Simulation of MST Tokamak Discharges with Resonant Magnetic Perturbations

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Abstract

Nonlinear MHD modeling of MST tokamak plasmas with an applied resonant magnetic perturbation (RMP) reveals degradation of flux surfaces that may account for the experimentally observed suppression of runaway electrons with the RMP. Runaway electrons are routinely generated in MST tokamak discharges with low plasma density. When an $m = 3$ RMP is applied these electrons are strongly suppressed, while an $m = 1$ RMP of comparable amplitude has little effect. The computations are performed using the NIMROD code and use reconstructed equilibrium states of MST tokamak plasmas with $q(0) < 1$ and $q(a) = 2.2$. Linear computations show that the $(1, 1)$-kink and $(2, 2)$-tearing modes are unstable, and nonlinear simulations produce sawtoothing with a period of approximately 0.5 ms, which is comparable to the period of MHD activity observed experimentally. Adding an $m = 3$ RMP in the computation degrades flux surfaces in the outer region of the plasma, while no degradation occurs with an $m = 1$ RMP. The outer flux surface degradation with the $m = 3$ RMP, combined with the sawtooth-induced distortion of flux surfaces in the core, may account for the observed suppression of runaway electrons.
Runaway electrons are routinely generated in MST tokamak discharges with low plasma density.

- Discharge parameters: $B_t \approx 0.14$ T, $I_p \approx 50$ kA, $q(0) \approx 0.7$, $q(a) \approx 2.2$, $n_e \approx 10^{18} \text{ m}^{-3}$, $T_e \approx 150$ eV

Resonant magnetic perturbations (RMP) have been applied to suppress runaway populations in these discharges.

- Each RMP applies a single “$m$ mode” whilst having a broad $n$ spectrum.
- An RMP with $m = 3$ can be made to strongly suppress the measurement of runaway electrons.
- An RMP with $m = 1$ of similar magnitude does not lead to similar suppression.

More detailed experimental discussion and results are covered on poster GP10.00098 “Suppression of runaway electrons with a resonant magnetic perturbation in MST tokamak plasmas.” by S. Munareto.
Resistive MHD in the zero-$\beta$ limit is used for nonlinear computations.

- We use NIMROD to solve linear and nonlinear initial-value problems.
- For nonlinear calculations we use the set of equations:

\[
\begin{align*}
\frac{\partial B}{\partial t} &= \nabla \times (v \times B - \eta J) + \kappa_{dib} \nabla \nabla \cdot B \\
\mu_0 J &= \nabla \times B \\
\rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) &= J \times B - \nu \nabla^2 v
\end{align*}
\]

- All nonlinear calculations presented here have dimensionless parameters $S = 10^5$ and $P_m = 1$. 
The equilibrium used for computations is deduced from experimental measurements.

- The equilibrium profiles are imported from the MSTFit for shot 1160212070.
- The equilibrium is re-solved with a native NIMROD representation to remove interpolation errors.
- The MST discharges have low plasma-$\beta$, and we set $P = 0$ for convenience.
The (1, 1)-kink and (2, 2)-tearing modes are linearly unstable in these discharges.
The computationally applied RMP satisfies \( \int_{\partial R} B \cdot dS = 0 \).

- A “tophat” function is taken for the RMP in the toroidal coordinate.

\[
B \cdot \hat{n}|_{\partial R} = \frac{B_{RMP \text{ gap}}}{2\pi R} \cos(m\theta) + \sum_{n=1}^{N_{\text{max}}} \frac{B_{RMP R_0}}{n\pi R} \sin\left(\frac{n\text{ gap}}{2R_0}\right) \cos(m\theta) \cos(n\phi)
\]

- \( B_{RMP} \) is the magnitude of the field as seen by poloidal pick-up coils in the experiment.
- \( \text{gap} \) is the width of the region at the plasma boundary that exhibits normal poloidal flux.
- \( N_{\text{max}} \) is generally taken to be half of the maximum Fourier mode present in the computation.
RMP vacuum fields are found by imposing normal field component boundary conditions and diffusing the field throughout the domain.

$m = 1$

Re BR and BZ

$m = 3$

Re BR and BZ
Time traces of fluctuation energy spectra demonstrate sawtoothing behavior.

No RMP

$m = 1$

Magnetic Energy vs. Time

Kinetic Energy vs. Time
We are working on finding the cause of noise in these time traces when an RMP is included in the calculations.

\[ m = 2 \]

\[ m = 3 \]
Even for the initial conditions, different RMP modes produce qualitatively different field topologies.
The initial condition for these simulations is the sum of the equilibrium from MST data and the RMP vacuum field.

\[ m = 2 \]

\[ m = 3 \]
At the height of $n = 1$ fluctuation kinetic energy there is strong distortion of the core flux surfaces.
During this ”built-up” state, the $m = 3$ RMP case maintains only a small width of circular, nested, flux surfaces.
When $n = 1$ fluctuation kinetic energy reaches a minimum, the flux surfaces return to a somewhat similar configuration to the initial state.
Throughout the sawtoothing activity, the outer region of the $m = 3$ RMP case exhibits stochastic magnetic fields.
A scan of RMP strength for the $m = 3$ case was conducted on the initial state of the system.
The stochastic edge field is a result of overlapping island chains excited by the $m = 3$ RMP.

- A weak $m = 3$ RMP excites (3, 3), (3, 2), (8, 5), (5, 3), (4, 2) island chains.
- Applying a stronger $m = 3$ RMP field produces stochastic regions of magnetic fields through overlapping island chains.
We propose a mechanism of runaway electron suppression for the MST tokamak discharges via RMP.

- Assuming seed runaway electrons are born in the core,
  
- Certain RMP modes, e.g., $m = 3$, of sufficient strength produce a stochastic edge region of substantial width due to excitation of multiple rational surfaces resulting in overlapping island chains.
  
- Periodic magnetic reorganization of the core, i.e., sawtooth activity, allows transport of seed runaway electrons to the outer regions.
  
- Runaway electrons in the edge are readily lost to the wall from the combination of transport from magnetic reorganization and a stochastic edge.
  
- This is consistent with the results of the experiments since the $m = 1$ RMP, which lacks a stochastic edge based on simulations, does not produce runaway electron suppression like the $m = 3$ RMP, which exhibits a stochastic edge region in simulation.
Conclusions

- Nonlinear simulations of MST tokamak discharges including RMP fields were conducted with the caveat of minor numerical issues.
- Based on flux surface plots from these calculations we have proposed a mechanism for runaway electron suppression that is consistent with the experimental findings.
- We have shown that in this case stochastic field regions can be produced by the $m = 3$ RMP field via overlapping island chains from excitation of multiple rational surfaces.
Future Work

- Produce more robust simulations including RMPs.
  - There is unwanted numerical noise in current computations, although qualitative magnetic topologies appear to be insensitive to this.

- Calculate realistic relativistic electron trajectories to get a clearer understanding of transport mechanisms.
  - Ideally, this would include time-dependent fields if the MHD activity timescale is comparable to relevant runaway electron timescales.
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