Particle Energization via Tearing Instability with Global Self-Organization Constraints

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November 2015

APS Division of Plasma Physics Annual Meeting
Magnetic reconnection in self-organized plasmas can lead to fast, efficient particle energization

- Tearing modes in the Reversed Field Pinch drive a set of self-organization processes that regulate field, flows, and non-thermal particle dynamics.
- The process relies on details of the global geometry (such as non-linear mode interaction) but impacts dynamics from the global to ion gyro-scale through a turbulent cascade.
- The extensive diagnostic set on the MST Reversed-Field Pinch has been used to identify characteristics of energization processes such as anisotropy, particle species dependence, and energetic tail formation.

**Challenge**: develop a self-contained framework that spans the global-to-micro scales, allowing for possibly multiple dissipation processes. This must include the nonlinear self-organization feedback that influences the plasma’s global structure.
Outline

• Brief description of the Madison Symmetric Torus

• A description of tearing mode instabilities and the turbulence cascade created by their non-linear interaction

• Particle heating and transport

• Particle acceleration
The Reversed-Field Pinch (RFP) produces a toroidal magnetically-confined laboratory plasma.

- Plasma is ohmically heated by a large toroidal plasma current
- Only a small toroidal magnetic field is applied
- Magnetic field confines the plasma sufficiently to attain temperatures of 200-2000 eV
MST is a moderate current RFP which confines a high-temperature deuterium plasma.

\[ R/a = 1.5 \text{ m} / 0.5 \text{ m} \]
\[ I_p < 0.6 \text{ MA} \]
\[ |B| \leq 0.6 \text{ T} \]
\[ n_e \sim 10^{19} \text{ m}^{-3} \]
\[ 200 \text{ eV} < T_e, T_i < 2 \text{ keV} \]
\[ \tau_{\text{pulse}} < 0.1 \text{ s} \]
\[ S \sim 10^5 - 10^6 \]
\[ \beta < 25\% \]
Tearing instability of core-resonant modes drive quasi-periodic bursts of rapid magnetic reconnection

The toroidal geometry creates multiple resonance locations:

\[ 0 = k \cdot B = \frac{m}{r} B_\theta + \frac{n}{R} B_\phi \quad \implies \quad q(r_s) = \frac{r_s B_\phi}{R B_\theta} = \frac{m}{n} \]

- \( m = \) poloidal mode number
- \( n = \) toroidal mode number

\[ q(r_s) \approx 0.2 \] minor radius

Multiple Islands

\( m = 1, n \geq 6 \) resonances

\( m = 0, \) all \( n \)

\( m = 1, n = 6 \)

\( n = 7, m = 1 \)

\( n = 1, m = 0 \)
Tearing instability in the RFP saturates through dynamo-like feedback on the current density profile.

Mean-field representation:

\[ \mathbf{B} = \langle \mathbf{B} \rangle + \tilde{\mathbf{B}} \]

- **toroidal average**
- **3D spatial fluctuation (tearing)**

\[
\langle E \rangle_\parallel - \eta \langle J \rangle_\parallel = -\langle \tilde{V}_e \times \tilde{B} \rangle_\parallel \approx \langle \tilde{E} \cdot \tilde{B} \rangle_\parallel / B
\]

- **nonlinear emf from tearing fluctuations**

The presence of an edge resonant m=0 mode facilitates global self-organization.

Schematic of 3-wave cascade:

(1,6) \rightarrow (0,1)
(1,7) \rightarrow (0,1)
(1,8) \rightarrow (0,1)
(1,9) \rightarrow (0,1)

etc.

Same m = 0 couples to many m = 1

m ≥ 2 branch drives a cascade to smallest scales

Linear Eigenfunctions (normalized)

\tilde{B}_r

\begin{align*}
m &= 1 \\
n &= 6
\end{align*}

\begin{align*}
m &= 0 \\
n &= 1
\end{align*}

Minor radius, r/a
Tearing instability at the global scale drives a cascade to gyro-scale turbulence

\[ P(f) \propto \frac{T^2}{Hz} \]

\[ \approx \frac{\omega_{ci}}{2\pi} \]
The turbulent cascade is not as strong when global self-organization is absent.
The cascade is anisotropic and hints at a non-classical dissipation mechanism

- The $k_\perp$ spectrum is well-fit by a dissipative cascade model
- Onset of exponential decay occurs at a smaller $k_\perp$ than expected for classical dissipation

\[ \tilde{B}^2(k) \quad \text{(T}^2\text{-cm)} \]

\[ k^{-5/3} e^{-3/2(k_\perp/0.8)^{4/3}} \]

\[ k_\parallel^{-5.4} \]

P.W. Terry PoP 16, 082305 (2009)
Particle Heating
The most striking feature of the non-collisional dissipation is the resulting ion heating.

- Instantaneous heating rate can be as large as 10 MeV/s (50 MW!)

\[ \frac{1}{2\mu_0} \int \left( B^2_{\text{before}} - B^2_{\text{after}} \right) d^3 x \]

M.S. Cartolano PoP 21, 012510 (2014)
Heating is anisotropic and species dependent

- MST’s advanced diagnostics reveal key features of the heating process:
  - Rutherford scattering for majority ion temperature
  - Charge-exchange recombination spectroscopy (CHERS) for minority ions
  - Neutral particle energy analyzers (energetic neutral loss from plasma)

Mineral and minority ions hotter than majority ions

Heating is Anisotropic

\begin{align*}
\text{Temperature} & \approx 1400 \\
\text{Time (ms)} & \approx 2.0
\end{align*}

\begin{align*}
\text{Temperature} & \approx 1300 \\
\text{Time (ms)} & \approx 2.0
\end{align*}
Heating depends on mass and charge

\[ \frac{\Delta E_{\text{ion}}}{\Delta E_{\text{mag}}} \approx M_i^{0.52} \]

Majority Ions

- He^{++}
- D^+
- H^+

Minority Ions

- (varied fueling gas)

\[ \Delta T (eV) \]

\[ Z/\mu \]
Ion energization is diminished when the coupling between the tearing modes in the core and edge regions is weak

- Reveals the importance of global self-organization
Cyclotron-resonant heating may explain anisotropy and ion species dependance

- **Cyclotron-resonant heating:**
  - Feeds off the turbulent cascade to gyro-scale
  - Preferential perpendicular heating, but with collisional relaxation
  - Preferential minority ion heating, since $\tilde{B}^2(\omega_{ci})$ is larger where $\omega_{ci}$ is smaller
  - Mass scaling is predicted with dominant minority heating and collisional relaxation

(similar to Cranmer et al)

R.M. Magee, PRL 107, 065005 (2011)
Stochastic heating is also a strong source of heating

- **Stochastic heating:**
  - Feeds off large electrostatic electric field fluctuations and the distinct stochastic magnetic diffusion process
  - Monte Carlo modeling yields MST-like heating rates (Fiksel et al, PRL 2009)
  - Predicts mass scaling close to that observed

\[ \text{RMS } \tilde{E}_r \]

![Graph of RMS \( \tilde{E}_r \)](image)

![Graph of Non-Alfvenic Cascade](image)

\[ \frac{1}{2} m_i n_i \tilde{V}_{E \times B_0}^2 \]

\[ \tilde{B}^2 / 2 \mu_0 \]
Thermal electrons, conversely cool during the reconnection events

- Unlike ions, the external heating is large for the electrons $\int \eta J_\parallel^2 \approx 5-10$ MW
  - Could this mask a non-collisional process?
- Flattening of the $T_e(r)$ profile evidences enhanced electron heat transport
Electron heat diffusivity is consistent with expectations for transport in a stochastic magnetic field

- Magnetic stochasticity is global since there are many simultaneous tearing modes
- The good agreement between experimental power balance and stochastic diffusion expectation suggests rapid loss of heat through the electrons

\[ \chi_{st} = v_{th} D_m \sim \sqrt{T_e} \left( \frac{\tilde{B}}{B} \right)^2 \]

**Figure 4.17:** A comparison of the measured vs predicted values of the electron thermal diffusivity without (left) and with (right) the reduction in thermal diffusivity due to trapped particles taken into account. Average electron thermal diffusivity for the core region (0 ≤ r/a ≤ 0.45) from MAL (equation 4.11) from MAL is shown with the black solid line and from the fit to experimental data shown with the green dashed line. Note that the value of \( \chi_{st} \) decreases between -4 and -1 ms because the thermal velocity is decreasing.
Particle Acceleration
An energetic ion tail is generated and reinforced at each reconnection event

- Distribution is well-fit by a Maxwellian plus a power-law tail

\[ f_{D^+}(E) = A e^{-E/kT} + B E^{-\gamma} \]
“Neutral beam injection” creates energetic test ions that are accelerated during the reconnection events.
Energy gain of the energetic ions is consistent with runaway acceleration.

![Friction on test ion graph]

- **$V_{Ti}$**: Ion friction velocity.
- **$E^*$ before/after event**: Reference electric field.
- **$E^*$ during event**: Electric field during the event.

**Figure 3.3**: The frictional force for a test electron (a) and a test ion (b) is plotted for a typical MST plasma. Test electron slowing is always dominated by electron-electron collisions, while test ions transition from primarily ion drag at low energies to primarily electron drag at high energies.

The friction is thus functionally dependent on the electron density and temperature, the ion density and temperature, $Z_{\text{eff}}$, and the mass and velocity of the test ion.

The frictional force for a test electron and a test ion in a 300 kA, $F=-0.2$ plasma with $n_e=n_i=0.7 \times 10^{13}$ cm$^{-3}$, $T_e=450$ eV, and $T_i=400$ eV is shown in Figure 3.3. The friction profile for electrons is monotonic as the high speed of electrons with respect to the thermal ion speed means that the argument for $x_i$ is greater than 1 for electrons and ions. However, thermal ions have a speed much less than the thermal electron speed, meaning that the argument of $x_t$ transitions from much less than 1 to much greater than 1 as ion energy increases. This leads to the double-hump structure seen in Figure 3.3(b). The effects of $Z_{\text{eff}}$, density, and temperature are illustrated in Figure 3.4.

To imagine the effects of an applied electric field to the test particles, one can draw a horizontal line on the plots in Figure 3.3 or 3.4 at the appropriate effective electric field strength. In regions where the electric field is greater than the friction, the test particles are accelerated.

Fast x-ray spectrum measurements reveal the formation of a transient energetic electron tail.
Summary

- Tearing reconnection in the Reversed Field Pinch leads to fast, powerful particle energization.

- The global-scale magnetic field geometry regulates the rate of reconnection through nonlinear mode coupling and feedback processes
  - Magnetic flux generation
  - Turbulent cascade from tearing mode scale to ion gyroradius
  - Particle heating and acceleration

- Significant tasks remain in creating a complete framework
  - Models need to address fluctuations from global instabilities to micro-turbulence scales
  - Multiple dissipation processes are possible
  - Boundary conditions, anisotropy, and inhomogeneity are key ingredients