Nonlinear 3D MHD verification study: SpeCyl and PIXIE3D codes for RFP and Tokamak plasmas

Daniele Bonfiglio\textsuperscript{1} \\
with S. Cappello\textsuperscript{1} and L. Chacon\textsuperscript{2} \\
\textsuperscript{1}Consorzio RFX, Italy \textsuperscript{2}ORNL, Tennessee

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Nonlinear verification benchmark of the two codes

- Mandatory step after numerical tools upgrade at RFX
  - From the visco-resistive MHD code SpeCyl
    - [S. Cappello & D. Biskamp, NF 1996]
  - …to the extended MHD code PIXIE3D
    - [L. Chacon, PoP 2008]

- Motivated by the need for RFP extended MHD modeling
In the past, the nonlinear 3D visco-resistive MHD model:
- Provided the theoretical discovery of the RFP helical self-organization
- Was instrumental to its experimental study

Extended MHD modeling of the RFP is needed for:
- Physical understanding
  - Need to address the interplay between MHD & transport:
    - Better description of momentum transport (than $\nu_\eta$-MHD)
    - Particle & energy transport
  - Besides of two fluid, toroidal effects and so on
- Predictive (quantitative) capability
  - As a long term goal
  - Strong emphasis on Verification & Validation of numerical tools
SpeCyl:
- Nonlinear visco-resistive 3D MHD in a cylinder

PIXIE3D:
- Extended physics:
  - Finite beta
  - Temperature equation
  - Two fluid effects
- General curvilinear geometry:
  - Toroidal effects
### SpeCyl vs PIXIE3D: discretization schemes

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<th>Spatial discretization</th>
<th>Time discretization</th>
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<td>Finite differences ($r$)</td>
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<td>Spectral ($\theta, z$)</td>
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<td>PIXIE3D</td>
<td>3D (curvilinear)</td>
<td>Finite volume</td>
<td>Fully implicit</td>
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**Fully implicit scheme:**

- $\Delta t \cong \tau_A$ allowed without accuracy degradation
Excellent scalability properties, close to optimal scaling:

- Increasing the problem resolution and the number of processors by the same factor
- Computation time remains nearly constant up to 4092 processors
Successful nonlinear verification benchmark performed:

- In the common limit of visco-resistive MHD in a cylinder

Helical (2D) symmetry:
- Tokamak-like & RFP configurations

Full 3D case:
- RFP configuration: MH – QSH – SH regimes

Details in:
- [D. Bonfiglio, L. Chacon & S. Cappello, PoP 2010]

Some examples in next slides:
- Look at the differences between black and red curves
Temporal evolution of magnetic energy by SpeCyl (—) and PIXIE3D (—).

- m=1, n=-8 helical symmetry: SHAx

\[(a) \quad 2\text{D RFP, } S=3 \times 10^4, \ P=300, \ m=1, \ n=-8\]
Verification benchmark: (2D) helical Tokamak

Temporal evolution of magnetic energy by SpeCyl (—) and PIXIE3D (―).

- \( m=1, n=-1 \) symmetry:
  - Stationary “snake” equilibrium
    - \( S=3 \times 10^4, P=30 \)
  - Sawtooth oscillations
    - \( S=3 \times 10^5, P=30 \)
Temporal evolution of magnetic energy by SpeCyl (—) and PIXIE3D (—).

- $m=1$, $n=-8$ helical symmetry: SHAx

(b) 3D RFP, $S=3 \times 10^4$, $P=100$, $128 \times 32 \times 256$
PIXIE3D: good accuracy with large $\Delta t$ (helical RFP)

Temporal evolution by PIXIE3D with $\Delta t=0.01$ (—) and large $\Delta t$ (—).

$\Delta t = 1 \, \tau_A$

$\Delta t = 4 \, \tau_A$

$\Delta t > \gamma^{-1/2}$

$\Delta t = 2 \, \tau_A$

$\Delta t = 8 \, \tau_A$
Conclusion & perspectives

Successful benchmark concluded between 3D MHD codes

Now ready to extend RFP self-organization studies with extended physics & geometry effects:

- Self-consistent temperature
- Toroidal geometry effects
- Two fluid effects

- See poster TP9.00035 by L. Chacon on Thursday morning