Understanding the role of turbulence on current generation in the Madison Dynamo Experiment

Mark Nornberg
Cary Forest, Kian Rahbarinia, Eliot Kaplan, Zane Taylor
University of Wisconsin-Madison
• Designing a liquid metal dynamo experiment
  – Optimizing flow for magnetic amplification and feedback
  – MHD Simulations of the experiment
  – Magnetic tomography to model the flow

• Effects of a turbulent EMF
  – A restatement of Mean Field Theory (side-stepping scale separation)
  – Enhanced resistivity
  – Mean fields due to helical eddies

• Elimination of the turbulent EMF by suppressing large-scale flows
  – Enhanced flux compression by mean flow
  – Stronger wind up of toroidal magnetic field due to Ω-Effect
  – Direct measurements of the turbulent EMF and a comparison to previous modeling

• Plans for further tailoring of the flow with rotatable baffles
We would like to demonstrate the mechanisms of magnetic field generation due to the flow of a homogeneous conducting fluid.

The sodium sphere (1000 l liquid sodium)

- sphere radius: 0.533 m
- 2 independent impellers (75 kW each)
- external fields: \( \leq 0.01 \) T (axial and transverse)
- cavitation suppression: \( \sim 7 \) bar capacity
- Reynolds number: \( \text{Re} = 10^5 \text{Rm} \)
  \( (\text{Rm}_{\text{Tip}} \leq 180) \text{ highly turbulent flow} \)

Diagnostics at the sodium sphere

Goals:
- Supply a sufficiently large, fast, helical flow of conducting fluid to excite the dynamo instability due to the large scale flow
- Study the effects of turbulence on magnetic field generation by measuring the response due to imposed magnetic fields
A laminar model predicts the excitation of transverse dipole magnetic field from the double-vortex flow.

The saturated state is predicted both by linear kinematic calculation and dynamical simulation to be dominantly a dipole field transverse to the flow symmetry axis.
This dynamo can be understood as a process of stretching and twisting field lines to provide amplification and feedback.

A lagrangian simulation following the development of a pair of field lines assuming they are frozen to the flow.
The impellers have been designed to produce the model flow as measured in an identical scale water experiment.
Both measurements in the water experiment and MHD simulations show that the driven flows are highly turbulent.

- Velocity measurements show the development of an inertial range with Kolmogorov scaling.
- Simulations show the evolution of small scale vorticity.
Simulations of the experiment show that while the mean flow remains an efficient dynamo, turbulence inhibits growth.

Mean-field theory:

\[
\frac{\partial \langle B \rangle}{\partial t} = Rm \nabla \times \left( \langle V \rangle \times \langle B \rangle + \langle \tilde{v} \times \tilde{b} \rangle \right) + \nabla^2 \langle B \rangle
\]

turbulent electromotive force

\[
\varepsilon_{turb} = \langle \tilde{v} \times \tilde{b} \rangle = \alpha \langle B \rangle + \beta \nabla \times \langle B \rangle
\]

\(\alpha\)-effect: Turbulent helicity

\(\beta\)-effect: Reduction of conductivity

\[
\sigma_{turb} = \frac{\sigma}{1 + \frac{\mu_0 \sigma}{3} \beta}, \quad \beta = \frac{\tilde{v} l_v}{3}
\]

Self-Excitation:

\[
Rm_{turb} = \mu_0 \sigma_{turb} V L \geq Rm_{crit}
\]

(in experiment: \(\sigma_{turb} = 0.5 \sigma\))
• Gain: ratio of magnitude of induced field to applied field
• Increasing speed doesn’t improve dynamo gain as much as predicted by laminar model
• Alignment of induced transverse field with applied field is improved moving from near perpendicular to full alignment
In the absence of a dynamo we can

• Use the induced magnetic field to probe the flow in order to optimize it for dynamo growth
• Measure the magnetic spectrum to understand how magnetic flux is transported in the turbulent flow
Given a model of the velocity field we can predict the magnetic fields induced by the flow when an external field is applied.
We perform experiments in which various external magnetic field geometries are applied and we measure the induced field.
A mean-field EMF can be extracted from the experimental measurements by separating out the currents due to the mean flow:

\[ \langle J \rangle = \sigma \left( \langle E \rangle + \langle V \rangle \times \langle B \rangle + \langle \tilde{v} \times \tilde{b} \rangle \right) \]

What form is Ohm’s law? $\sigma$ is the conductivity.

$B$ due to $\langle V \rangle \times \langle B \rangle$

$B$ due to $\langle \tilde{v} \times \tilde{b} \rangle$

$\sigma_T \approx 0.2 \sigma_{\text{sodium}}$

What do the turbulent magnetic fluctuations look like?

- Large scale
- Small scale
- Resistive dissipation scale

Power Spectrum [Gauss²] vs. $k$ [m⁻¹]

- $Rm=100$
- $Rm=60$
- $Rm=30$

$k^{-5/3}$

$k^{-11/3}$
Let’s use the measured spectrum to motivate the assumptions for modeling the turbulent EMF for our experiment.

The large scale shear necessary for a dynamo also is the driving scale for the turbulent cascade.

- Mean-field theory assumes a scale separation between mean field and fluctuations *which we don’t have*.
- Rather than making these assumptions we assume that the mean flow is the axisymmetric $t_2s_2$ flow.
- Non-axisymmetric flows are fluctuating with correlation times shorter than the resistive diffusion time.
We take advantage of the spherical geometry of the experiment by using a spherical harmonics decomposition of the flow and field.

\[ \mathbf{B}(r, \theta, \phi) = \sum \nabla \times \nabla \times S_{l_i}^{m_i}(r)Y_{l_i}^{m_i}(\theta, \phi) \hat{\mathbf{r}} + \nabla \times T_{l_i}^{m_i}(r)Y_{l_i}^{m_i}(\theta, \phi) \hat{\mathbf{r}} \]

\[ \mathbf{V}(r, \theta, \phi) = \sum \nabla \times \nabla \times s_{l_i}^{m_i}(r)Y_{l_i}^{m_i}(\theta, \phi) \hat{\mathbf{r}} + \nabla \times t_{l_i}^{m_i}(r)Y_{l_i}^{m_i}(\theta, \phi) \hat{\mathbf{r}} \]

**Poloidal harmonics**  **Toroidal harmonics**

\[ \frac{\partial \mathbf{B}}{\partial \tau} = R_m \nabla \times \mathbf{V} \times \mathbf{B} + \nabla^2 \mathbf{B} \]

**Induction Equation:**

\[ \partial_t S_k = \frac{1}{\mu_0 \sigma} \nabla_k^2 S_k + \sum_{i,j} s_i S_j S_k + s_i T_j S_k + t_i S_j S_k + t_i T_j S_k \]

\[ \partial_t T_k = \frac{1}{\mu_0 \sigma} \nabla_k^2 T_k + \sum_{i,j} s_i S_j T_k + s_i T_j T_k + t_i S_j T_k + t_i T_j T_k \]

**Interaction terms quantifying magnetic induction**

magnetic mode \( i \) interacts with flow \( j \) to produce magnetic mode \( k \)

**Boundary Conditions and Geometry Matter!**
Selection rules limit the number of possible interactions

$S, T$ poloidal and toroidal magnetic modes

$s, t$ poloidal and toroidal flow modes
Terms identical to the transport coefficients in Mean Field Theory arise in the Bullard and Gellman formalism of the turbulent EMF.

Correlated helical fluctuations convert toroidal flux into poloidal flux:

\[
\partial_t \langle S_j \rangle = - \frac{L^2_{ijk} p_i p_k}{N_j N_k r^4} \langle \tilde{s}_i \tilde{t}_i \rangle \tau_{\text{cor}} \langle T_k \rangle + \eta \nabla^2_j \langle S_j \rangle
\]

\(\alpha\) Effect

Auto-correlations of the velocity fluctuations act to enhance resistive dissipation:

\[
\partial_t \langle T_j \rangle = - \frac{L^2_{ijk} p_i p_k}{N_j N_k r^4} \langle \tilde{t}_i^2 \rangle \tau_{\text{cor}} \langle T_j \rangle + \eta \nabla^2_j \langle T_j \rangle
\]

\(\beta\) Effect
Experiment: Introduce baffles to eliminate the undesired large scale flows and measure the changes in the induced magnetic field.

- **Equatorial baffle** (turbulence reduction)
  - Equatorial baffle installed last summer:
  - The equatorial baffle suppresses the large-scale flow eddies.

- **Poloidal baffles** (optimize flow-ratio)
  - Driven flow
  - Shear layer instability

**Diagram:**
- Graphical representation showing the effect of equatorial baffle on flow patterns.
A computational fluid dynamics model shows a 10-fold reduction in the turbulent energy and kinematic calculation shows lower $R_{m_{\text{crit}}}$

Reduction of avg. turbulent kinetic energy at 800 rpm

0.78 m²/s²  
0.42 m²/s²  
0.06 m²/s²

Reduction of the critical magnetic Reynolds number with poloidal baffles at 45°
Measurement of the dynamo gain and decay rate of a pulsed applied transverse dipole field show drastic improvement.

\[ \text{Gain} = \frac{B_1 \cos(\delta) + B_0}{B_0} \]

With Equitorial Baffle:
- The predicted gain of the expected dynamo mode is experimentally confirmed.
- Increasing decay times with increasing Rm observed.

With baffle

Without baffle
The flow compresses the axial magnetic field and amplify it by a factor of 20 in the core.
The toroidal windup due to the Omega-effect provides an amplification factor of 2.

- Applied field (axial dipole)
- Applied flow (two-vortex flow)
- Toroidal field generation

Magnetic field wind up from $\Omega$ effect

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Magnetic Field (gauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coils turn on</td>
<td>$B_{TOR}$</td>
</tr>
<tr>
<td>motor ramp start</td>
<td>$B_{POL}$</td>
</tr>
<tr>
<td>motor ramp end</td>
<td>$B_{TOR}/B_{POL}$</td>
</tr>
</tbody>
</table>

- a) Measured fields
- b) Toroidal gain

Gain = 2.0
A comparison of the induced magnetic fluctuations show that the harmonics associated with the turbulent EMF are decimated.

Predicted suppression of the largest-scale turbulent eddies $s_1^0$ and $t_1^1$ experimentally confirmed.

By elimination of $t_1^1$ the $\alpha$-effect induced dipole moment has been drastically reduced.

The elimination of large-scale eddies has reduced the effective electrical resistivity by 57%. This allows the mean flow to more effectively amplify and transport magnetic field.

We can now directly measure the turbulent emf using a combination of electrodes and Hall sensors.

Potential between two electrodes proportional to the flow between them:

\[ \Phi_{ab} = \int_{a}^{b} (V \times B) \cdot dl \]

Three-axis velocity probe

NdFeB permanent Magnet
silver coated copper wire
3-axis Hall sensor

In combination with three-axis measurements of the magnetic field fluctuations via close-by Hall sensors, the localized EMF is obtained.

The first direct measurements of the turbulent emf show it is weaker than induction by the mean flow by an order of magnitude.

\[
\langle J \rangle = \sigma \left( \langle E \rangle + \langle V \rangle \times \langle B \rangle + \langle \tilde{v} \times \tilde{b} \rangle \right)
\]

With Equitorial Baffle:
Turbulent EMF is an order of magnitude smaller than mean-field part.

Previous Result without baffle:
Turbulent fluctuations generate significant part of the measured magnetic field.
Spence et al., PRL 98 164503 (2007)

First-time measurement of the local turbulent EMF in the Madison Dynamo Experiment.

\[
\varepsilon_{turb} = \langle \tilde{v} \times \tilde{b} \rangle
\]
Summary and outlook

- The elimination of large-scale flow modes has reduced the turbulent emf by an order of magnitude in both local and global measurements.
- The new flow is capable of compressing axial flux and amplifying it by a factor of 20.
- The toroidal field windup factor, a measure of the gain from the Omega-effect is about 2.

- Near term plans include introducing rotatable baffles to further constrain the large-scale flows and to directly modify the helical pitch of the mean flow:
  - Optimize the flow for dynamo growth.
- In the absence of a turbulent emf we can proceed with magnetic tomography to map out the flow from measured induced fields for optimization.
- With our new diagnostics we can directly measure the turbulent emf to see how it is modified as we tailor the large scale flow.