Momentum and Current Transport in the MST Reversed Field Pinch


Department of Physics & Astronomy, University of California Los Angeles
Department of Physics, University of Wisconsin, Madison
Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas, University of Wisconsin, Madison
1. On MST reversed field pinch, plasma flow (ions) and current (electrons) can simultaneously arise from fluctuating magnetic fields.

2. Self-generated plasma flow and current can be coupled via density fluctuations associated related to finite pressure fluctuations (finite $\beta$ effects);

3. Density fluctuation induced kinetic stress is observed to account for the observed intrinsic flow (ions);

4. Density fluctuation induced kinetic dynamo (self-generated plasma current - electrons) is found to be significant, motivating further theoretical and experimental investigation.
Fluctuation-Induced Current and Momentum Transport

Hall Dynamo
\[ \langle \delta \vec{j} \times \delta \vec{b} \rangle \]

MHD Dynamo
\[ \\]
\[ \langle \delta \vec{v} \times \delta \vec{b} \rangle \]

Kinetic Dynamo
\[ \delta_{\text{skin}}^2 l_c \left( \frac{\delta b_r}{B} \right)^2 \]
\[ \langle \delta p_{\parallel, e} \delta b_r \rangle \]
\[ \frac{B}{B} \]

\[ \nabla J_{\parallel} (r) \]

Current transport/ Ohm’s law

Momentum transport

MHD Effects
\[ \langle \delta \vec{b} \delta \vec{b} \rangle \]
\[ \langle \delta \vec{v} \delta \vec{v} \rangle \]

Kinetic Effects
\[ l_c \left( \frac{\delta b_r}{B} \right)^2 \]
\[ \langle \delta p_{\parallel} \delta b_r \rangle \]
\[ \frac{B}{B} \]
MST Reversed-Field Pinch (RFP) is toroidal configuration with relatively weak toroidal magnetic field $B_T$ (i.e., $B_T \sim B_p$)

For plasma w/o current profile control

$R_0 = 1.5 \text{ m}, a = 0.51 \text{ m}, I_p \sim 400 \text{ kA}$

$n_e \sim 10^{19} \text{ m}^{-3}, T_e \sim T_i \sim 400 \text{ eV}$
Measurement of Electron and ion current profile

*Imbalance* between applied inductive electric field and measured electric current

*Imbalance* between momentum input (zero) and measured ion flow

Plasmas self-generate both electron current and ion current
Momentum and Current Transport Due to Fluctuations in a torus

Parallel Ohm’s law can be obtained by mean field theory

\[ \eta J_{||} - E_{||} = < \delta \vec{V} \times \delta \vec{B} >_{||} - < \delta \vec{J} \times \delta \vec{B} >_{||} + < \delta p_{e,||} \delta b_r > /enB_0 - \mu_{e,||}^T \nabla \times \nabla \times J_{||} + ... \]

MHD dynamo  Hall dynamo  Pressure effect  Electron viscosity

Parallel momentum equation can be obtained by mean field theory

\[ \rho \frac{\partial}{\partial t} < V_{||} > = < \delta \vec{J} \times \delta \vec{B} >_{||} - \rho < \delta \vec{V} \cdot \nabla \delta \vec{V} >_{||} - < \delta p_{||} \delta b_r > /B + \mu^T \nabla^2 < V_{||} > + ... \]

Flow Change  Maxwell Stress  Reynolds Stress  Pressure effect  Flow dissipation
Finite pressure effect (kinetic stress or dynamo)

Parallel pressure (or momentum flux) projected to the radial direction in fluctuating magnetic field

\[ \Pi = \frac{\left< p_\parallel \vec{B} \cdot \vec{e}_r \right>}{B} \]

\[ p_\parallel = p_{0\parallel} + \delta p_\parallel \]

\[ \vec{B} = \vec{B}_0 + \delta \vec{b} \]

\[ \delta p_\parallel = T_\parallel \delta n + n_0 \delta T_\parallel \]

\[ \Pi = \frac{\left< \delta p_\parallel \delta b_r \right>}{B} = T_\parallel \frac{\left< \delta n \delta b_r \right>}{B} + n \frac{\left< \delta T_\parallel \delta b_r \right>}{B} \]

Density fluctuations \[ \Pi^n \]

S. Prager, PPCF (1995)
Multiple Diagnostics Enable Measurement of Flow, Kinetic Stress and Stochastic magnetic field

Investigate relations between flow and fluctuation-induced forces

\[ \rho \frac{\partial}{\partial t} \langle V_{||} \rangle \]

\[ - \nabla \cdot \left[ \frac{T_{||} \langle \delta n \delta b_r \rangle}{B_0} \right] \vec{e}_r \]

\[ D^{RR} \sim l_c \left[ \frac{\delta b_r}{B} \right]^2 \]

\[ V_{||} \quad \text{Rutherford Scattering, mode velocity, ion Doppler spectroscopy} \]

\[ B_0 \quad \text{Motional Stark effect+Faraday rotation} \]

\[ \delta b_r(r) \quad \text{Laser Faraday rotation} \]

\[ n \quad \delta n \quad \text{Laser (differential) interferometer (} \nabla \delta n \text{)} \]

\[ T_{||,i} \quad \text{Rutherford Scattering (bulk deuterium ions), CHERS (impurity)} \]

\[ T_{||,e} \quad \text{Thomson scattering} \]
\[ \phi \sim \int ndl + \int \delta ndl \]

**Interferometer density fluctuations**

\[ \Psi \sim \int n\vec{B} \cdot d\vec{l} + \int n\delta\vec{b} \cdot d\vec{l} + \int \delta n\vec{B} \cdot d\vec{l} \]

**Faraday rotation magnetic field fluctuations**

11 chords, \( \Delta x = 8 \text{ cm}, \) phase \( \sim 0.01 \) degree, time response \( \sim 1 \mu \text{s} \)

32+8 magnetic coils toroidal-poloidal array \((m,n)\)

Space-Time Evolution of Fluctuations using combined interferometry and Faraday polarimetry

\[ \delta \phi \propto \int \delta n \, dl \]

\[ \delta \psi \propto \int \delta n \vec{B}_0 \cdot d\vec{l} + \int n_o \delta \vec{b} \cdot d\vec{l} \]

Time [msec]

\[ 2\pi R \]

\[ m=1, n=6 \]

(a) Interferometry [10^{19} m^{-2}]

(b) Polarimetry [Deg.]
Typical Core Mode Density and Magnetic Fluctuation Spatial Profile (from fluctuation fitting)

Correlated product of fluctuations is obtained by flux surface average
Kinetic Stresses Spatial Profile

\[ -\nabla \cdot \Pi^n \]

Force density profile (kinetic stress) reverses direction near \( r/a = 0.5 \)

\[ \Pi^n = \left[ \frac{T_{/\parallel} < n \delta b_r >}{B_0} \right] \]

Momentum flux has a maximum near \( r/a = 0.5 \)
Overlapping of magnetic islands lead to stochastic magnetic field
Magnetic Diffusivity Coefficient (R-R theory) is Inferred from Measurements

Magnetic diffusivity coefficient from field line tracing

\[
D_m = \frac{\langle (\Delta r)^2 \rangle}{2\Delta l} \sim 1.0 \times 10^{-4} \text{ m}
\]

(Hudson and Fiksel, 2006)

Rechester & Rosenbluth (1978) derived a quasi-linear coefficient

\[
D_m \sim \pi R_0 \sum_{m,n} q \left( \frac{\delta b_{r}^{m,n}}{B_0} \right)^2 \sim 1.0 - 2.0 \times 10^{-4} \text{ m}
\]

Momentum diffusion rate is \( D^{ST} = D_m c_s \) (Finn, et al. PoP, 1990)

Using MST parameters: we expect flow confinement time \( \tau_m \sim 1-2 \text{ ms} \)
Observation of Flow Damping in Biasing Experiment

Almagri, et al. 
PoP, 1998

External bias induced flow damps in 1-2 ms.

Momentum confinement time is 1-2 ms (~energy confinement time), much less than classical dissipation time (~250 ms), consistent with stochastic magnetic field diffusion time.
Kinetic stress is comparable to stochastic magnetic field diffusion in quasi-steady state

\[ \Gamma^{ST} = -\rho D^{ST} \frac{\partial V_{//}}{\partial r} \sim -2.0 \times 10^{-2} \text{ N} / \text{m}^2 \]

Stochastic field dissipation

\[ \Pi^n = T \frac{\langle \delta n \delta b_r \rangle}{B} \sim 1.8 \times 10^{-2} \text{ N} / \text{m}^2 \]

Kinetic stress

Experimental implication:

\[ \rho \frac{\partial \langle V_{//} \rangle}{\partial t} \sim -\nabla \cdot \Pi^n + \rho D^{ST} \nabla^2 \langle V_{//} \rangle \]

Momentum Source (Kinetic stress)

Momentum Change

Momentum Diffusion (Stochastic magnetic field)
Comparison between Parallel Flow and Kinetic Stress Profile

Kinetic stress has same sign and spatial distribution as parallel flow

Parallel flow reverses direction at mid-radius as well

Flow reverses sign, consistent with total momentum conservation in a conducting shell.

Ding, et al. PRL 2013
Momentum source and stochastic magnetic field diffusion

\[ \Gamma = T \frac{\langle \delta n \delta b_r \rangle}{B} \]

Momentum flux associated with density fluctuations peaks at mid-radius

\[ D \sim c_s D_m \sim c_s \frac{\langle (r - r_0)^2 \rangle}{2L} \]

Stochastic field diffusivity has similar spatial profile.

*from Reusch 2011*
Two important Results

- Measurements show that magnetic fluctuations have **TWO** kinetic effects on plasma flow:
  
  \[ \text{momentum source (or sink) - kinetic stress} \]
  
  \[ \text{-independent of flow} \]
  
  \[ \text{momentum dissipation - stochastic magnetic diffusion} \]
  
  \[ \text{-dependent of flow} \]

\[
\text{Momentum flux } \sim \frac{\langle \delta p_{\parallel} \delta b_r \rangle}{B} - \rho D^{ST} \frac{d \langle V_{\parallel} \rangle}{dr}
\]

\[
[D^{ST} \sim c_s l_c \left( \frac{\delta b_r}{B} \right)^2]
\]
Assuming a fraction of particle \( f_1 \) momentum loss \( f_2 \) is due to magnetic and density fluctuation

(1) Particle balance
\[
\Gamma_r \approx V_{\parallel,e} \frac{< \delta n_e \delta b_r >}{B} \sim f_1 n_e \nu_{\text{ion}} a
\]

(2) Momentum balance
\[
f_2 \frac{< \delta p_{\parallel} \delta b_r >}{B} = \rho D^{ST} \frac{d < V_{\parallel} >}{dr}, \quad [D^{ST} \sim c_s \pi q R \left( \frac{\delta b_r}{B} \right)^2]
\]

(3) Adiabatic
\[
p \rho^{-\gamma} = C \quad \delta p = c_s^2 \delta \rho
\]

\[
\frac{V_{\parallel}}{c_s} \sim f_1 f_2 \nu_{\text{ion}} \frac{\mu_0 e}{2\pi^2} \times \left( \frac{n_e \pi a^2}{B} \right) \times \left[ \frac{\delta b_r}{B} \right]^{-2} \sim \left[ \frac{I_p}{N} \right]^{-1} S^{2\alpha} \quad S = \frac{\tau_R}{\tau_A} \sim \frac{I_p}{\eta \sqrt{n_e}}
\]

**Mach number increases with Lundquist number** \( S \) **for fixed** \( I/N \) \( (0<\alpha<0.3) \)**
Flow scaling on MST

\[ \frac{V_{\parallel}}{c_s} \sim f_1 f_2 V_{ion} \frac{\mu_0 e}{2\pi^2} \times \frac{(n_e \pi a^2)}{B} \times \left[ \frac{\delta b_r}{B} \right]^{-2} \sim \left[ \frac{I_p}{N} \right]^{-1} S^{2\alpha} \]

\[ S = \frac{\tau_R}{\tau_A} \sim \frac{I_p}{\eta \sqrt{n_e}} \sim \frac{I_p}{\sqrt{n_e}} T_e^{3/2} \]

\[ T_e(0) = 5.0 a^{0.83} I_p^{1.18} n_e^{-0.51} \]

\[ \frac{V_{\parallel}}{c_s} \sim \left[ \frac{I_p}{N} \right]^{-0.25} I_p^{0.9} \]

For MST at high I/N
\[ \sim 6 \times 10^{-14} \text{ A/m} \]

\( \alpha = 0.3 \)

(Assumption: \( Z_{\text{eff}} \) does NOT change with current and density)

**Plasma flow has favorable scaling with high Lundquist number in RFP**
Flow observation in Deuterium plasmas

MST electron temperature scaling
(Ion temperature scaling is unclear)

High current and low density (increasing S) shows increased flow
Mass dependence of flow
(sound speed vs ion thermal speed)

Flow in Helium is faster than in Deuterium.

Sound speed ~ ion thermal speed


\[
\frac{V_{He}}{V_D} \sim 1.40 \quad \frac{c_{s,He}}{c_{s,D}} \sim 1
\]

\[
T_{e,He}(0) \sim 350 - 400 \text{eV} , \quad T_{e,D}(0) \sim 350 \text{eV}
\]

\[
T_{i,He}(0) \sim 500 \text{eV} , \quad T_{i,D}(0) \sim 225 \text{eV}
\]
Enhanced density fluctuations in Helium may account for increased flow.

In Helium plasmas density fluctuations are larger than Deuterium plasmas, implying possible stronger kinetic drive force. However, momentum loss due to charge exchange is less in Helium.
Imbalance between electric field (E) and current (J)

\[ \eta J_{||} - E_{||} = \langle \delta \vec{v} \times \delta \vec{B} \rangle_{||} - \langle \delta \vec{J} \times \delta \vec{B} \rangle_{||} + \langle \delta p_{e,\|} \delta b_r \rangle / e n B_0 - \mu_{e,\|}^T \nabla \times \nabla \times J_{||} + \ldots \]

Density fluctuation induced dynamo

\[ T_{e,\|} \frac{\langle \delta n \delta b_r \rangle}{n e B} + \frac{\langle \delta T_{e,\|} \delta b_r \rangle}{e B} \]
Kinetic dynamo is measured to be finite value

Kinetic dynamo is consistent with electric current direction.

Measure kinetic dynamo in RFP plasmas implies that field reversal may arise from multiple dynamo effects.

\[
\frac{T_{e,\parallel}}{neB_0} < \delta n \delta b_r >
\]
Measurements show that magnetic fluctuations have two kinetic effects on plasma flow - driving and damping:

\[ \text{Momentum flux } \sim \frac{<\delta p_\parallel \delta b_r>}{B} - \rho D^{ST} \frac{d <V_\parallel>}{dr} \]

\[ [D^{ST} \sim c_s l_c (\frac{\delta b_r}{B})^2] \]

Flow scaling has a Lundquist number dependence:

\[ V_0 \sim c_s \left[ \frac{I_p}{N} \right]^{-1} S^{2\alpha}, \quad S = \frac{\tau_R}{\tau_A} \sim \frac{I_p}{\eta \sqrt{n_e}} \]

Momentum and current transport could be coupled through finite pressure fluctuations.