New Density Measurement by Differential Interferometry

W.X. Ding, D.L. Brower, B.H. Deng, T.Yates

Electrical Engineering Department, University of California, Los Angeles, CA 90095
A novel differential interferometer is being developed on MST Reversed Field Pinch to measure the local density gradient and its fluctuations. Spacing between adjacent chords has been reduced to <1 cm which is comparable to (or even less than) the FIR laser beam size. Two slightly offset beams with frequency difference ~1 MHz are coupled into a single mixer making a heterodyne measurement. Phase difference between these two beams with respect to a reference signal is determined directly. No local oscillator beam is required to bias the mixer. Initial measurements indicate a phase difference of a few degrees which corresponds to < 1% total plasma density. In this configuration, the system is insensitive to vibrations since both beams traverse nearly identical optical paths. In addition, the phase difference is much less than a fringe eliminating potential fringe skip errors, especially at high density. Modeling shows that a non-Gaussian beam intensity profile will modify the phase difference when two beams have a large overlap. With system time response of ~1 µs, it becomes possible to measure fluctuations in the density gradient which provides a new measurement relevant to momentum transport.
A new interferometer, referred to differential interferometer, is being developed on MST.

Differential interferometer directly measures the phase difference between two adjacent chords giving \( \Delta \phi / \Delta x \), chord spacing small such that \( \Delta \phi \ll 2\pi \).

Advantages of Differential Interferometer technique:

- Immune to fringe counting errors
- Vibration and path length effects cancel
- Improved spatial resolution
- Improved phase resolution
The Principle of Differential Interferometer

Conventional interferometer phase shift induced by plasma, in a cylindrical geometry:

\[ \phi(x) = r_e \lambda \int_{\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} n_e(r) dz \]

\[ n_e(r) = -\frac{1}{\pi r_e \lambda} \int_r^a \frac{\partial \phi(x)}{\partial x} \frac{dx}{\sqrt{x^2 - r^2}} \]

after Abel inversion

\[ \frac{\partial \phi(x)}{\partial x} \]

Measured directly by differential interferometer
Differential Interferometer Characteristics

2 probe beams ($\lambda=0.432$ mm, $f=694$ GHz) with

(1) offset $\delta < 5$ mm
(2) frequency difference $\Delta f \sim 1$ MHz
(3) beam width $\sim 1$ cm

Due to small offset and frequency difference of probe beams:

- probe beams use same mixer generating a signal at $\Delta f$
- system is insensitive to vibrations or pathlength changes, $\Delta f/f=10^{-6}$
- system phase resolution $\sim0.05$ deg at 50 kHz bandwidth
- total phase measured $\ll 2\pi$, no fringe counting errors
Experimental Arrangement for Differential Interferometer

Laser 1 (probe)

Laser 2 (probe)

Laser 3 (LO)

Beam diameter ~1 cm

δ ~ a few mm

\[ \omega_1 - \omega_2 \rightarrow \Delta \phi(x) \]

\[ \omega_1 - \omega_3 \rightarrow \phi(x_1) \]

\[ \omega_2 - \omega_3 \rightarrow \phi(x_2) \]

\[ \phi(x) = \frac{\phi(x_1) + \phi(x_2)}{2} \]
FIR Differential Interferometer System

MST Far-Infrared Differential Interferometer System

- 3 Wave FIR Laser 25mW Each \( \lambda = 432 \mu \text{m} \)
- Gating Tuned CO\(_2\) Pump Laser 125W Continuous \( \lambda = 9.27 \mu \text{m} \)

- Waveguides
- Signal Beam
- Local Oscillator Beam
- Translating Mirror
- Wire Mesh BeamSplitters
- Plasma
- Vacuum Duct
- Schottky Detectors
- 9 Chord \( \text{H}_2 \) Array
Far-infrared laser differential interferometer hardware system

Translating Mirror used to set $\delta$
First Test of Differential Interferometer

for $\delta=3.5$ mm
1. for collinear beams, $\delta=0$, and $\Delta \phi \sim 0$

2. for offset beams, $\delta \sim 3.5 \text{ mm}$, $\Delta \phi$ measures local density gradient and $\Delta \phi < 2\pi$, no fringe counting ambiguities
Spatial Profile of Differential Phase

\[ |\Delta \phi| \text{ [deg.]} \]

\[ x \text{ [cm]} \]

-40 -20 0 20 40

\[ t=16 \text{ ms} \]

Spline Fit
Calibration of Differential Interferometer

Actual separation between two probing beams, $\Delta x$, is difficult to physically measure.

Calibration procedure for differential interferometer has been developed to determine $\Delta x$

$$n_e(r) \mid_\delta = -\frac{1}{\pi \lambda r_e} \int_r^a \frac{\Delta \phi(x)}{\delta} \frac{dx}{\sqrt{x^2 - r^2}} = k \times n_e(r)$$

Effective separation

$$\Delta x = k \delta$$
Time History of Conventional Interferometer

Graphs showing the time history of conventional interferometer with time in milliseconds on the x-axis and electron density $n_e$ in $10^{13} \text{cm}^{-3}$ on the y-axis. The graphs illustrate data over time for different locations $x = 6 \text{ cm}$ and $x = -32 \text{ cm}$, indicating a decrease in electron density over time.

Fringe skips removed by software.
\( \Delta \phi / dx \) from standard interferometer matches differential interferometer
Density Profile Comparison

different measurement techniques give same profile
Differential Interferometer during Pellet Injection

Conventional Interferometer

Fringe counting errors!

Differential Interferometer

No Fringe counting errors!

Pellet injection effect on electron density and phase shift.
Differential Interferometer

Mathematically, determination of density profile from measurements of $\phi(x)$ or $\frac{d\phi}{dx}$ are equivalent, because 

$$\phi(x) = \sum_i \frac{\partial \phi}{\partial x_i} \Delta x_i$$

However, technically, the differential interferometer has some distinct advantages over conventional interferometer approach. $\Delta x$

$$\phi(x) = r_e \lambda \int n_e(r) \, dz + \frac{2\pi}{\lambda} \delta L$$

$$\Delta \phi(x) = \phi_1 - \phi_2 = r_e \lambda \left[ \int n_1(r) \, dz - \int n_2(r) \, dz \right] + 2\pi \delta L \frac{\Delta \lambda}{\lambda}$$

$$\Delta \phi(x) \approx r_e \lambda \left[ \int \Delta n(r) \, dz \right]$$

where $\lambda_1 \approx \lambda_2 \approx \lambda$, $\Delta \lambda = \lambda_1 - \lambda_2$ and $\frac{\Delta \lambda}{\lambda} = 10^{-6}$

$\Delta \phi(x)$ is insensitive to path length changes
Inversion of differential Interferometer

\[ \Delta x = \frac{N_1 L_3 + N_2 L_1}{N_3} \]

\[ \Delta x = \frac{N_2 L_2 + N_3 L_1}{N_3} \]

\[ \phi(x_1) - \phi(x_2) = (N_2 - N_3) \times \Delta x \]

\[ \phi(x_1) = N_3 L_3 + N_2 L_1 \]
\[ \phi(x_2) = N_3 L_2 \]
\[ \phi(x_1) - \phi(x_2) = N_2 L_1 - N_3 (L_2 - L_1) \]
Effect of finite beam width on phase measurement

Assumed $\phi(x)$ profile

Beam has a Gaussian profile

Two beams are shifted a distance less than beam width

Average Phase 1 = 27.297
Average Phase 2 = 25.737

Phase difference can be tracked as long as two beams have similar profile
Summary

(1) Differential Interferometer has been successfully implemented on MST to determine $n_e(r,t)$
   - profile consistent with conventional interferometer
   - Total phase measured $<<2\pi$, immune to fringe counting errors
   - Low phase noise since vibrations and path length changes automatically cancel

(2) This novel interferometer is well suited to a harsh plasma environment where phase noise or fringe counting errors can occur due to large refractive effects, path length changes, vibrations
Future Plans

1. Test differential interferometer with
   - pellet injection
   - path length changes (by scanning mirror)
   to further confirm claim of no fringe counting errors
2. Establish spatial resolution
3. Use *Differential Interferometer* to measure density gradient fluctuations and their correlation with magnetic fluctuations; a measurement relevant to fluctuation-induced momentum transport

\[
\text{momentum flux : } \frac{\langle \nabla \tilde{p}_\parallel \rangle \tilde{b}_r}{\langle B \rangle} \sim \frac{\nabla \tilde{n}_e \tilde{b}_r}{\langle B \rangle}
\]