Nonlinear dynamics in RFP experiment and computation

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Main ideas

- RFP is a nonlinear dynamical system studied in experiment and computation.

- Single-fluid MHD qualitatively reproduces experimental equilibrium evolution.

- Extended MHD, e.g. two-fluid MHD with ion gyroviscosity, may be needed for better quantitative matching of magnetic fluctuation amplitudes.

- Studies of flow relaxation with extended MHD are underway.

- Applying AC loop voltages as in OFCD allows us to study the plasma’s dynamical response.
Outline

- Standard RFP
  - Magnetic relaxation
  - Flow relaxation

- OFCD (oscillating-field current drive)

- Relaxation cycles with AC loop voltages

- Conclusion
Standard RFP magnetic relaxation
Magnetic tearing modes resonant in RFP

- Core-resonant $m = 1$ modes
  - Innermost often unstable in standard MST plasmas
- Edge-resonant $m = 0$ modes
  - Stable in standard MST plasmas

![Diagram showing Safety Factor vs. Radius](image)
Standard RFP sawtooth relaxation is a limit-cycle phenomenon

- Ohmic drives $\lambda \propto \frac{J_\parallel}{B}$ more peaked: flatness parameter $\alpha$ decreases

- $m = 1$ modes become unstable

- $m = 0$ stable but nonlinearly driven by $m = 1$ at sawtooth crash

- Crash EMF generates core toroidal flux $\Phi$, flattens $\lambda$ profile
Single-fluid MHD in cylindrical geometry with the DEBS code

- \( \partial A/\partial t = S V \times B - \eta J \)

\[ \rho \partial V/\partial t = -S \rho V \cdot \nabla V + S J \times B + \nu \nabla^2 V \]

- This run:
  - \( \beta = 0 \)
  - Lundquist number \( S \equiv \tau_R/\tau_A = 3.8E6 \)
    * Matches the MST experiments to be compared
  - Prandtl number \( Pr \equiv \nu/\eta \sim 100 \)
    * Large \( Pr \) damps sub-grid-scale fluctuations
  - Reusch APS-DPP (2010), \textit{et al.}, PRL (2011)
Single-fluid MHD qualitatively reproduces MST relaxation

**Simulated \( \tilde{b} \)**

**Measured \( \tilde{b} \)**
Single-fluid MHD matches MST equilibrium evolution per relaxation event

- Radial equilibrium profiles also agree well
Single-fluid MHD overpredicts MST magnetic-fluctuation amplitudes by 2x

Bt spectrum before sawtooth

- Simulation
- Experiment

Edge
$m=0$

Core resonant
$m=1$ modes
Extended MHD with the NIMROD code

\[ E = - \nabla \times B + \frac{J \times B}{en} - \frac{\nabla p_e}{en} + \eta J + \frac{m_e}{e^2 n} \partial J / \partial t \]

\[ \rho_i \frac{dV}{dt} = J \times B - \nabla p - \nabla \cdot \Pi_{gv} - \nabla \cdot \nu \rho_i \mathbb{W} \]

- Hall term \( J \times B / (en) \)

- Ion gyroviscous stress \( \Pi_{gv} \)

- These runs:
  - Cylindrical geometry
  - Single fluid or two-fluid with cold or warm ions
  - Lundquist number \( S \equiv \tau_R / \tau_A \leq 8E4 \)
    * Much smaller than MST experiments
Computed magnetic island and flow contours projected onto a helical plane

- Warm-ion case has smaller islands due to ion gyroviscous damping
- King et al., POP (2011)
Two-fluid MHD with ion gyroviscosity has saturated magnetic-fluctuation amplitudes 2x smaller than single-fluid computation

- Tends to agree with similar MST experiments
- King et al., POP (2012)
Standard RFP flow relaxation
In MST, core mode rotation slows and profile tends to flatten at sawtooth crash for all combinations of $I_p$ and $B_t$ polarities.

- MST has background toroidal flow in same direction as $I_p$
- Typical $\mathbf{J} \cdot \mathbf{B} < 0$
With background flow modeled on experimental measurements, flow changes are driven by relaxation in NIMROD at $S = 5E3$

- Flow becomes more peaked during relaxation event
- Does not match MST results
- Investigation continues
- Note single-fluid MHD shows no such flow changes
OFCD (oscillating-field current drive)
OFCD adds 10% plasma current in MST

- DC helicity injection from poloidal and toroidal AC loop voltages
- Injection couples to 3D magnetic reconnection to drive global plasma current
- Drive or anti-drive chosen by phase $\delta$ between the two AC loop voltages
- McCollam et al., PRL (2006)
OFCD in MST is optimum at phase $\delta \approx \pi/8$

- DEBS simulations ($S = 1E5$) agree with experimental $\Delta I_p$

- Conundrum:
  - Resistivity the same for all phases in DEBS runs
  - In experiment, resistivity’s phase dependence a major cause of $\Delta I_p$’s phase dependence

- $B_{m=0}$ decreases and $\tau_E$ increases at the optimum $\Delta I_p$ phase

- McCollam et al., PoP (2010)
Relaxation cycles with AC loop voltages
Standard sawtooth cycle a clockwise trajectory in $(\Theta, F)$ space

- Pinch parameter $\Theta = B_\theta(a) / \langle B_\phi \rangle$
- Reversal parameter $F = B_\phi(a) / \langle B_\phi \rangle$
Oscillating poloidal loop voltage alone entrains sawteeth

- Sawteeth recur at the same phase of the oscillation
- Zigzag \((\Theta, F)\) trajectory
Higher amplitude completely changes relaxation cycle

- Now the \((\Theta, F)\) trajectory is counterclockwise
- Sawtooth shape has become nearly sinusoidal
- Large \(m = 0\) during deep \(F\) excursions likely due to instability
Oscillating toroidal loop voltage results in similar effects.
OFCD at $\delta = \pi/2$, where $\langle K_{\text{inj}}' \rangle$ is maximum

- $(\Theta, F)$ trajectory is clockwise
- Large $m = 0$ during deep $F$ excursions
OFCD at $\delta = \pi/8$, where $\Delta I_p$ is maximum

- $(\Theta, F)$ trajectory subtends a minimum area
- $m = 0$ amplitudes are minimum
Conclusion
Summary

- Single-fluid MHD can qualitatively reproduce the RFP equilibrium’s overall nonlinear dynamics

- Recent extended-MHD comparisons:
  - Better quantitative match with magnetic fluctuation amplitudes
  - Qualitative disagreement in flow relaxation

- Single-fluid simulations of OFCD experiments quantitatively match the added plasma current, but it is not yet clear why

- General applied AC loop voltages can change and control the RFP relaxation cycle, suppress fluctuations, and provide an interesting case for MHD comparisons