, and the inflection point criterion reflects a constraint imposed by invariance of vorticity.

Shear flow can be stabilized by rotation and buoyancy, and can be incorporated into a generalized inflection point criterion¹⁰ based on the gradient of a generalized vorticity known as the potential vorticity.⁸ In ionized fluids, the magnetic field also stabilizes shear flow. Because the magnetic (Lorentz) and rotation (Coriolis) forces enter the momentum equation isomorphically, the modification of the inflection point criterion by magnetic field is analogous to that of rotation. To consider rotation, we examine the flow of a planetary atmosphere. For fluid motions whose typical time scale is longer than the planetary rotation period, motion is 2D with negligible variation in the vertical direction. The invariant generalized vorticity is the total vorticity \hat{Q} , which includes the vertical component of the planetary vorticity (vorticity of flow component corotating with the planet) and the relative vorticity (vorticity of motion in the rotating frame). Thus,

$$\frac{d\hat{Q}}{dt} = \frac{d}{dt} [2\Omega \sin \theta - \nabla^2 \Phi] = 0, \tag{3}$$

where Ω is the planetary rotation rate, θ is the latitude, and $\nabla^2 \Phi$ is the relative vorticity, expressed in terms of a stream function Φ and the Laplacian of displacements perpendicular to the vertical. The advective derivative d/dt is with respect to relative motion. We introduce a tangency plane in which to describe the 2D motion. The latitude of the tangency point is θ_0 , $y = a(\theta - \theta_0)$ is the northward displacement about the tangency point, x is the eastward displacement, and a the radius of the planet. The vertical component of the planetary vorticity is

$$\begin{aligned} 2\Omega \sin \theta &= 2\Omega \sin \left(\theta_0 + \frac{y}{a} \right) \\ &= 2\Omega \sin \theta_0 + \frac{y}{a} 2\Omega \cos \theta_0 + O(y^2/a^2) \\ &\equiv f_0 + y\beta + O(y^2/a^2), \end{aligned} \tag{4}$$

valid for small displacements ($y \ll a$). Here β is the lowest order gradient in the tangency plane of planetary vorticity $2\Omega \sin \theta$. The tangency plane is referred to as a β -plane. The relative vorticity is the curl of the relative flow $\mathbf{u} = \nabla \Phi \times \mathbf{z}$, where \mathbf{z} is normal to the β -plane. If the stream function has mean and turbulent components $\Phi = \Phi_0(y) + \phi(x, y)$, the flow $\mathbf{u} = \bar{u}(y)\mathbf{x} + \nabla \phi \times \mathbf{z}$ has a mean zonal (east-west) component with north-south shear $\bar{u}(y) = \partial \Phi_0 / \partial y$ and a turbulent component $\nabla \phi \times \mathbf{z}$. Specifying these flows in the advective derivative, Eq. (3) becomes

$$\left(\frac{\partial}{\partial t} + \bar{u}(y) \frac{\partial}{\partial x} + \nabla \phi \times \mathbf{z} \cdot \nabla \right) \nabla^2 \phi - \frac{d^2 \bar{u}}{dy^2} \frac{\partial \phi}{\partial x} + \beta \frac{\partial \phi}{\partial x} = 0. \tag{5}$$

This is the equation for quasigeostrophic turbulence in a β -plane. Comparing with Eq. (1), the stabilizing effect of planetary rotation through the β term is evident in its creation of an effective zero-point shift of the mean flow curvature. The Rayleigh inflection point criterion is modified so that a necessary condition for instability stipulates that the gradient of mean total vorticity,

$$S_\Omega \equiv \frac{d\hat{Q}}{dy} = \beta - d^2 \bar{u} / dy^2, \tag{6}$$

change sign in the domain of the flow. If β is sufficiently large, there is no instability. In the stratosphere, large-scale shear flows such as the equatorial jet¹¹ and the Antarctic and Arctic polar vortices^{12,13} occasionally become unstable under episodic disturbances called wave breaking events.¹⁴ These events redistribute the flow curvature, and stability is reestablished.

In an ionized fluid with a magnetic field, fluid parcels are subjected to the Lorentz force. The Lorentz force associated with a mean magnetic field enters the momentum equation of an ionized fluid in the same way the Coriolis force enters the momentum equation of a rotating neutral fluid. From the respective momentum equations, $\rho(d\mathbf{u}/dt + 2\mathbf{\Omega} \times \mathbf{u}) = -\nabla p$ and $\rho d\mathbf{u}/dt = e\rho m^{-1}(\mathbf{u} \times \mathbf{B}) - \nabla p$, the dynamical equivalence of $2\mathbf{\Omega}$ and $e m^{-1} \mathbf{B}$ is evident. Consequently, a magnetic field induces two-dimensional dynamics and has a stabilizing effect on shear flow, just as rotation does in a neutral fluid. The equations of reduced magnetohydrodynamics (MHD) provide a convenient and simplified representation of the essential physics,

$$\rho \frac{d}{dt} \nabla_\perp^2 \frac{\Phi}{B_0} = B_0 \nabla_\parallel J_\parallel, \tag{7}$$

$$B_0 \nabla_\parallel \frac{\phi}{B_0} = -\eta J_\parallel, \tag{8}$$

where ρ is the plasma mass density, $\nabla_\perp^2 \Phi B_0^{-1}$ is the total vorticity, obtained as the curl of the $E \times B$ flow, J_\parallel is the plasma current along the equilibrium field B_0 , and η is the resistivity. The derivative d/dt includes advection by the mean $E \times B$ flow $\bar{u}(x) = -B_0^{-1} \nabla \Phi_0 \times \mathbf{b}$ and fluctuating $E \times B$ component $-B_0^{-1} \nabla \phi \times \mathbf{b}$, where \mathbf{b} is the unit vector along B_0 . Magnetic field fluctuations have been neglected in Eqs. (7) and (8). Note that the left-hand side of Eq. (7) is identical in form to the neutral fluid case [Eq. (1)]; the right-hand side is the curl of the Lorentz force, and is analogous to the curl of the Coriolis force. The second equation is Ohm's law for current along the mean magnetic field. Parallel current enters the dynamics provided there is a perturbation (finite wave number) along the field.

Dropping the nonlinearity, combining Eqs. (7) and (8) by eliminating J_\parallel , and introducing the Fourier transform for y and the parallel displacement, the vorticity equation becomes

$$\left(\frac{\partial}{\partial t} + ik_y \bar{u}(x) \right) \left(\frac{\partial^2}{\partial x^2} - k_y^2 \right) \frac{\phi_{ky}}{B_0} - ik_y \frac{d^2 \bar{u}}{dx^2} \frac{\phi_{ky}}{B_0} = \frac{B_0^2}{\rho \eta} k_\parallel^2 \frac{\phi_{ky}}{B_0}, \tag{9}$$

where a uniform magnetic field in the \mathbf{z} direction has been assumed. The last term of Eq. (9) represents field line-bending energy and couples to Ohmic dissipation. Its similarity to the β term of Eq. (5) suggests an inflection point criterion based on $S_M = iB_0^2 k_{\parallel}^2 / \rho \eta k_y - d^2 \bar{u} / dx^2$ changing sign. However, any fluctuation that is truly 2D (with $k_{\parallel} = 0$) experiences no stabilizing effect from the magnetic field. Recalling that β is the gradient of rotation, we are promoted to consider a magnetic field with shear. For a sheared slab with $\mathbf{B}_0 = B_0 z + (x/L_s) B_0 y$, $k_{\parallel} \rightarrow (x/L_s) k_y$, and any mode that extends from $x=0$ has a finite parallel wave number. The line-bending term now forms a quadratic eigenmode potential well for the fluctuation eigenmode. When magnetic shear is strong, the large line-bending energy associated with large x forces the mode to localize to small x . However, if $x=0$ is the point of inflection of the mean flow $\bar{u}(x)$, magnetic shear can force a localization that is so strong that the fluctuation is unable to sample sufficiently large regions of opposite curvature to access the free energy of the shear flow, thus leading to stability.

Numerical evaluation of Eq. (9) for a model flow profile $\bar{u}(x) = V_0 \tanh(x/L_E)$ indicates that the mode is stabilized for all wave numbers of the system if the line-bending term is larger than the curvature term at $x \cong L_E$ for k_y evaluated at the minimum poloidal wave number of the system.¹⁵ Omitting a multiplier of order unity, stability thus requires

$$\frac{d^2 \bar{u} / dx^2}{u} \cong \frac{1}{L_E^2} < \left(\frac{B_0^2 k_{\min}^2}{\rho V_0 \eta L_s^2} \right)^{1/2} = (\text{Lu})^{1/2} (\text{Al})^{1/2} a^{-1} L_s^{-1}, \quad (10)$$

where $\text{Lu} = \mu_0 a v_A / \eta$ is the Lundquist number, $\text{Al} = v_A / V_0$ is the Alfvén number, and k_{\min} is taken to be the inverse minor radius a^{-1} . Because the line-bending term of Eq. (9) is quadratic in x , it dominates the driving term at large x . An inflection point criterion can thus be formulated as a necessary condition for instability. It follows from insisting that the stabilizing term become weak at some value of x , i.e., that

$$S_M = \frac{B_0^2 k_y x^2}{\rho \eta L_s^2} - \frac{d^2 \bar{u}}{dx^2} \quad (11)$$

change sign in the domain of x . This is equivalent to Eq. (6), the instability criterion for horizontal motion in a rotating atmosphere, i.e., that $S_{\Omega} = \beta - \partial^2 \bar{u} / \partial y^2$ change sign in the flow domain. (Note: standard conventions are used, making x and y the directions of mean flow and flow shear in the atmospheric examples, with x and y the directions of flow shear and mean flow in the fusion example.)

Comparison of these criteria indicates that planetary rotation gradient β and magnetic shear $B_0^2 k_y x^2 / \rho \eta L_s^2$ play comparable roles in stabilizing shear flows. For typical tokamak parameters, Eq. (10) is readily satisfied and shear flow is stable. Consideration of rotation likely would indicate additional stabilization. On the other hand, β is not overwhelmingly large in the atmosphere, with wave breaking events a result. Because wave breaking events are among the largest scale dynamical events in the atmosphere, their massive re-

distribution of potential vorticity makes it difficult to detect the role of flow shear in suppressing fluctuations and transport.

B. Long time dynamics

Stable flow shear suppresses turbulence⁴ when the rate τ_s^{-1} at which mean shear strains eddies exceeds the eddy turnover rate τ_e^{-1} , or a suitable surrogate such as an instability growth rate. There is a second temporal condition that is less well known but equally important: turbulence must remain in the physical region of flow shear for longer than an eddy turnover time.² These two conditions can be expressed as

$$\tau_s < \tau_e < \tau_D, \quad (12)$$

where the domain time τ_D is the time turbulence remains in the region of flow shear. A large class of engineering flows do not satisfy this criterion, but instead satisfy⁷

$$\tau_s \approx \tau_D < \tau_e. \quad (13)$$

In this regime, the nonlinearity has insufficient time to modify the flow. The flow can therefore be modeled by linearized equations, and features of the initial state are retained. The modeling procedure, known as rapid distortion theory (RDT), maps fluctuation structure incident at a region of flow shear into an evolving pattern downstream.^{7,16} We show here that BDT is the nonlinear, long time extension of RDT.

BDT and RDT regimes can be obtained as long and short time asymptotic limits of the nonlinear advection process associated with suppression of turbulence by flow shear. Consider the advection of a scalar ξ by a sheared mean flow $U_1 = \alpha x_3$, where U_1 is in the x_1 direction, and advection by 2D turbulence is represented with a turbulent diffusivity. Under a Fourier–Laplace transform involving time and the x_1 direction, the evolution of the scalar is given by

$$\begin{aligned} (-\gamma + i\alpha x_3 k_1) \xi_{k_1, \gamma} - \frac{\partial}{\partial x_3} \left(D_{k_1} \frac{\partial \xi_{k_1, \gamma}}{\partial x_3} \right) + k_1^2 D_{k_1} \xi_{k_1, \gamma} \\ = \sigma_{k_1, \gamma}(x_3) - \xi_{k_1}(x_3, t=0), \end{aligned} \quad (14)$$

where $\xi_{k_1, \gamma}(x_3)$ is the amplitude of the Fourier–Laplace transformation of $\xi(x_1, x_3, t)$,

$$\xi_{k_1, \gamma}(x_3) = \int_0^{\infty} dt \exp(-\gamma t) \int_{-\infty}^{\infty} dx_1 \exp(ik_1 x_1) \xi(x_1, x_3, t), \quad (15)$$

$\xi_{k_1}(x_3, t=0)$ is the Fourier amplitude at the initial time, $u = \alpha x_3$ is the mean shear flow, $\sigma_{k_1, \gamma}(x_3)$ is a source for the scalar ξ , and D_{k_1} is the turbulent diffusivity. This is the type of model solved by BDT using a two-point theory.⁴ The two-point approach preserves nonlinear invariance properties such as energy conservation. While the dissipative form of the nonlinearity in the one-point representation [Eq. (14)] does not preserve energy conservation, it does accurately represent the turbulent response, which governs spatial and temporal structure. In the asymptotic limit of large shear

($\alpha \rightarrow \infty$), this structure can be found from the unique combinations of terms that dominate the left-hand operator of Eq. (14).

There are two consistent asymptotic balances in the limit of large shear. Before the lapse of the first nonlinear interaction time, $\gamma \gg D_{k_1} / \Delta x_3^2, D_{k_1} k_1^2$, where Δx_3 is a turbulent eddy scale in the direction of shear. With insufficient time for the nonlinear interaction to modify the flow, γ must balance $\alpha x_3 k_1$. This balance leads to secular evolution of ξ , with the scale in the direction of the shear shrinking continuously with time at a rate proportional to αk_1 . If the initial state is $\xi_{k_1, \gamma}(x_3, t=0) = \exp[-1/2 x_3^2 \Delta k^2 - k_1^2 / 2 \Delta k^2]$, the solution for later times is (assuming zero source)

$$\xi_{k_1}(x_3, t) = \int_{-\infty + i\gamma_0}^{\infty + i\gamma_0} d\gamma \exp(\gamma t) \frac{\xi_{k_1, \gamma}(x_3, t=0)}{(-\gamma + i\alpha x_3 k_1)}, \quad (16)$$

which becomes $\xi(x_1, x_3, t) = \exp\{-1/2[x_3^2 + (x_1 - \alpha x_3 t)^2] \times \Delta k^2\}$. Under a Fourier transformation of both spatial directions, $\xi_{k_1, k_3}(t) \propto \exp\{-[k_1^2 + (k_3 + \alpha k_1 t)^2] / 2 \Delta k^2\}$. This is equivalent to a structure that evolves from the initial Fourier state $\exp[-(k_{30}^2 + k_{10}^2) / 2 \Delta k^2]$ under a mapping of wave numbers,

$$k_1 = k_{10}, \quad k_3 = k_{30} - \alpha k_1 t. \quad (17)$$

In this solution the wave number along the flow is unmodified, while the wave number in the shear direction increases secularly with time. This corresponds to a continuous decrease of the turbulence scale in the shear direction.

Equation (17) is identical with the results of RDT.¹⁶ The linearized evolution equation is solved by Fourier transformation in both x_1 and x_3 , to yield

$$\frac{\partial}{\partial t} \xi_{k_1, k_3} - \alpha k_1 \frac{\partial}{\partial k_3} \xi_{k_1, k_3}, \quad (18)$$

and the method of characteristics is introduced by writing $d\xi/dt \equiv \partial\xi/\partial t + (\partial\xi/\partial k_j)(dk_j/dt)$. The wave numbers thus evolve according to

$$\frac{dk_1}{dt} = 0; \quad \frac{dk_3}{dt} = -\alpha k_1, \quad (19)$$

with solutions given by Eq. (17).

The secular increase of k_3 cannot continue indefinitely, because eventually the nonlinearity becomes important. After several nonlinear interaction times $\gamma \ll D_{k_1} / \Delta x_3^2$. In the asymptotic limit $\alpha \rightarrow \infty$, Eq. (14) is singular because the highest derivative drops out unless a singular layer develops in which there are rapid variations of ξ over x_3 , allowing $i\alpha x_3 k_1 \sim D_{k_1} \partial^2 / \partial x_3^2$. In this limit, memory of the initial spatial structure is lost due to nonlinear decorrelation. The structure is governed by the eigenmode of the homogeneous equation, which describes variation in the singular layer. From Wentzel–Kramers–Brillouin (WKB) theory, the leading order asymptotic eigenmode structure in the limit $\varepsilon_s^{-1} = \alpha k_1 \Delta x_3^3 / D_{k_1} \rightarrow \infty$ is

$$\xi_{k_1, k_3} \sim (x_3)^{-1/2} \left(\frac{\alpha k_1 x_3}{D_{k_1}} \right)^{-1/4} \exp \left[\pm \frac{2}{3} \left(\frac{-i\alpha k_1}{D_{k_1}} \right)^{1/2} x_3^{3/2} \right] \quad (\varepsilon_s^{-1} \rightarrow \infty). \quad (20)$$

From WKB ordering, the rapid variation in x_3 that allows $D_{k_1} \partial^2 / \partial x_3^2$ to balance $i\alpha x_3 k_1$ in the singular limit $\alpha \rightarrow \infty$ is dominated by the exponential function. Note that this limit corresponds to $\varepsilon_s < 1$, i.e., the strong shear limit of BDT, and the spatial scale of the rapid variation in the shear direction is $\Delta x_3|_s = (D_{k_1} / \alpha k_1)^{1/3}$, precisely the reduced correlation length derived by BDT. This value of Δx_3 , or equivalently $k_3|_s = (D_{k_1} / \alpha k_1)^{-1/3}$, represents the nonlinear saturation of the secular growth of Eq. (17). BDT is thus an extension of RDT to the nonlinear, long time regime. Note that RDT, while incapable of giving the saturated value of the scale in the direction of shear, does show that this scale diminishes from nominal values in turbulence with no mean flow shear. It also shows that the scale along the flow is unchanged in the linear regime. In nonlinear treatments it has generally been assumed that this scale remains unchanged.

C. 2D turbulence

Two-dimensional dynamics provides an ideal circumstance for detecting suppression of turbulence by flow shear. To understand why, consider a three-dimensional mean flow with shear given by $U_1 = \alpha_1 x_1, U_2 = \alpha_2 x_2, U_3 = \alpha_3 x_3$, where α_1 can vary in time but not in space. Incompressibility constrains $\alpha_1 + \alpha_2 + \alpha_3 = 0$. This type of flow occurs in ducts with changing cross section. As in the example of the prior section, the tendency of flow shear to enlarge or diminish scales nonlinearly is already apparent in the linear evolution of RDT. We therefore employ RDT to examine the evolution of vorticity,

$$\frac{\partial \omega_i}{\partial t} + U_l \frac{\partial \omega_i}{\partial x_l} = \omega_l \frac{\partial U_i}{\partial x_l}, \quad (21)$$

where $\omega = \nabla \times \mathbf{U}$ is the vorticity, viscosity has been assumed to be negligible, and only linear evolution is retained, consistent with RDT. Under the Fourier expansion $\omega_i = \sum_k \Omega_i(k, t) \exp(i\mathbf{k} \cdot \mathbf{x})$, the characteristic procedure yields

$$\frac{d\Omega_1}{dt} = \alpha_1 \Omega_1, \quad \frac{dk_1}{dt} = -\alpha_1 k_1, \quad (22)$$

where the other two components are governed by identical equations with the appropriate change of subscripts. It is evident that along directions in which the flow moves outward from the origin ($\alpha_j > 0$), the vorticity intensifies and the wave number decreases. The opposite is true along directions in which the flow moves inward. This is a simple manifestation of a basic process known as vortex tube stretching.¹⁷ The outward directed flow velocity increases with distance from the origin and thereby stretches vortex tubes whose axes align with the flow velocity. The increase of vortex tube length requires a decrease of cross-sectional area because the vortex volume must remain invariant. The smaller cross-sectional area requires an increase in vorticity to maintain the invariance of circulation ($\oint \mathbf{u} \cdot d\mathbf{l} = \text{constant}$, for inviscid

flow). In a flow with $\alpha_1 = -\alpha_3 = \text{constant}$, $\alpha_2 = 0$, vorticity in the x_1 direction increases while its cross-sectional area, parameterized by k_3 , decreases. This means that the wave number k_3 increases. Similarly, Ω_3 and k_1 decrease. This intensification process is a 3D phenomenon. If the turbulence is 2D, the vorticity is solely in the x_2 direction. With $\alpha_2 = 0$, the vorticity is unchanged. In this case only the scales are modified. In 3D flows, the intensification of vorticity aligned with the mean flow is a competing effect to the reduction of vorticity perpendicular to the flow. While there is suppression in one direction, the signals are dominated by the amplified components in the other directions, making suppression of vortical turbulence, even in the long time domain, difficult to detect.

There is, however, a function of the vorticity, known as potential vorticity, that responds to flow shear in compressible, rotating, 3D turbulence the way advected scalars respond in 2D turbulence. The latter is described in BDT, which treats a scalar quantity whose total concentration is invariant, i.e., is governed solely by advection. In the invariant situation, advection incorporates just two processes, the straining by mean flow shear and straining by the turbulence. These lead to suppression when the rate of mean straining exceeds the turbulent straining rate. Vorticity, in contrast, is not invariant, but has sources associated with compressibility, rotation, and stretching. These effects can be treated, along with the straining of the turbulence and mean flow shear, by considering the potential vorticity, which remains invariant in the presence of these effects. The potential vorticity is $Q = \rho^{-1} \omega \cdot \nabla \vartheta$, where ρ is the density, ω is the vorticity, and ϑ is the potential temperature, defined as the temperature acquired by a fluid parcel under an adiabatic change from a given temperature and pressure to a reference pressure. The potential vorticity is an invariant of the motion because its definition (through the factor $\omega \rho^{-1}$) offsets the increase of vorticity ω when fluid within a filament is compressed. The increases of vorticity as a filament is stretched by lofting in a stable stratified medium is offset by $\nabla \vartheta$, which acts as a Lagrangian marker, adjusting the metric for the stretching of material lines in the flow.⁸ The invariance of potential vorticity,

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) Q = 0, \tag{23}$$

holds for a system governed by $\partial \mathbf{u} / \partial t + (\mathbf{u} \cdot \nabla) \mathbf{u} + 2 \boldsymbol{\Omega} \times \mathbf{u} = -\rho^{-1} \nabla p - \nabla \Phi$, $p = p(\rho, \eta)$, $\partial \rho / \partial t + \nabla \cdot (\rho \mathbf{u}) = 0$, and $\partial \eta / \partial t + (\mathbf{u} \cdot \nabla) \eta = 0$, where \mathbf{u} is a 3D flow comprised of mean and turbulent components, Φ is the potential for external forces, $\boldsymbol{\Omega}$ is the rotation rate, η is the specific entropy, and p is the pressure.⁸ The potential temperature can be replaced with any function of η and Eq. (23) still holds. Equation (23) is like the scalar evolution equation of BDT in the limit of weak dissipation, i.e., there are only two time scales, the shear and turbulent straining rates. Hence, we immediately conclude that the fluctuation scale of potential vorticity in the direction of shear and the correlation time are reduced when the shear straining rate exceeds the turbulent decorre-

lation rate. Moreover, in the presence of a source associated with advection of mean potential vorticity, fluctuations in the potential vorticity will be suppressed.

III. SUPPRESSION IN NONIONIZED FLOWS

A. Coherent vortices in Navier–Stokes turbulence

Suppression of turbulent vorticity transport by flow shear leads to spatial intermittency in decaying 2D Navier–Stokes turbulence.^{18,19} Intermittency is manifested in the emergence of coherent vortices in simulations that initialize homogeneous turbulence from a Gaussian random distribution of vorticity with no mean flow.²⁰ As the turbulence decays, certain eddies emerge as coherent vortices, avoiding mixing by ambient fluctuations and thus persisting for a large number of eddy turnover times. The vortices are patches of intense localized vorticity characterized by a particular spatial profile of a quantity called the Gaussian curvature (GC). The GC is the difference of the mean squared shear stress, $(\partial V / \partial x - \partial U / \partial y)^2 + (\partial U / \partial x + \partial V / \partial y)^2$, and the mean squared vorticity. (U and V are the total flow velocities in the x and y directions.) The GC is strongly positive in the vortex core, and strongly negative in the edge. Coherent vortices avoid turbulent mixing by suppressing turbulence in their periphery. Localized vorticity fluctuations have a flow profile in which flow shear is largest at their edges. Those whose initial vorticity is stronger than that of ambient fluctuations possess an edge flow shear whose shear straining rate exceeds the ambient turbulent decorrelation rate. This shear flow suppresses the ambient turbulence and its transport of vorticity, and yields the observed profile of GC. Vortices whose initial vorticity is comparable to that of ambient turbulence cannot suppress ambient turbulence. They participate in the cascade of energy to the dissipation scale and decay in an eddy turnover time.

The interaction of an intense symmetric vortex with the ambient turbulence can be described with a two-time scale analysis of the Navier–Stokes equation.^{18,19} The origin of a polar coordinate system is placed at the center of a vortex. With a Fourier–Laplace transform of the turbulent vorticity $\xi(r, \theta, t)$, $\xi_{n,\gamma} = \int dt \exp(-\gamma t) \int d\vartheta \exp(in\vartheta) \xi(r, \vartheta, t)$, the $n=0$ component is the symmetric vortex and $n \geq 1$ is the turbulence. The $n=0$ component evolves on a slow time scale under the action of turbulent mixing. On the rapid time scale the vortex can be treated as stationary. The turbulence structure at the edge of the vortex is subjected to the strong shear of the vortex, and has an exponentially decaying envelope moving inward from the vortex edge. The envelope function has the same form as that of Eq. (20), i.e., $\xi_{n,\gamma} \sim \exp[\pm(2/3)(-i\Omega'_n/D_n)^{1/2}(r-r_0)^{3/2}]$, where $\Omega'_n = n \partial(\bar{u}/r) / \partial r|_{r_0}$ is the differential rotation of $\bar{u}(r)$, the vortex flow, r_0 is a radial position in the vortex edge, and D_n is the turbulent diffusivity. On the long time scale, the vortex is subject to mixing by turbulence, with an eddy viscosity given by

$$D_v \sim \int d\gamma \sum_n \left(\frac{-in^2 |\phi_{n,\gamma}|^2}{r^2} \right) \frac{S\Omega_n'^{-1}}{(r-r_0)^{1/4}(a-r_0)^{3/4}} \times \exp \left[\frac{2}{3} \left(\frac{-iQ_n}{D_n} \right)^{1/2} (r-r_0)^{3/2} \right], \quad (\varepsilon_s^{-1} \gg 1), \quad (24)$$

where S is a weakly varying structure function of order unity, a is the vortex radius, and for the present parameters $\varepsilon_s = D_n/a^3\Omega_n'$. Due to phase mixing in summing the exponential of a complex argument, the effective eddy viscosity is dominated by $n=1$. Moreover, the real part of the argument of the exponential makes D_v different from zero only within a narrow exponential layer of thickness $(D_n/\Omega_n')^{1/3}$ at the vortex edge. Stronger vortices (relative to ambient fluctuations) have a larger value of ε_s^{-1} , and therefore a smaller effective viscosity. For turbulence to mix the vortex, it must diffuse into the vortex, extending the edge layer inward. This process is greatly slowed by the weakness of the eddy viscosity, and its localization within the narrow layer at the vortex edge.

The condition $\varepsilon_s = D_n/a^3\Omega_n' < 1$ is dimensionally equivalent to the condition that the vortex vorticity exceed the rms turbulent vorticity, $(\Xi_0/\langle \xi^2 \rangle^{1/2} > 1)$, where the rms average is computed for an ensemble of fluctuations in the vicinity of the vortex and Ξ_0 is the vortex vorticity at $r=0$. This inequality indicates that an initial vorticity fluctuation becomes coherent if it lies in the tail of the initial vorticity probability distribution function (pdf). For decaying turbulence, fluctuations initially in the core of the pdf, distinguished by $\Xi_0/\langle \xi^2 \rangle^{1/2} < 1$, decay as part of the Kolmogorov cascade. Fluctuations in the tail of the pdf ($\Xi_0/\langle \xi^2 \rangle^{1/2} > 1$) decay at a far slower rate, causing the tail probability to become enhanced with time. A Kolmogorov cascade with no coherent vortices implies a Gaussian pdf. Thus, an initial Gaussian pdf will evolve so that its core remains Gaussian while its tail develops an enhanced non-Gaussian feature. This type of evolution is observed in intermittent turbulence.

The observed GC profile in the region of a coherent vortex directly indicates that the shear suppression criterion $\varepsilon_s < 1$ is satisfied. The GC of the vortex flow is $C_v = r^2 n^{-2} \Omega_n'^2 - \Xi^2$, where Ξ is the vorticity of the vortex, while the total GC is

$$C_T = \frac{r^2 Q_n'^2}{n^2} - (\Xi^2 + \langle \xi^2 \rangle). \quad (25)$$

The turbulent vorticity is included in Eq. (25) to account for the total squared vorticity. It is of importance near $r=a$, where the vortex vorticity is zero. (The vanishing of the vortex vorticity at the vortex radius is implicit in the stipulation that the vortex is localized.) The turbulent shear stress is not included in Eq. (25) because it is dominated by the vortex shear stress in the edge, and fluctuations are small near the center. The observed negative GC near the centers of the coherent vortices¹⁹ reflects the fact that $r^2\Omega_n'^2 n^{-2}$ vanishes there. At the edge, Ξ^2 vanishes and positive GC implies that $a^2\Omega_n'^2 n^2 > \langle \xi^2 \rangle \approx D_n^2 a^{-4}$, reproducing $\varepsilon_s < 1$.

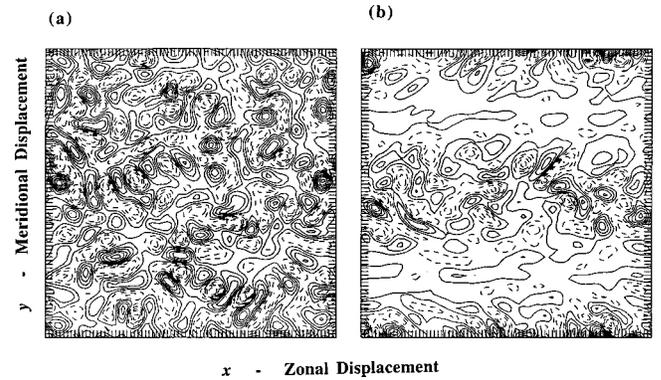


FIG. 1. Contours of constant vorticity for β -plane turbulence with (a) no mean flow, and (b) a cosine jet in the x direction with minima at the top and bottom and a maximum in the center.

As apparent from the similarity of the 2D Navier–Stokes equation and the Hasegawa–Mima equation, flow shear is a vehicle for intermittency and coherent structure formation in drift wave turbulence.^{21,22}

B. Quasigeostrophic turbulence

A number of observations of constituent concentrations (aerosols, chloroflourocarbons, ozone, etc.) in the stratosphere show steep gradients coincident with regions of strong horizontal shear.⁶ It is possible that these represent a suppression of turbulent transport due to flow shear. This statement is speculative at present because other competing processes, such as large-scale wave breaking events, must also be assessed, and source and sink configurations of the constituents and the turbulence itself must be determined. One of the most striking examples is that of aerosol injected into the tropical stratosphere by volcanic eruptions.²³ The aerosols are quickly spread along the equator by the equatorial jet, a zonal flow in the lower part of the stratosphere. North-south spreading extends to about $\pm 20^\circ$ latitude where a sharp poleward gradient of aerosol concentration forms. This gradient is closely aligned with the flanks of the jet, a region of strong flow shear. The gradients at $\pm 20^\circ$ persist for years after a major volcanic eruption. Strong constituent gradients also appear at the edge of the polar vortices, again a region of strong flow shear.¹¹ Moreover, fluctuations in the wind velocity itself appear to be smaller in the region of shear than on either side, where the mean wind speed profile is flatter.¹²

In simple models of geostrophic turbulence it is possible to specify what conditions are present and thus unequivocally determine the role of flow shear. The β -plane model, Eq. (5), is ideal because the physics it contains is analogous to that of plasma models while it applies to large-scale motions in the stratosphere. Two studies have documented suppression of turbulence^{5,6} and transport⁶ in regions of strong mean flow shear. In these studies a mean zonal flow with meridional shear was specified as a model for flows such as the equatorial jet and polar winter vortices. For computational simplicity, the mean flow profile was a one-period sinusoid, allowing periodic boundary conditions in both

zonal and meridional directions. The turbulence was initialized and allowed to decay, or driven by an external force. In the simulations, turbulence is not driven by the mean flow because β is chosen sufficiently large relative to the second derivative of the mean flow to make the flow stable. The largest scales of the turbulence were smaller than the meridional scale of the mean flow by a factor of ~ 5 . This turbulence is representative of fluctuations created by wave breaking, or driven by other processes, such as waves propagating upward from the troposphere.

A reduction of turbulent vorticity where the zonal mean flow has maximum shear was observed in Ref. 5, but no explanation was offered. The shear suppression criterion, Eq. (12), was not a part of the analysis or interpretation, and it is difficult to determine how strongly it was satisfied. In Ref. 6 parameters were chosen to make τ_s/τ_e as small as 0.1 in the region of maximum shear. Figure 1 shows contours on constant vorticity for two cases, one with no mean flow initialized in the simulation, and one with a cosine jet yielding $\tau_s/\tau_e \cong 0.1$ in the region of strongest flow shear. There is a near absence of turbulence where $\tau_s/\tau_e < 1$ and strong turbulence elsewhere. For the case with no jet the turbulence is homogeneous and isotropic. Transport can be examined for the β -model of Eq. (5) by using tracer particles and calculating an effective spatial diffusivity from the separation of pairs of tracer particles as a function of time. The diffusivity is found to be minimum where τ_s/τ_e is minimum. The diffusivity decreases as the jet amplitude is raised for fixed mean flow scale length. Above a critical amplitude, however, the jet becomes unstable ($\partial^2 \bar{u}/\partial y^2 > \beta$), and the diffusivity rises abruptly. Above the instability threshold diffusivities in the x and y directions are comparable, while below the threshold, the diffusivity in the direction of shear is much smaller than the diffusivity along the flow.

IV. CONCLUSIONS

Flow shear is able to suppress turbulence and transport in nonionized flows, just as it does in plasmas. However, the process is unfamiliar in nonionized flows because three conditions required for suppression, beyond the standard strong shear condition,⁴ are rarely satisfied. These conditions are summarized in Table I. They stipulate that shear flow must be stable, that turbulence must remain in the domain of flow shear for longer than an eddy turnover time, and that the dynamics should be 2D.

Consideration of these conditions indicates that rotation and the magnetic field play analogous roles in stabilizing shear flow. In fusion plasmas, magnetic shear has typically been credited for stabilizing shear flows; however, rotation may also contribute to the stability of the shear flows associated with experiment. The suppression of turbulence by flow shear is a nonlinear effect that operates when turbulence is subjected to shear for longer than a nonlinear correlation time. The short time regime, of interest when turbulence passes transiently through strong flow shear in less than a nonlinear interaction time, is a complementary limit for which the linearized rapid distortion theory (RDT) was developed. For unidirectional plane shearing, RDT predicts a

secular growth of wave number in the direction of the shear. This wave number growth saturates nonlinearly after a nonlinear interaction time at the value predicted by BDT.⁵ For the wave number along the flow there is no change in magnitude in the short time regime, indicating that there is no evolution to saturate. This provides the rationale for an assumption implicitly made in BDT that the scale along the flow is not modified by the flow shear. The restriction to 2D dynamics eliminates vortex tube stretching, whose amplification of vorticity in directions along the flow and along the shear competes with suppression in the other direction. However, an invariant function of vorticity, the potential vorticity, responds to shear in 3D compressible flows the way invariant scalars do in 2D flows.

Two nonionized flows that exhibit suppression of turbulence by flow shear were considered. In 2D decaying Navier–Stokes turbulence, the suppression of ambient turbulence by the shear flow at the edge of large amplitude vorticity fluctuations allows them to escape mixing.^{18,19} They emerge as the coherent structures that account for the spatial intermittency and non-Gaussian nature of the flow. Quasigeostrophic turbulence in a β -plane satisfies the three conditions for suppression of turbulence by flow shear, with rotation providing stability and two dimensionality. Simulations of externally driven turbulence in a cosine jet show suppression of both turbulence and transport.^{5,6} This behavior is speculated to play a role in transport barriers in the stratosphere.⁶

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