Anomalous particle pinch for collisionless plasma

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The particle transport arising from the convection of nonadiabatic electron density by ion temperature gradient driven turbulence is examined for trapped electron collisionality regimes. In the lower temperature end of this regime, trapped electrons are collisional and the particle flux is outward (in the direction of the gradients). When the trapped electrons are collisionless, there is a temperature threshold above which the electron temperature gradient driven particle flux changes sign and becomes inward. The magnitude of the nonadiabatic electron contribution to the growth rate is found to be potentially of the same order as the ion contribution.

It has long been recognized from efforts to model equilibrium density profiles in tokamak discharges that inward particle flow is frequently needed to counter particle losses resulting from outward diffusion in regions where the particle source is negligibly small. This requirement follows straightforwardly from the radial particle balance, $\partial n/\partial t + r^{-1}\,\partial(r\Gamma)/\partial r = S$, where n is the density, Γ is the flux, and S is the particle source. In the absence of a source, the steady-state solution of this equation constrains the net particle flux to be uniform. A flux, which consists only of outward diffusion, $\Gamma = -D\,\partial n/\partial r$, is inconsistent with this constraint. It has been popular to model the flux with an inward convection in addition to the outward diffusion: $\Gamma = -D\,\partial n/\partial r - V_0 nr/a$. Under a model assumption of constant D and V_0, a Gaussian steady-state density profile $n = n_0 \exp[-(V_0/2D)r^2/a]$ follows from the particle balance when $S = 0$. While a transport model representing outward fluxes with diffusion, inward fluxes with convection, and using constant coefficients is not entirely consistent with observed transport, this modeling exercise nevertheless illustrates the need for inward particle flow in characterizing steady-state particle transport which incorporates outward diffusion in regions without any source.

This view is further supported by nonstationary transport analysis techniques. Using a modulated gas feed, and assuming there is no source, coefficients for D and V_0 have been determined independently.[1] The diffusivity is found to be on the order of 1 m^2/sec while the convection velocity lies between 1 and 20 m/sec. Both have similar scalings with respect to global parameters. The magnitudes of these transport coefficients are sufficiently large that a description in terms of neoclassical diffusion and the Ware pinch is inadequate, suggesting that anomalous processes play the dominant role. It is worth noting that the above statements regarding the need for an inward particle flux follow from analyses, which assume that the particle source away from the edge is zero. A nonzero source of particles could effectively reduce the magnitude of inward flux inferred from stationary and nonstationary transport analyses. At present, the particle source in interior regions has not been measured experimentally, although neutral recycling codes suggest that it is not zero.

One mechanism for inward particle flux is the ion mixing mode,[2] which incorporates the effect of nonadiabatic electrons in ion temperature gradient driven turbulence. Because the dissipative nonadiabatic electron response comprises the only part of the fluctuating density which is out of phase with the potential, these electrons determine the particle transport (both ion and electron since the transport is ambipolar). It is assumed that the nonadiabatic electrons produce only minor changes to the basic ion temperature driven growth (an assumption that must be verified *a posteriori*). This is an important consideration since the phase shift of density relative to potential required for inward transport also gives a stabilizing contribution to the basic ion growth rate. Ion mixing mode transport has previously been considered for electron dissipation associated with passing electrons,[2–4] including the collisionless parallel electron Landau resonance and collisionless dissipation, and for dissipative trapped electrons.[4] For passing electrons it was found that the part of the flux proportional to the electron density gradient produces outward flux while the part proportional to the electron temperature gradient gives inward flux. Thus the direction of the net flux depends on the ratio of gradients η_e as well as the real part of mode frequency. For dissipative trapped electrons the flux is outward for all centrally peaked temperature and density profiles.[4]

The dissipation mechanisms associated with passing electrons apply for relatively low temperatures corresponding to $v_* = v_e/\epsilon\omega_b > 1$, where v_e is the collision frequency of thermal electrons, ϵ is the aspect ratio, and ω_b is the electron bounce frequency. Since electron temperatures in the regions where the particle source is negligible tend to be larger in present-day and future tokamaks, it is important to examine the ion mixing mode for temperature regimes with $v_* < 1$. For these temperatures, trapped electrons dominate the dissipative response and their dynamics must be considered in order to determine the particle transport.

In this Brief Communication, quasilinear particle fluxes for the ion mixing mode in trapped electron regimes are documented.[5] The calculation of fluxes from nonlinear theory will be considered elsewhere. Preliminary examination of resonance broadening effects indicate that the fluxes in both trapped electron regimes are not significantly altered by nonlinear decorrelation, unless the rate greatly exceeds ω_{*e}. The fluctuation level, $|e\phi/T_e|^2$, which enters the quasilinear flux formulas, is taken to be a known quantity. Its value can be obtained either from experimental observation, as has

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been done for comparison with measured fluxes,[6] or from theoretical predictions for ion temperature gradient driven turbulence.[4,7,8] Attention is focused on the sign of the thermal gradient driven flux term, which in the quasilinear description does not depend on fluctuation level. For modes rotating in the ion diamagnetic direction (or the electron direction with rotation below the electron diamagnetic frequency) the density gradient driven flux is always outward. Hence the thermal gradient driven flux offers the only possibility for inward particle flow. Both trapped electron regimes are considered, the collisional regime, $\nu_{\rm eff} \equiv \nu_e/\epsilon > \omega_*$, which corresponds to the dissipative trapped electron branch of electron modes, and the collisionless regime, $\nu_{\rm eff} < \omega_*$. Transport in the dissipative regime ($\nu_{\rm eff} > \omega_*$) has been shown to be outward,[4] as would be expected on the basis of the instability of dissipative trapped electron modes. For the collisionless trapped electron limit, drift wave fluxes have also been calculated.[9] However, there has not been, to the knowledge of the author, any discussion in the literature of mixing mode transport in the collisionless trapped electron limit, where, as will be shown, an inward pinch is possible.

The quasilinear particle flux is given by the $E \times B$ convection of the fluctuating density,

$$\Gamma_e = \langle V_E \tilde{n}_e \rangle = -\Sigma_k n_0 \left(\frac{cT_e}{eB}\right) k_y \left(\frac{e\tilde{\phi}_{-k}}{T_e}\right) \mathrm{Im} \int d^3\mathrm{v}\, h_k(\mathbf{v}), \quad (1)$$

where $h_k(\mathbf{v})$ is the nonadiabatic part of the linearized electron phase space density, $f_k(\mathbf{v}) = (e\tilde{\phi}_k/T_e)F_{\max} + h_k(\mathbf{v})$. The nonadiabatic electron density satisfies a gyrokinetic equation, which for the trapped electron regime ($\nu_* < 1$) is given[9] by

$$\left(\omega - \omega_D \frac{\mathrm{v}^2}{\mathrm{v}_e^2} + \frac{i\nu_e}{\epsilon}\frac{\mathrm{v}_e^3}{\mathrm{v}^3}\right) h_k(\mathbf{v}) = \left\{\omega - \omega_{*e}\left[1 + \eta_e\left(\frac{\mathrm{v}^2}{\mathrm{v}_e^2} - \frac{3}{2}\right)\right]\right\} \frac{e\tilde{\phi}_k}{T_e} F_{\max}, \quad (2)$$

where $\omega_{*e} = (cT_e/eB)k_y/L_n$, $\eta_e = L_n/L_T = d \ln T_e/d \ln n_e$, $\omega_D \cong \omega_{*e}(L_n/R)$, and the other symbols have their usual definitions. Toroidal effects are not considered in this simple treatment but are left to future work.

First the dissipative trapped electron regime is considered. Because the dissipative trapped electron mode is unstable, it follows that the ion mixing mode particle flux in this regime is outward. Both the density gradient and the temperature gradient driven fluxes have this property, provided the gradients are in the same direction ($\eta_e > 0$). Assuming that the mode rotates in the ion direction (no drift resonance), for $\nu_{\rm eff} > \omega_{*e}$, the imaginary part of the propagator $\mathrm{Im}(\omega - \omega_D + i\nu_e \mathrm{v}_e^3/\epsilon \mathrm{v}^3)^{-1}$ is approximately $-\epsilon \mathrm{v}^3/\nu_e \mathrm{v}_e^3$ for the bulk electrons. Using this propagator for electrons of all energies, the flux is given approximately by

$$\Gamma_e = \frac{4n_e\epsilon^{3/2}}{\pi^{1/2}}\left(\frac{cT_e}{eB}\right)\sum_{k_y} k_y \left(\frac{\omega_{*e}}{\nu_e}\right) \times \left(1 - \frac{\omega}{\omega_{*e}} + \frac{3}{2}\eta_e\right) \left|\frac{e\tilde{\phi}_k}{T_e}\right|^2. \quad (3)$$

This expression differs from the exact quasilinear flux (unapproximated integration over velocity) by terms of order $\nu_*^{1/2}\omega_{*e}/k_\parallel \mathrm{v}_e$ and $\nu_*^{1/2}\omega_{*e}/\nu_{\rm eff}$, which are required to properly take account of low energy (passing electrons), and terms of order $(\nu_{\rm eff}/\omega_{*e})\exp[-(\nu_{\rm eff}/\omega_{*e})^{1/2}]$, which account for high-energy collisionless trapped electrons. As found previously,[4] Eq. (3) indicates that the density gradient driven flux is positive for frequencies $\omega < \omega_{*e}$ (which includes modes propagating in the ion direction) and that the temperature gradient driven flux is always positive for $\eta_e > 0$.

When the electron temperature is such that the bulk of the electron distribution scatters less than once in a wave period ($\nu_{\rm eff} < \omega_{*e}$), the flux is determined by electrons with $\mathrm{v}^2 > \mathrm{v}_e^2(\nu_{\rm eff}/\omega_{*e})^{2/3}$. For these electrons the propagator is given approximately by

$$\mathrm{Im}\left(\omega - \omega_D \frac{\mathrm{v}^2}{\mathrm{v}_e^2} + \frac{i\nu_e}{\epsilon}\frac{\mathrm{v}_e^3}{\mathrm{v}^3}\right)^{-1} \cong -\frac{\nu_e}{\epsilon}\frac{\mathrm{v}_e^3}{\mathrm{v}^3}\left(\omega - \omega_D\frac{\mathrm{v}^2}{\mathrm{v}_e^2}\right)^{-2},$$

and the nonadiabatic distribution is expressed as

$$\mathrm{Im}\, h_k(\mathbf{v}) = -\frac{\nu_e}{\epsilon}\frac{\mathrm{v}_e^3}{\mathrm{v}^3}\left(\omega - \omega_D\frac{\mathrm{v}^2}{\mathrm{v}_e^2}\right)^{-2}\left\{\omega - \omega_{*e} \times \left[1 + \eta_e\left(\frac{\mathrm{v}^2}{\mathrm{v}_e^2} - \frac{3}{2}\right)\right]\right\}\frac{e\tilde{\phi}_k}{T_e}F_{\max}. \quad (4)$$

Substituting this expression into Eq. (1) and integrating from $\mathrm{v} = \mathrm{v}_e(\nu_{\rm eff}/\omega_{*e})^{1/3}$ to infinity gives the particle flux, correct to order $(\omega_{*e}/\nu_{\rm eff})\exp[-(\omega_{*e}/\nu_{\rm eff})^{2/3}]$. While there is no simple closed-form expression for this integral when $(\nu_{\rm eff}/\omega_{*e})^{2/3}$ is of order unity, it can be expressed in terms of special functions. When $(\nu_{\rm eff}/\omega_{*e})^{2/3}$ becomes small, this flux can be approximated by a power series expansion with leading-order logarithmic terms.[9] For arbitrary $\nu_{\rm eff}/\omega_{*e}$, the flux is given by

$$\Gamma_e = \frac{2n_e\epsilon^{1/2}}{\pi^{1/2}}\left(\frac{cT_e}{eB}\right)\sum_{k_y} k_y\left(\frac{\nu_{\rm eff}}{\omega_{*e}}\right)\left(\frac{\omega_{*e}^2}{\omega^2}\right)\left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \times \left\{\left(1 + \frac{|\omega|}{\omega_{*e}}\right)E_1(t_3) - \eta_e \times \left(\frac{3}{2}E_1(t_3) - \exp(-t_3)\right)\right\}, \quad (5)$$

where $E_1(t_3) = \int_{t_3}^\infty dt\, t^{-1}\exp(-t)$ is the exponential integral, $t_3 = (\nu_{\rm eff}/\omega_{*e})^{2/3}$, and rotation in the ion direction, $\omega = -|\omega|$, has been assumed. Because of this negative rotation, there is no curvature drift resonance, and contributions from the curvature drift, of order $\omega_D/\omega_{*e} \sim O(L_n/R)$, have been neglected.

From Eq. (5), it is clear that the density gradient driven particle flux is outward (the lowest-order curvature drift correction decreases this flux, but it remains positive as long as $L_n/R < 0.6$). The temperature gradient driven flux, on the other hand, can be either inward or outward. For sufficiently small t_3, $E_1(t_3) \sim -\ln t_3$, while $\exp(-t_3) \sim 1$ and the flux is clearly inward. For $t_3 = 1$, however, $\exp(-t_3) > E_1(t_3)$ and the flux is outward. The value of t_3

at which the temperature gradient driven flux becomes inward is $t_3 = 0.85$. Accordingly, this part of the flux is inward for $v_{\rm eff}/\omega_{*e} < 0.78$. On TEXT,[10] this threshold corresponds to a temperature in the neighborhood of 0.6 keV for discharges with a significant trapped particle population ($L_n = 15$ cm, $n_e = 1\times 10^{13}$ cm^{-3}, $k_\theta \rho_s = 0.1$; for higher density discharges the wavenumber increases, so the threshold does not increase significantly). Because of the factor $v_{\rm eff}/\omega_{*e}$ outside the curly brackets in Eq. (5), it is clear that the magnitude of the pinch decreases with increasing temperature above the threshold (however, so does the density gradient driven outward flux). Thus the temperature gradient driven particle pinch is largest at temperatures just above the threshold. Numerical evaluation of the flux in trapped particle regimes has also been carried out and is in agreement with the analytic results given here.[11]

It is useful to write down an approximate expression for the particle flux that is valid over the entire trapped particle regime, $v_* < 1$. This is accomplished by performing the energy integral for the entire trapped particle regime, but using the approximate propagators in the collisionless and dissipative subintervals. This corresponds to retaining upper or lower limits at the transition from one regime to the other. The flux is

$$\Gamma_e = \frac{2n_e\epsilon^{1/2}}{\pi^{1/2}}\left(\frac{cT_e}{eB}\right)\sum_{k_y} k_y\left(\frac{v_{\rm eff}}{\omega_{*e}}\right)\left(\frac{\omega_{*e}^2}{\omega^2}\right)\left|\frac{e\tilde{\phi}_k}{T_e}\right|^2 \times\Bigg(\frac{1}{t_3^3}\Bigg\{\left(1+\frac{|\omega|}{\omega_{*e}}\right)[2-(t_3^2+2t_3+2)\exp(-t_3)] + 3\eta_e\left[1-\left(\frac{1}{3}t_3^3+\frac{1}{2}t_3^2+t_3+1\right)\exp(-t_3)\right]\Bigg\} + \left(1+\frac{|\omega|}{\omega_{*e}}\right)E_1(t_3) - \eta_e \times\left(\frac{3}{2}E_1(t_3)-\exp(-t_3)\right)\Bigg). \quad (6)$$

It is apparent from this expression that bulk electrons must be collisionless in order to have an inward temperature gradient driven flux.

We now discuss these results. Consideration of trapped particle electron responses for the ion mixing mode, appropriate for higher temperature conditions than those of previously considered responses,[2–4] allows for the possibility of an inward pinch. This fact may be of some help in accounting for the pinch needed to model steady-state discharges in regions of outward diffusion where the particle source is negligible. However, this pinch only occurs for the higher temperature collisionless extreme ($v_{\rm eff}/\omega_{*e} < 1$) of the trapped electron regime. In TEXT, this threshold is in the neighborhood of 600 eV. This leaves significant portions of the plasma volume in the dissipative trapped electron regime where this model predicts a flux that is positive definite (all portions of the flux are outward). It should also be cautioned that while such temperatures are achieved on TEXT, the radial location is sufficiently close to the $q = 1$ surface that it is not clear what relevance this result has to TEXT, even in the higher temperature part of the plasma. Moreover, because $v_{\rm eff}/\omega_{*e}$ varies as L_n/r, the collisionless regime does not necessarily extend to the $q = 1$ surface in all discharges (although for low-density discharges there is definitely a collisionless trapped electron regime with temperatures above the threshold for the inward pinch). Thus, on the basis of these considerations, it must be concluded that a viable theoretical model for anomalous inward particle transport has yet to be established.

It is also important to check to what extent the basic ion growth is counteracted by the stabilizing electron contribution, which accompanies a pinch in the ion mixing mode. From Eq. (5), it is seen that $v_{\rm eff}/\omega_{*e}$ is the determining factor in the magnitude of the phase shift between n and ϕ. This is to be compared with $\omega_*/k_\parallel v_i$ in order to estimate the relative importance of the ion and electron contributions.[3] For TEXT parameters, $\omega_*/k_\parallel v_i > v_{\rm eff}/\omega_{*e}$ at temperatures near the pinch threshold; however, the difference in magnitude is not large. This raises the possibility that in the collisionless trapped electron collisionality regime, the instability is significantly weakened by electron effects. However, it should also be pointed out that as far as transport and the requirements for stationary density profiles are concerned, it is sufficient for the inward flux component to balance the outward component. In this case, the electron contribution to the growth rate is marginal and the ion instability mechanism is unaffected by the nonadiabatic electrons.

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