AC loop voltages and MHD stability in RFP plasmas

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Main ideas

- MHD phenomena are central to standard RFP behavior, where magnetic relaxation helps determine equilibrium and confinement through self-organization processes, *e.g.* sawtooth cycles

- Oscillating-field current drive (OFCD), the chief candidate for efficient, steady-state RFP sustainment, also provides a useful experimental platform for studying such issues in MST

- Applying AC poloidal and toroidal loop voltages as in OFCD allows measurement of the MHD response to controlled perturbations

- AC loop voltages may also allow some steady-state control of relaxation activity, as opposed to transient control, which has already been demonstrated with pulsed voltages
Outline

• Introduction

• OFCD concept and main results

• Relaxation cycles and sawtooth entrainment

• Relaxation cycles in OFCD

• Linear stability analysis

• Conclusion
Introduction
Madison Symmetric Torus (MST) reversed-field pinch experiment

- $R_0/a = (1.5 \text{ m})/(0.52 \text{ m})$
- Aluminum shell 5 cm thick
- $I_p \lesssim 600 \text{ kA}$
- $T_{e,i} \lesssim 2 \text{ kV}$
- $n_e \sim 10^{19}/\text{m}^3$
RFP magnetic fields mostly due to plasma currents

- Present RFPs sustained by transient toroidal induction

- Bootstrap current is small, motivating OFCD for steady state

- Basic equilibrium parameters:
  - Reversal parameter
    \[ F = \frac{B_\phi(a)}{\langle B_\phi \rangle} \]
  - Pinch parameter
    \[ \Theta = \frac{B_\theta(a)}{\langle B_\phi \rangle} \]
Magnetic tearing modes resonant in RFP

- Resonant surfaces for \((m, n)\) modes where safety factor \(q \equiv (rB_\phi)/(R_0 B_\theta) = m/n\)

- \(m = 1\) modes
  - Innermost resonant in core
  - Innermost often unstable in standard MST plasmas

- \(m = 0\) modes
  - Resonant in edge at reversal surface
  - Stable in standard MST plasmas

![Safety Factor vs. Radius](image)
Normalized parallel current profile easily modeled with two-parameter cylindrical model

- \( \lambda \equiv \left( \nabla \times \mathbf{B} \right) \cdot \mathbf{B} / B^2 = \mu_0 J_{||} / B \)

  - A negative \( d\lambda / dr \) is a source of free energy for tearing modes

- ‘Alpha model’

  - \( \lambda(r) a = 2\Theta_0 [1 - (r/a)^\alpha] \)

  - \( \Theta_0 \) and \( \alpha \) determined by measured \( F \) and \( \Theta \) values

  - Output \( B \) fields scaled to match edge measurements

  - Core \( B \) values well match internal measurements
OFCD concept and main results
DC Sustainment by Applying AC Loop Voltages

- Poloidal $\hat{V}_\theta \sin(\omega t)$ and toroidal $\hat{V}_\phi \sin(\omega t - \delta)$

- Magnetic helicity
  $K \equiv \int \mathbf{A} \cdot \mathbf{B} \, dv \sim 2\psi \Phi$
  ($\sim I_\phi^2$ in RFP)

- Helicity balance
  $K' = 2V_\phi \Phi - 2 \int \eta \mathbf{J} \cdot \mathbf{B} \, dv$
  (injection minus decay)

- Time-average OFCD helicity injection rate
  $\langle K'_{\text{inj}} \rangle = \left( \hat{V}_\theta \hat{V}_\phi / \omega \right) \sin \delta$
  (maximum at $\delta = \pi/2$)
Current driven by fluctuation-induced EMF

- EMF generated at edge by OFCD modulations:
  \[ \mathcal{E}_\parallel = \langle \hat{\mathbf{V}} \times \hat{\mathbf{B}} \rangle_\parallel \]

- Edge current destabilizes magnetic fluctuations \( \tilde{\mathbf{B}} \)

- Fluctuations generate EMF throughout plasma:
  \[ \mathcal{E}_\parallel = \langle \tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \rangle_\parallel \]

- See Ebrahimi et al., PoP (2003) for more information
10% plasma current added in MST

- Drive or anti-drive chosen by phase $\delta$

- Optimum plasma current at $\delta \approx \pi/8$, not that of maximum helicity-injection rate, $\delta = \pi/2$

- See McCollam et al., PRL (2006) for more information
MHD activity in OFCD depends strongly on phase

**Graph:**
- **Axes:**
  - Horizontal: Time (t [ms])
  - Vertical: Various measurements
  - Toroidal Loop Voltage
  - Poloidal Loop Voltage
  - Magnetic Fluctuations

**Legend:**
- OFCD OFF
- OFCD ON ($\delta = \pi/8$)
- OFCD ON ($\delta = \pi/2$)

**Measurements:**
- $B_{m=0}$ [G]
- $V_\varphi$ [V]
- $V_\theta$ [V]
- $\alpha$

**Notes:**
- Different phases of operation are shown with distinct waveforms.
OFCD in MST is optimum at $\delta \approx \pi/8$

- DEBS MHD simulations agree with experiments
- Note resistivity the same for all phases in DEBS runs, unlike experiment
- $m=0$ magnetic fluctuations minimized at the optimum phase
- See McCollam et al., PoP (2010) for more information
$m = 0$ may be unstable during OFCD at $\delta = \pi/2$

- Peaking in $m = 0$ amplitude
  - Flat $\lambda$ profile ($\alpha$ large)
  - Co-driving edge $E_\parallel$ ($\sim E \cdot B$)
  - Outward $E \times B$ motion

- Note $m = 0$ deemed stable in standard MST plasmas
Relaxation cycles and sawtooth entrainment
Standard RFP sawtooth relaxation is a limit-cycle phenomenon

- Ohmic drives $\lambda$ more peaked ($\alpha$ decreases, $\Theta_0$ increases)

- $m = 1$ modes become unstable

- $m = 0$ stable but nonlinearly driven by $m = 1$ modes

- Crash EMF generates core toroidal flux $\Phi$, flattens $\lambda$ profile
Sawtooth crash $m = 0$ driven nonlinearly by $m = 1$ modes

- Top and bottom panels show fluctuation amplitudes for $m = 0$ and $m = 1$, respectively.

- Middle two panels show coupling terms expected to drive the corresponding mode amplitudes via three-wave coupling.

- Note the $m = 0$ amplitudes at top are changing quickly when the corresponding coupling terms are large.
Standard RFP executes clockwise loops in $(\alpha, \Theta_0)$ space during sawtooth cycles
Antoni & Ortolani, NF (1986) calculated \( m = 1 \) stability for zero thermal pressure \( (\beta = 0) \)

- Their Figure 7 for \( m = 1 \) has MST data (assuming \( \beta = 7\% \)) overlaid

- \( m = 1 \) becomes resistively and then ideally unstable before sawtooth crash

- Average position is near marginal resistive stability

- See Antoni & Ortolani, PF (1987) for more information
Sawtooth cycle is also a clockwise trajectory in $(\Theta, F)$ space

- Steady toroidal loop voltage drives $\Theta$ more positive until sawtooth crash

- Note Antoni & Ortolani stability boundaries mapped into $(\Theta, F)$
Oscillating poloidal loop voltage alone entrains sawteeth

- Sawteeth recur at the same phase of the oscillation
- Zigzag ($\Theta, F$) trajectory
Higher amplitude completely changes relaxation cycle

- Now the \((\Theta, F)\) trajectory is counterclockwise.
- Sawtooth shape has become nearly sinusoidal.
- Large \(m = 0\) during deep \(F\) excursions.
Oscillating toroidal loop voltage also entrains sawteeth

- Oscillation-driven motion is in $\Theta$ direction and doubles back on itself in this example
At higher amplitude, sawtooth cycles are similar to standard RFP but entrained to be more frequent.
Relaxation cycles in OFCD
OFCD allows more detailed control of relaxation cycle

- Poloidal loop voltage oscillation moves the RFP equilibrium in \((\Theta, F)\) space obliquely, roughly parallel to \(m = 1\) stability boundaries.

- Toroidal loop voltage oscillation moves the RFP mostly along curves of constant \(F\).

- Applied together, the two loop voltages move the RFP equilibrium according to their relative phase and amplitudes.
OFCD at $\delta = \pi/2$, where $\langle K'_{inj} \rangle$ is maximum

- $(\Theta, F)$ trajectory is clockwise
- Large $m = 0$ during deep $F$ excursions
OFCD at $\delta = \pi/8$, where $\Delta I_p$ is maximum

- $(\Theta, F)$ trajectory subtends a minimum area
- $m = 0$ amplitudes are minimum
OFCD at $\delta = -\pi/2$, where $\langle K'_{\text{inj}} \rangle$ is minimum

- $(\Theta, F)$ trajectory is counterclockwise
- Large $m = 0$ during deep $F$ excursions
- Very large sawtooth $m = 0$
Linear stability analysis
AC loop voltage experiments motivate stability analysis

- As seen above, large $m = 0$ modes often appear with AC loop voltages when the RFP is well within the predicted $m = 1$ stability region, suggesting $m = 0$ instability.

- Resistive and ideal MHD stability is calculated for $m = 1$ and $m = 0$ modes using a Newcomb's analysis of the mode eigenfunctions that includes the effect of an edge vacuum layer.

- MST vacuum layer due to limiter is $d = 1$ cm thick compared to the $a = 52$ cm conducting wall radius.

- Initial results are consistent with previous results for $m = 1$, but agreement with $m = 0$ experimental results is less straightforward.

- Results are sensitive to the choice of equilibrium profile model.
Analysis method

- Based on Newcomb, Ann. Phys. (1960), this follows Robinson, NF (1978) as implemented with the RESTER code by Sovinec.

- Integrates $\psi'' + A\psi = 0$ across the radial domain, where $\psi$ is the mode eigenfunction and the equilibrium-dependent $A$ is singular at the resonant surface(s) and does not include thermal pressure.

- Known solutions to a comparison equation with matching singular behavior are used to proceed through the singularities of $A$.

- Both resistive and ideal stability are then determined from the properties of the integrated solutions.
Modifications to RESTER stability code

- We include an edge vacuum layer in the equilibrium calculation of the quantity $A$, which has a marked effect on stability results.

- As suggested by Sovinec, we also include the proper Bessel eigenfunctions in the vacuum region instead of merely a plasma carrying zero equilibrium current.
  - This in itself does not change the results noticeably except in cases where a resonant surface resides in a thick vacuum layer.
\(m = 1\) stability results consistent with Antoni & Ortolani, who used no vacuum layer
For the same \((\Theta, F)\) value, resistive \(m = 1\) slightly stabilized for increasing vacuum width
The $\lambda$ gradient is slightly shallower at a dominant $m = 1$ resonant surface for thicker vacuum layers.
$m = 0$ resistive stability quickly decreases with increasing vacuum width
The $\lambda$ gradient is steeper at the $m = 0$ resonant surface for thicker vacuum layers.

- Manifests first at shallower $F$, as then the resonant surface is nearer to the steepest $\lambda$ gradient.
$\Delta'$ values are much smaller for $m = 0$ than for $m = 1$ modes

- For $\gamma = 0.55 (\Delta')^{4/5} \left( \frac{\eta}{4\pi} \right)^{3/5} \left( \frac{dF}{dr} \right)^{2/5} /(4\pi \rho)^{1/5}$ (see Robinson, 1978), this implies relatively small growth rates for $m = 0$
Other simple $\lambda$ models are predicted $m = 0$ resistively unstable for all $F < 0$ even without a vacuum layer.

- These have steep $\lambda$ gradients in their concave edge regions around the $m = 0$ resonant surface.
Discussion of stability results

- The $m = 1$ results are qualitatively consistent with experiment.

- The $m = 0$ results are not clearly consistent.
  - Experimentally, $m = 0$ seems unstable only at large reversal with large applied electric fields.
  - The present analysis for the MST case predicts widespread $m = 0$ instability, albeit with small growth rates, with an island of stability just about where it looks unstable in the experiment.
  - Simple cylindrical models are likely not accurately capturing edge profiles.
  - Thermal pressure may be important but is not included in the present analysis.
Conclusion
Summary

- In addition to OFCD experiments on efficient current drive, AC loop voltages are used to investigate MHD stability properties on MST.

- Applied AC fields can dramatically alter the character of relaxation cycles in the RFP.

- OFCD may be useful in controlling MHD stability, which is also discussed by Ebrahimi & Prager, PoP (2004).

- In certain experiments, $m = 0$ appears to be destabilized by the AC fields, but so far this is not clearly reproduced in linear stability analysis.
Future Work

• Experiments
  – Current profiles and $m = 0$ stability with probe measurements
  – Advanced waveform control with programmable power
  – Nonlinear correlations from surface magnetic data

• Theory
  – Linear analysis with more realistic profiles and thermal pressure
  – Nonlinear 3D numerical MHD
    • Linear instabilities
    • Sawtooth precursors and triggering
    • Investigate advanced waveforms