Non-axisymmetric Flows and Transport in the Edge of MST

by

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NON-AXISYMMETRIC FLOWS AND TRANSPORT IN THE EDGE OF MST

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Magnetic reconnection occurs in plasmas all throughout the universe and is responsible for spectacular and perplexing phenomena. In the Madison Symmetric Torus (MST) reversed field pinch (RFP), reconnection occurs as quasi-periodic bursts of tearing instabilities (saw-teeth), which give rise to a number of processes that affect the RFP’s global behavior and confinement. This work examines the structure of turbulent plasma flow in the edge region and its role in affecting momentum and particle transport through the use of several insertable probes and novel ensemble techniques.

Very few measurements exist of tearing mode flow structures. The flow structure has now been measured for $m = 0$ modes and is in good agreement with theoretical expectations for nonlinear resistive MHD calculated for the RFP using DEBS and NIMROD. The flows are predicted and measured to be different than the classical Sweet-Parker picture with symmetric inward flows.

The flow fluctuations have a profound effect on momentum transport, which is transported rapidly at the crash. This work advances the understanding of this process by measuring the Reynolds stress associated with turbulent flow. Combined with measure-
ments of the Maxwell stress, a new picture for magnetic self-organization in the RFP via two-fluid physics has emerged. The Reynolds and Maxwell stresses are measured to be an order of magnitude larger than the rate of change in inertia but oppositely directed such that they almost cancel. Two-fluid effects are significant because of the relationship between the Maxwell stress and the Hall dynamo, a term only existing in two-fluid theories. This relationship inextricably couples the momentum dynamics with the current dynamics. Indeed, the parallel momentum profile exhibits a relaxation at the crash akin to the relaxation seen in the parallel current density profile.

Tearing modes also drive particle transport. Fluctuation-induced particle flux is resolved through a crash by measuring it directly as \( \langle \tilde{n}_e \tilde{v}_r \rangle \). The flux increases dramatically during a crash and is non-axisymmetric. Between crashes, the transport from tearing is small, which agrees with previous measurements that identified electrostatic transport as dominant at that time.

John S. Sarff (Advisor)
Acknowledgements

Herein lies the culmination of the last eight and a half years of my—academic—life: Two years of graduate physics classes and teaching general physics 103/104, one more year of plasma classes, one more year of additional classes and the pursuit of a Masters Degree in Nuclear Engineering, and several years of designing probes, conducting experiments, running analysis, and working with my colleagues to parse out interesting conclusions. To date, this work is the largest project I have ever worked on and the longest manuscript I have ever produced. It is a monograph on the plasma physics I’ve learned. But! Try as I did, it contains a small fraction of the total plasma physics I’ve learned, an even smaller fraction of the plasma physics I’ve been exposed to, and a much much smaller fraction of the total knowledge I’ve accumulated during my stay at UW-Madison. Along the way I had a lot of help. To ignore this fact would be to lie to myself and the world and a blatant insult to those who supported me. Acknowledgments are in order.

First and foremost, I must thank Steward Prager for accepting me as a research assistant and for heading the MST group for most of my graduate life.

A huge thanks goes to Gennady Fiksel, my advisor for most of graduate school, who
once “kicked me in the butt” and told me to get back to work. I dare say I could have used a few more such kicks. He painfully taught me self-reliance, and perhaps it finally stuck. Many of the topics in this thesis were his ideas. Pulling the reconnection flow patterns out of a noisy signal by correlation with the toroidal array was one such idea he nonchalantly sketched on the white board once. That idea was fleshed out much later and further inspired a look into the spatial structure of particle transport.

I’d like to especially thank John Sarff who taught me a LOT about writing in a very short amount of time. My thesis is tremendously better thanks to John, and I also thank him for many insightful physics discussions.

I thank Vladimir Mirnov for help with theoretical comparisons and his wealth of knowledge. He knows way more about tearing modes than I do, and I am thankful he was able to teach me a thing or two.

The entirety of the MST staff should be thanked individually, but I must at least mention a few. For help with all things probe related and much guidance, I thank Abdulgader Almagri, who also taught me where meat comes from. For the collaboration on the stress measurements I thank Alexey Kuritsyn. For help with spectroscopic measurements, I thank Darren Craig, Dave Ennis, and Sanjay Gangadhara. I would also like to thank Jay Anderson and Weixing Ding for physics discussions. I don’t know how I would have gotten through gradschool without the help of the “hackers”. Paul Wilhite, especially, fixed almost all my computer related problems.

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Finally, I cannot thank enough my beloved Althea Archer. Not only has she scrutinized this thesis, offering many great suggestions, but she has held me up when I was down. mln.

Matthew Miller

Madison, Wisconsin

December, 2010
In memory of my mother

Carol L. Miller
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## MST Plasma Parameters

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<th>Description</th>
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<th>Edge value</th>
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<td>Poloidal Magnetic field</td>
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<td>$B_\phi$</td>
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<td>Toroidal Magnetic field</td>
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<td>$0.2 \times 10^{19}$ m$^{-3}$</td>
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<td>$\lambda_D$</td>
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<td>$\omega_p$</td>
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<td>$\omega_{ci}$</td>
<td>$eB/m_i$</td>
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<td>$9.6 \times 10^6$ s$^{-1}$</td>
<td>$3.8 \times 10^6$ s$^{-1}$</td>
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Chapter 1

Introduction

Magnetic reconnection occurs in plasmas all throughout the universe and is responsible for spectacular and perplexing phenomena. In the Madison Symmetric Torus (MST) reversed field pinch (RFP), reconnection occurs as quasi-periodic bursts of tearing instabilities (saw-teeth), which give rise to a number of processes that affect the RFP’s global behavior and confinement. This work examines the structure of turbulent plasma flow in the edge region and its role in affecting momentum and particle transport through the use of several insertable probes and novel ensemble techniques. A Mach probe, a spectroscopic probe, and a triple tip Langmuir probe are used to measure components of the plasma flow as well as plasma density and electron and ion temperature, thereby, providing a comprehensive picture of the plasma. Correlations are performed with a toroidal array of magnetic pickup coils to reconstruct a signal’s spatial variation in the rotating reference frame of the plasma from single point probe measurements. This allows for a detailed examination of the flow
dynamics and spatial structure during a sawtooth crash.

Very few measurements exist of tearing mode flow structures. The flow structure has now been measured for $m = 0$ modes and is in good agreement with theoretical predictions calculated for the RFP using a nonlinear cylindrical DEBS code and a toroidal NIMROD simulation, which are the most realistic theoretical models for the RFP. The flows are predicted and measured to be different than the classical Sweet-Parker picture with symmetric inward flows. The structure is a single flow vortex that is radially inward through the $X$-point and outward through the $O$-point. Asymmetries in the reconnection geometry give rise to an asymmetry in the radial flow, making $\tilde{u}_r$ non-zero at the resonant surface.

The flow fluctuations have a profound effect on momentum transport, which is transported rapidly from the core to the edge during a crash much faster than a classical collision time. This work advances the understanding of this process by measuring the Reynolds stress associated with turbulent flow. Combined with measurements of the Maxwell stress associated with magnetic fluctuations, a new picture for magnetic self-organization in the RFP via two-fluid physics has emerged. The Reynolds and Maxwell stresses are measured to be an order of magnitude larger than the rate of change in inertia but oppositely directed such that they almost cancel. Two-fluid effects are significant because of the relationship between the Maxwell stress and the Hall dynamo, a term only existing in two-fluid theories. This relationship inextricably couples the momentum dynamics with the current dynamics. Indeed, the parallel momentum profile exhibits a relaxation at the crash akin to the relaxation seen in the parallel current density profile.
Tearing modes also drive particle transport. Fluctuation-induced particle flux is resolved through a crash by measuring it directly as \(\langle \tilde{n}_e \tilde{v}_r \rangle\). The flux increases dramatically during a crash and is found to be non-axisymmetric and correlated with tearing modes. Between crashes, the transport from tearing is small, which agrees with previous measurements that identified electrostatic transport as dominant at that time.

Tearing reconnection in a RFP plays a pivotal role in relaxing the plasma to its characteristic equilibrium. Sawteeth are also principally responsible for a degradation to its confinement properties. However, not all aspects of a sawtooth crash adversely thwart fusion goals. The ion heating has already been exploited to achieve high performance plasmas in MST. The advancement of the RFP as a thermonuclear fusion reactor hinges on a complete and fundamental understanding of the underlying mechanisms and dynamics occurring during a sawtooth crash. Furthermore, such an understanding can offer insights into explaining astrophysical phenomena.

1.1 The Madison Symmetric Torus

The Madison Symmetric Torus (MST)[18] is located at the University of Wisconsin–Madison and belongs to a class of toroidal plasma confinement devices known as reversed field pinches (RFPs). In comparison to a tokamak, where the toroidal component of the confining magnetic field \(B_\phi\) is much larger than the poloidal component \(B_\theta\), in a RFP, \(B_\phi\) is comparable to \(B_\theta\). The RFP magnetic configuration is one of high shear, as it is toroidal in the core and almost all poloidal in the edge. Indeed, the RFP gets its name because \(B_\phi\)
goes to zero at the reversal surface and changes sign in the extreme edge, see Figure 1.1.

Figure 1.1: Magnetic field configuration of a RFP. The toroidal field reverses sign near the edge of the plasma.

MST is a large RFP with a major radius $R_0$ of 1.5 m, a minor radius $a$ of 0.52 m, and a circular poloidal cross section. The vacuum vessel is a highly conducting 5 cm thick aluminum wall that also serves as a single-turn toroidal field coil. A schematic of MST is shown in Figure 1.2.

A MST shot begins by charging up large capacitor banks to 2000–5000 V. A puff of deuterium gas is puffed into the vacuum chamber and the shot is seeded with an initial $B_\phi$ by discharging one of the capacitor banks to drive current around the conducting shell. Then at $t = 0$ s, the main banks are fired; driving current through windings around the iron core. This inductively drives up to half a million amperes of current toroidally through the plasma creating $B_\theta$ and ohmically heats the plasma up to millions of degrees Kelvin. This impressive release of energy ($\sim 4$ MJ) is over in about a tenth of a second.

For the work described in this dissertation, the plasma current was limited from the full
MST capability in order to decrease the plasma temperature and allow for the insertion of diagnostic probes. Plasmas were made from a mixture of deuterium and helium. Deuterium was puffed mainly at the beginning of shots with helium being puffed from a single puff valve during the shot. However, deuterium fueling from the wall occurred during the shot as per usual MST operation. Helium was puffed in order to study He II line emission. To get decent spectroscopic signal strength, overall line average density was made higher than typical running conditions \( n_e = 1.2 \times 10^{19} \text{ m}^{-3} \). About half of the electron density resulted from the helium puff, verified by shots without the helium. A summary of experimental running conditions for this work is summarized in Table 1.1. Precise values varied from these nominal values depending on the specific run conditions.

This thesis consistently uses a right-handed coordinate system \((r, \theta, \phi)\) as shown in Figure 1.3. The radial direction \( \hat{r} \) is outward, the poloidal direction is \( \hat{\theta} \) is outboard.
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_p$</td>
<td>200 kA</td>
</tr>
<tr>
<td>$B_\phi$ (average)</td>
<td>450 G</td>
</tr>
<tr>
<td>$F$</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>1.7</td>
</tr>
<tr>
<td>$n_e$</td>
<td>$1.2 \times 10^{19}$ m$^{-3}$</td>
</tr>
</tbody>
</table>

Table 1.1: Experimental conditions for typical MST plasmas in this study.

down, and the toroidal direction $\mathbf{\hat{\phi}}$, as viewed from the top, is counterclockwise (the same direction as $B_\phi$ in the core). In a cylindrical approximation, $\phi$ is treated as a “$z$” coordinate with value $R\phi$. This coordinate system is perhaps the most commonly used right-handed coordinate system on MST. One must carefully note which coordinate system is being used when comparing past results or looking at analysis codes. For instance, the mode analysis procedures are written in a left-handed coordinate system consistent with MST naming convention. The ensembles for this work were performed in this coordinate system so that the pseudospectrum would be correctly calculated. Quantities were then converted back into the right-handed coordinate system for plotting and further analyses.

### 1.2 Sawteeth

As previously mentioned, this work focuses on the dynamics occurring at the sawtooth crash. Before discussing sawtooth behavior in MST, one must first introduce the concept of a tearing mode. Magnetic plasma confinement devices with a toroidal geometry and
a strong plasma current gradient are susceptible to an instability known as the tearing mode.[32] A tearing mode is a particularly dynamic example of magnetic reconnection where previously nested flux surfaces gain an island structure. A finite amount of plasma resistivity allows these modes to develop where ideal MHD would forbid them. Magnetic field lines “tear” and reconnect releasing magnetic tension and abruptly transferring energy stored in the magnetic field to plasma kinetic and thermal energy. For a more complete description of tearing modes see, for instance, Bateman[5]. Interacting tearing modes are believed to be the cause of disruptions in tokamaks[61].

Tearing modes can develop on rational flux surfaces—surfaces where a helical magnetic field line bites its tail. If $m$ is the poloidal mode number and $n$ is the toroidal mode number,
then a rational surface exits where the safety factor $q$ is a rational value $m/n$.

$$q = \frac{rB_\phi}{RB_\theta} = \frac{m}{n} \quad (1.1)$$

Because the $q$ profile is decreasing, many magnetic tearing modes are resonant in RFPs like MST, see Figure 1.4. However, it is empirically found that the lowest $n$ modes dominate[1]. Due to their proximity, these island structures can overlap. This is especially the case during a sawtooth crash. When islands overlap, the magnetic field lines become stochastic over large portions of the plasma volume. This leads to a rapid loss of energy and particles because particles stream freely along field lines.

![Figure 1.4: A plot of a typical q profile courtesy of D. Ennis[25]. Shown are the locations of the largest n rational surfaces where tearing modes are resonant.](image)

Discharges in MST (and other RFPs) exhibit a sawtooth cycle, which is a quasi-periodic burst in tearing mode amplitudes, see Figure 1.5. The toroidal current is driven inductively and ohmically heats the plasma, with the core getting much hotter than the edge. Plasma resistivity is proportional to $T^{-3/2}$, and, therefore, the current profile becomes peaked and provides a free energy source for tearing modes. As the gradient in the current profile
Figure 1.5: Various signals depicting sawteeth in a typical MST shot. (a) Average toroidal field. MHD dynamo generates toroidal field. (b) Stored magnetic energy. Energy is released through magnetic reconnection. (c) Core velocity calculated from $n = 6$ mode rotation. Momentum is transported rapidly at the crash. (d) $n = 6$ mode amplitude. Core mode amplitudes begin increasing before the crash. (e) $n = 1$ mode amplitude. Edge modes are nonlinearly driven by core mode coupling.
increases, the core resonant modes with $m = 1$ become unstable and begin growing in amplitude. The edge modes resonant at the reversal surface with $m = 0$, on the other hand, are predicted to be stable, but in a sudden burst, they are nonlinearly driven by core mode coupling and grow to large amplitude[14, 11, 12]. Dynamo action drives a large parallel current in the edge and suppresses it in the core—converting poloidal flux into toroidal flux. This is seen as a sawtooth shape in the average $B_\phi$ signal. However, the reconnection of field lines during a crash relaxes the plasma toward a lower energy state as evidenced by a sharp decrease in the stored magnetic energy. The cycle then repeats itself.

1.3 Thesis overview

Chapter 2 discusses the diagnostics used in this work. A Mach probe, a spectroscopic probe, and a triple tip Langmuir probe are used to measure components of the plasma flow as well as plasma density and electron and ion temperature, thereby, providing a comprehensive picture of the plasma.

Chapter 3 goes into the analysis techniques used throughout the thesis. It introduces the concept of sawtooth ensemble averaging and relates the ensemble average to a flux surface average. Then it discusses how one calculates the pseudospectrum of a diagnostic signal. Because the plasma is rapidly rotating, many of the fluctuations measured in time correspond to spatial variations rotating around the torus. The pseudospectral method allows one to extract the spacial fluctuation spectrum from a single point measurement and correlation with the toroidal array. Tearing modes lock together during the crash,
so the pseudospectral components can be added together in a meaningful way to reveal a
measurement in the rotating reference frame of the plasma.

Chapter 4 discusses flux surface averaged measurements. Ensembles were performed
over many sawtooth events to attain the sawtooth behavior of each measured signal at
various radii. The ensemble averages over phase of the tearing mode structure, and, there-
fore, can be thought of as the symmetric component. The rest of the thesis is devoted to
fluctuations and quantities derived from products of fluctuations.

Chapter 5 presents the first measurement in MST of the flow structure driven by $m = 0$
tearing reconnection resonant in the edge. The flows are predicted and measured to be
different than the classical Sweeet-Parker picture. The structure is a single flow vortex that
is radially inward through the $X$-point and outward through the $O$-point. Asymmetries in
the reconnection geometry give rise to an asymmetry in the radial flow, making $\tilde{v}_r$ non-zero
at the resonant surface.

Chapter 6 presents the measurements of turbulent stresses that have advanced our un-
derstanding of momentum transport processes in the RFP. The stresses are measured to be
an order of magnitude larger than the rate of change in inertia but oppositely directed such
that they almost cancel. Previously, explanations of the observed momentum transport
invoked a single fluid picture. We now understand that two-fluid effects are signicant due
to the relationship between the Maxwell stress and the Hall dynamo, a term only existing
in two-fluid theories. This relationship inextricably couples the momentum dynamics with
the current dynamics. Indeed, the parallel momentum profile exhibits a relaxation at the
crash akin to the relaxation seen in the parallel current density profile.

Chapter 7 discusses fluctuation induced particle (and energy) transport. During the crash, the density profile flattens much faster than a collision time. Magnetic field lines become stochastic over much of the plasma volume due to overlapping islands. Particles are free to stream along field lines and are quickly lost. The Particle fluxes increase dramatically during a crash and are shown to have a spatial structure correlated to the magnetic structure resulting in non-axisymmetric particle losses. The flux between crashes, however, was found to be small, which agrees with previous measurements that identified electrostatic transport as dominant at that time.

Finally, Chapter 8 provides a brief conclusion of this thesis and a discussion of potential future work. Appendix A presents derivations using kinetic theory mentioned in the text. Appendix B extends the discussion of Langmuir probe theory to include effects of fast electrons and particle shields.
Chapter 2

Diagnostics

A set of three probes was used to obtain a comprehensive picture of the plasma. The Radial Ion Dynamics Spectrometer Probe (RIDSP) is an optical probe that collects light from atomic line emission from which can be obtained a local measurement of temperature and radial velocity fluctuations in the plasma. I designed the RIDSP under the tutelage of Dr. Gennady Fiksel and based its design on an earlier probe that he and Dr. Paul Fontana developed. The helicity probe is a combination of a triple tip Langmuir probe (TLP) and a magnetic probe. Doug Stone and I designed it, but it was constructed primarily by Stone. Its main purpose is to measure “helicity” injected by oscillating field current drive (OFCD), however, I have used it primarily for the TLP measurements. A bidirectional Mach probe was also employed to make velocity measurements in the poloidal and toroidal directions. It was designed by Dr. Genady Fiksel, commissioned by Dr. Alexey Kuritsyn, and I became a primary user and steward of it. The combined use of these three probes
provide a measure of all three components of velocity, \( v_r, v_\theta, \) and \( v_\phi \); all three components of magnetic field, \( B_r, B_\theta, \) and \( B_\phi \); ion and electron temperatures, \( T_e \) and \( T_i \); plasma density \( n_e \) (\( n_i \approx n_e \) assuming quasi-neutrality); and floating potential \( \Phi_f \).

### 2.1 Radial Ion Dynamics Spectrometer Probe

The radial ion dynamics spectrometer probe, RIDSP, is an optical probe used to passively collect light emitted from the edge plasma of MST. It views the plasma in the radial direction from a spatially localized area. Light collected by the RIDSP is sent through a high resolution spectrometer, the Ion Dynamics Spectrometer version II (IDSII)[16], to analyze impurity line emission. Radial velocity and temperature of the bulk ions are then inferred from the Doppler shift and broadening of the spectral line.

The RIDSP is based on the earlier design, the IDSP[28], but is different in some key features. It is much smaller than the previous probe. It has better spatial resolution (smaller collection volume) and is less of a perturbation to the plasma. Equilibrium radial flows measured by IDSP[31] were unexpected and could have been due to its large perturbation to the plasma. Furthermore, RIDSP’s small size allows for it to be inserted through any of MST’s 1.5 in portholes.

Whereas the IDSP had two intersecting viewing cords, the RIDSP only observes the radial direction. Consequently, the RIDSP is optimized for measuring radial velocity fluctuations and is not sensitive to the equilibrium radial velocity.

The last major difference is that the RIDSP was coupled to the new IDSII, which has
higher spectral resolution (0.234 Å/Ch versus 0.70 Å/Ch) and bigger optics to provide a higher throughput (×100 higher). Reliable temperature measurements were not previously possible due to the lower resolution of the IDS. The new probe, coupled with the IDSII, provides radially localized ion temperature for the first time in the edge of MST.

2.1.1 RIDSP design

The RIDSP was designed to collect light radiated by the plasma while both minimizing its disturbance on that plasma and surviving within it. A picture of it is shown in Figure 2.1. The RIDSP is 5.75 in long with a diameter of 1.25 in. All plasma facing components are machined from Boron Nitride. Its relatively simple design makes it very robust and it has withstood thousands of plasma discharges.

![Figure 2.1: Radial Ion Doppler Spectroscopy Probe](image)

To collect the light, the probe base houses a bundle of fused silica optical fibers that pipes the light out towards the IDSII. The fibers observe the plasma through a carefully designed collimator that extends away from the base. It extends out in order to push the
measurement area away from the more bulky probe base. The collimator was turned down in the middle to further minimize its perturbation. At the exit plane, the collimator angles away more steeply than the viewing angle so that the probe’s plasma interaction surface is not seen by the fiber. Inside, the collimation channel has grooves to minimize reflection into the collection fibers. A fused silica window resides between the fiber and the collimation channel to block hard UV light and prevent solarization of the optical fibers.

Supported out in front of the collimating channel is a view dump providing a spacial resolution of $\sim 3 \text{ cm} > r_i \approx 1 \text{ cm}$. It also was designed to minimize plasma interaction in the light collection area.

A cone shaped depression (not shown) was machined out of the view dump on the probe side so that light emitted from the collection volume towards the view dump is reflected away from the collection fiber. This depression was also painted black with carbon paint to further minimize reflection from the view dump.

The RIDSP is fixed to a 0.75 in diameter stainless steel shaft. The shaft holds the MST vacuum with a double O-ring sliding seal. This allows the RIDSP to be inserted in and out during MST operations without breaking vacuum. The RIDSP fiber bundle threads through the shaft to where it exits through a vacuum tight flange at the end of the probe assembly. Once outside the machine, the RIDSP fiber bundle is coupled to the IDSII fiber by a pair of lenses.
2.1.2 RIDSP theory

The light collected by the RIDSP provides information about the plasma. Random thermal motions spread out the line emission, an effect known as Doppler broadening. By measuring the line width, one can infer the plasma temperature. A shift in the centroid of the line gives the average velocity of the ions, Doppler shift. The light distribution in wavelength space corresponds to the ion distribution in velocity space. If the velocity distribution were non-Maxwellian, it would be apparent in the measured spectrum.

Ideally, we want to directly measure the temperature and velocity of the deuterium ions, but fully stripped deuterium does not produce line emission. For this work, the plasmas were doped with helium and the IDSII was tuned to center on the He\textsc{ii} line at 4685.7 Å. Partially ionized ions still have orbital electrons that can get excited and make transitions, thus, producing line spectra. From measurements of impurity ions we infer the characteristics of the bulk ions.

The data is fit to a Gaussian plus a linear background, equation (2.1). The linear part takes care of radiation that would be additive to the line spectrum, for instance Bremsstrahlung. The velocity and the temperature are calculated from moments of the Gaussian. At each time point, there are 16 channels in wavelength space.

\[
y(\lambda) = A_0 \exp\left[-\frac{1}{2} \left(\frac{\lambda - A_1}{A_2}\right)^2\right] + A_3 + A_4 \lambda \tag{2.1}
\]

Velocity and temperature are calculated from moments of the fitted Gaussian. Velocity is related to a shift in the centroid, \(\Delta \lambda = A_1 - \lambda_0\), and temperature is related to the line
width, $A_2$.

$$v = \frac{c \Delta \lambda}{\lambda_0}$$  \hspace{1cm} (2.2)

$$T_i = m_i c^2 \left( \frac{A_2^2 - w_{\text{trans}}^2}{\lambda_0^2} \right)$$  \hspace{1cm} (2.3)

The instrumental broadening of the line, $w_{\text{trans}}$, must be subtracted from the thermal broadening, see section 2.1.3 for details.

### 2.1.3 Calibration

Doppler physics is very well understood. However, the measurement of the light convolves the emission line with the instrumental function. If the ions were at zero temperature, the distribution should show up as a delta function. In other words, we would only see light in one channel. However, the optics spreads out the light, and it does not necessarily spread it out symmetrically. The instrumental function must be accurately measured and modeled so that we can back out the desired plasma parameters from the collected data, as done in equation (2.3). A detailed description of calibration procedures can be found in Appendix B of Dave Ennis’s PhD thesis[25], where he describes calibrating the IDSII for CHERS measurements of the C\textsc{vi} line at 3434 Å. Here I describe calibrating the IDSII for measurements of the He\textsc{ii} line at 4686 Å.

Spectra from standard gas discharge lamps (He, Cd, and Ne) are used to calibrate the spectrometer at or near wavelengths to be examined in MST’s plasma. The IDSII fibers are mounted in front of the lamp, and neutral density filters are used to attenuate the intensity. We then assume that the spectral line shape observed is entirely due to
instrumental effects. This is a good approximation because the gas inside these lamps is essentially at room temperature, and, hence, the thermal broadening is small. We then need to ascertain three things: the relative wavelength spacing of each channel, the relative gains of the photomultiplier tubes (PMTs), and the instrumental function.

**Channel spacing**

Diffraction gratings disperse light into its constituent wavelengths. The 16 channels of the IDSII sample a small band of the dispersed spectrum. In order to fit the line shape, the relative channel spacing in wavelength must be known. Since the angular dispersion of the gratings is a function of wavelength, the channel spacing must be measured at each wavelength of interest, namely, around 4686 Å for the He II line.

The IDSII grating motor rotates the gratings at constant angular velocity. This sweeps the spectrum across all channels. There is a time delay between when a line passes the first channel and when it passes each subsequent channel. The channel spacing is determined by measuring the time delay $\Delta t$ and converting it into a wavelength separation knowing the motor speed, $\Delta \lambda = v_M \Delta t$.

Two proximate lines are required to determine the motor speed. Knowing the wavelength separation of the two lines and their temporal separation on a given channel as they are swept across allows for the determination of the motor speed in Å/s. Even though the neon lamp’s spectrum is much more complicated than that of the other lamps, it was used because it contained two bright well separated lines at 4704.3950 Å and 4715.3453 Å, which
are near the He\textit{\textit{ii}} line. The measured Ne spectrum was carefully compared to the NIST database to identify the lines, see Figure 2.2.

Figure 2.2: Neon spectrum identification. The top plot shows the measured spectrum (smoothed to remove the 120 Hz oscillation). The lower plot was calculated from relative intensities given in the NIST database; assuming a temperature of about 40 eV.

The motor speed measurement was further complicated because our lamps are powered by an AC supply, so there is a 120 Hz oscillation in their luminosity. The motor speed was measured on all 16 channels and found to be constant but vary within about 1.2%. Most of this variation, however, can be attributed to error in identifying the location of the line’s peak due to the 120 Hz oscillation. The average motor speed for each scan of the gratings was used in subsequent calculations.

The time delay between when the strongest line at 4704 Å passed the first channel and
when it passed each subsequent channel was converted into a wavelength separation using the previously found motor speed. Then a linear fit was performed on the wavelength separations versus channel number. The slope of this line is the channel separation.

\[ y = -0.291 \pm 0.038 + (0.241 \pm 0.004) \times \text{channel} \]

Figure 2.3: Example fit for channel separation at 4707.4 Å

Figure 2.3 shows an example fit where the channel spacing was found to be 0.241 ± 0.004 Å/ch. Three scans were made, and the channel spacing measured for each scan. A weighted average based on the \( \sigma \) of the fit was performed to arrive at the final value used,

\[ D = 0.235 \pm 0.003 \text{ Å/ch.} \quad (2.4) \]

The uncertainty cited is the weighted standard deviation. The final answer is consistent with a ray tracing code, which predicted 0.227 Å/ch.
Relative gains

The overall sensitivity of each channel is determined by several factors. PMT gains are a function of the supplied high voltage, the wavelength of the incident photons, and other effects. When photons hit the photocathode, they are converted into photoelectrons with some quantum efficiency, which is quoted by the manufacturer for each PMT at several wavelengths. The photoelectron current is amplified at stages of dynodes through the PMT. The secondary emission coefficient of the dynodes can vary over time. There is also some variation in the gains of the external amplifiers. In order to get the correct line profile, the relative channel gains must be measured.

A cadmium lamp was used and the grating was scanned at a constant rate across the Cd$_{\text{i}}$ line at 4678 Å. The Cd$_{\text{i}}$ line is a single, well isolated, and bright line. The relative gains are determined by measuring the line’s amplitude on each channel and calculating the factors required to normalized them to the average amplitude.

The spectral line shows up in each channel with a 120 Hz oscillation due to the AC power supply on the lamp, and it is also skewed, see Figure 2.4. There is an imaging property that transforms a straight vertical entrance slit into a curved image at the exit. To compensate for this, the IDSII entrance slit was designed with a curve so that the image at the exit would be straight for measurements of C$_{\text{vi}}$ at 3433 Å. However, since the Cd$_{\text{i}}$ line is at 4678 Å, its image will be curved at the exit plane. This may account for the skewed image.

Fortunately, the measurement of the relative gains is insensitive to what function is
chosen to fit the line, so long as the amplitudes are measured consistently. First, the baseline was subtracted off from each channel. The signal was broken up into two halves on either side of the peak. Each half was reflected to make a symmetric signal, and a Gaussian was fit to each symmetrized half. Then the fit was done again while forcing the two Gaussians to have the same amplitude. Figure 2.4 shows a typical fit. The line amplitudes on each channel were normalized by the average of all 16 channels, and the required factors to do so were saved to a text file for use by the fitting algorithm. Multiple scans of the Cd I line were averaged to increase the accuracy of the measurement.

Transfer function

The IDSII instrumental function was modeled as a single Gaussian. Using the helium lamp, the IDSII was centered on the He I line at 4713 Å. The PMT voltage was converted to photoelectrons and summed over several seconds. Photoelectrons should follow Poisson distribution.
statistics, so a Poisson weighting \( (\sqrt{N}) \) was given to each channel going into the Gaussian fit. The fit is shown in Figure 2.5.

![Figure 2.5: The transfer function is modeled as a single Gaussian to subtract instrumental broadening from Doppler broadening.](image)

The width of the line is almost entirely due to instrumental broadening because the gas in the lamp is cold. The instrumental broadening at 4713 Å was found to be

\[
 w_{\text{trans}} = 1.2991 \pm 0.0011 \text{ ch},
\]

(2.5)

or about 0.3 Å \((1.2991 \text{ ch} \times 0.235 \text{ Å/ch})\). This instrumental broadening corresponds to a helium temperature of about 15 eV, which is subtracted from the temperatures measured in MST’s plasma. This instrumental temperature is of the same order as the ion thermal temperature at the edge of low current MST plasmas.
Error analysis

There are four main sources of error that should be considered when making measurements with the RIDSP. Probably the largest uncertainty comes from the shot to shot reproducibility of MST discharges and the dissimilarity of sawtooth events. To remedy that, large ensembles are taken with hundreds of events per ensemble. The standard deviation of the sawtooth average gives a measure of the shot to shot uncertainty in the final result.

More fundamentally, photon statistics follow a Poisson distribution. The emission comes from excited electrons making energy level transitions, which is an inherently random process. The PMT converts photons into photoelectrons that give rise to a current that is turned into the voltage that is eventually measured. The Poisson uncertainty in the photoelectron number $\sim 1/\sqrt{N}$. Poisson weightings are used in the Gaussian fitting algorithms.

Fit error represents how much the fit parameters can be varied while still maintaining a relatively good fit. In particular, the uncertainty in the centroid $\delta A_1$ and the line width $\delta A_2$ are relevant to physical parameters of the plasma, namely velocity and temperature.

A systematic error is introduced from imperfect calibrations. The uncertainty in the dispersion and instrumental broadening affects the uncertainty in the final fit. However, incorrect calibrations would manifests as a systematic shift in the RIDSP measurement and would have less of an effect on the fluctuations.

The standard deviation in the fit parameters and the uncertainty in the calibrations are propagated through the calculation of velocity and temperature to arrive at the uncertainty
in both those quantities.

\[
\delta v = \frac{c(\delta A_1)}{\lambda_0} \quad (2.6)
\]

\[
\delta T_i = \frac{2m_i}{\lambda_0^2} \sqrt{(A_2\delta A_2)^2 + (w_{\text{trans}}\delta w_{\text{trans}})^2} \quad (2.7)
\]

The above equations were found using a method outlined in Taylor’s book, *An Introduction to Error Analysis* [65].

### 2.2 Triple Tip Langmuir Probe

Langmuir probes were one of the first tools used to measure plasma characteristics. Developed by Irving Langmuir back in 1924, they are relatively simple in design. A Langmuir probe consists of one or more biased electrodes inserted into a plasma. The electrode draws current from the plasma and, thereby, samples the particle distribution.

Despite their simplicity, Langmuir probes are actively used in research today and are very useful in characterizing plasmas. Like other probes, they are typically used in cooler plasmas or in the edge of fusion devices where temperatures are cooler. Because Langmuir probes are so prevalent in current research, it is important to understand the theory behind how they work and their limitations.

This work utilized the helicity probe, which is a triple tip Langmuir probe instrumented with magnetic pickup coils. It consists of a boron nitride base with three 1/8 in molybdenum electrodes protruding from the end, as shown in Figure 2.6. The helicity probe features a particle shield to block a population of fast electrons that are known to exist in MST [63].
2.2.1 Langmuir probe theory

Langmuir probes come in several configurations that are generally classified according to the number of electrodes they utilize. The most common configurations being single, double, and triple tip Langmuir probes. The Mach probe is closely related to the Langmuir probe and will be reviewed in section 2.3. All of these probes are similar in that they consist of biased electrodes, but they differ in functionality as well as the assumptions and analysis required to produce the final measurement.

First, some basic probe theory is presented. Then the single probe will be discussed because understanding how it works naturally lends a better conception of how the others work. Following that, a theory for triple tip probes will be presented. This discussion will cater to an experimentalist. To that end it will endeavor to supply probe theory needed to understand the measurements, analysis techniques, and experimental guidelines and tips. However, the details of circuit design will be omitted and probe tips will be treated
abstractly as an electrode at some potential relative to the plasma potential, $\Phi_p$.

**Plasma sheaths**

A plasma is made up of ions and free electrons. When there is a perturbative charge present in a plasma, for example a voltage applied to a probe tip, the charged particles redistribute themselves in a way that zeros out the electric field caused by the perturbation. This shielding effect is known as Debye shielding. The redistribution is mainly done by the electrons because they are much faster and more mobile than the heavier ions. A sheath forms whose thickness is given by the Debye length,

$$\lambda_D = \sqrt{\frac{e_0 T_e}{n_\infty e^2}}.$$  

Here, $n_\infty$ is the electron density far from the probe tip and $T_e$ is the electron temperature. In general, the Debye length will be shorter than this because the ions also play a role, but their effect is smaller than that of the electrons. Beyond the Debye length the potential of the probe is not seen by the rest of the plasma—the entire potential drop is across the sheath.

**Floating potential**

An electrically insulated electrode in a plasma will “float” up to the floating potential, $\Phi_f$. This is understood because the total current is the sum of the electron and ion currents. Since the thermal electrons have a much faster average speed than the ions, the flux of electrons to the probe tip will initially be greater. In an isotropic kinetic gas, for example,
the flux of particles through a surface from one side only is given by $\Gamma = \frac{1}{4} n \bar{v}$, where $\bar{v}$ is the mean speed, see appendix A. The total current density is then given by $J = \frac{1}{4} e (n_i \bar{v}_i - n_e \bar{v}_e)$. Therefore, the electrode draws a negative current and quickly charges up negatively (relative to the unperturbed plasma potential, $\Phi_p$) until its potential drops sufficiently to repel the electrons. At $\Phi_f$, the electron and ion currents to the electrode are equal and the total current collected is zero.

Current collection

In general, the total electric current drawn by a probe tip is the sum of the electron and ion currents. The amount that each species contributes to the total current depends on the probe’s voltage $\Phi_{probe}$ relative to $\Phi_p$ and on the temperature and density of the plasma.

Ion current

If an electrode is biased sufficiently negative, it will collect what is called the ion saturation current $I_{i,sat}$. Electrons are repelled and the current to the probe tip is almost entirely due to ions. Ions move much slower than electrons, and it can often be assumed that they are cold, $T_i \ll T_e$. Recall that beyond $\lambda_D$, the potential is shielded out. Hence, for sufficiently negative biases, the current saturates at a value determined by the rate at which “cold” ions drift into the sheath and are subsequently accelerated to the electrode. Under these assumptions, the ion saturation current density is given by the Bohm formula,

$$J_{i,sat} = \exp \left( -\frac{1}{2} e n_{\infty} \left( \frac{T_e}{m_i} \right)^{1/2} \right). \quad (2.8)$$
The current drawn by the probe is the current density multiplied by the area of the sheath, $A_s J_{i,\text{sat}}$. Typically, $A_s$ is only slightly larger than the area of the probe tip.

These equations are valid when $T_i \ll T_e$. However, $J_{i,\text{sat}}$ depends only weakly on ion temperature. Computations have shown that the “$\exp\left(-\frac{1}{2}\right)$” factor out in front of equation (2.8) changes from 0.61 to 0.57 for monoenergetic ion energies of $0.01 T_e$ and to 0.54 for $0.5 T_e$ [41].

In our simplified model, we assume that the ions have zero thermal energy. That is to say, all ions are collected by a probe tip if it is biased lower than $\Phi_p$, and all ions are repelled by biases larger than $\Phi_p$. The Bohm formula (2.8), however, is modified for finite ion energies. The ion contribution to the probe current as a function of potential is then given by

$$J_i = \begin{cases} J_{i,\text{sat}} & \text{if } \Phi_{\text{probe}} < \Phi_p \\ 0 & \text{if } \Phi_{\text{probe}} > \Phi_p \end{cases}.$$ (2.9)

**Electron current**

In order to gain a physical intuition about electron current collection, let us start with a simple physical picture. We assume that the plasma is isotropic and homogeneous and that the collecting electrode is an infinite plane. MST plasmas are in an interesting regime where the ion gyrodiameter (about 2 cm) is large compared to a probe tip but the electron gyrodiameter (about 0.4 mm) is much smaller than a probe tip. From the probe’s perspective, electrons are constrained to a tight helical path along magnetic field lines, and we can, therefore, model the probe as an infinite plane, see Figure 2.7.
Figure 2.7: Electrons move along helical paths along magnetic field lines. The probe tip is modeled as an infinite plane.

The electrons are assumed to have a thermal, Maxwellian, distribution, so they are moving both towards and away from the probe tip. Under these conditions, the electron distribution function is a 1D Maxwellian given by

\[
f_e = n_e \left( \frac{m_e}{2\pi T_e} \right)^{1/2} \exp \left[ \frac{-m_e v_x^2}{2T_e} \right]. \tag{2.10}
\]

Let the potential drop across the plasma sheath \((\Phi_p - \Phi_{\text{probe}})\) be denoted \(\Phi_{\text{sh}}\). The minimum velocity for an electron to overcome \(\Phi_{\text{sh}}\) is found by conservation of energy.

\[
\frac{1}{2} m_e v_{e0}^2 = e\Phi_{\text{sh}}
\]

\[
v_{e0} = \sqrt{\frac{2e\Phi_{\text{sh}}}{m_e}} \tag{2.11}
\]

The current density to the electrode is calculated by integrating the distribution function from \(v_{e0}\) to infinity, see Figure 2.8.

\[
J_e = e n_e v
\]

\[
= e \int_{v_{e0}}^{\infty} f_e v_x \, dv_x
\]

\[
= e \int_{v_{e0}}^{\infty} n_e \left( \frac{m_e}{2\pi T_e} \right)^{1/2} \exp \left[ \frac{-m_e v_x^2}{2T_e} \right] v_x \, dv_x \tag{2.12}
\]
Integrating (2.12) by parts and substituting in the expression for $v_{e0}$ from (2.11) yields

$$J_e = e n_e \left( \frac{T_e}{2\pi m_e} \right)^{1/2} \exp \left[ -\frac{e\Phi_{sh}}{T_e} \right]. \quad (2.13)$$

The current to the probe tip is found by multiplying the electron current density by the effective current collection area, $A_{eff} J_e$. $A_{eff}$ should be twice the cross-sectional area of the probe sheath because the current is collected from both directions.

Equation (2.13) is valid for voltages at or below the plasma potential. At voltages above the plasma potential, all electrons drifting into the sheath are collected by the probe. This regime is known as electron saturation, and the current is given by the rate at which electrons drift into the sheath. In reality, the current does not perfectly plateau and there are other effects, such as secondary emission, that need to be taken into account. For simplicity, it will be assumed that the electron contribution to the total current is

$$J_e = \begin{cases} J_e & \text{if } \Phi_{probe} < \Phi_p \\ J_{e,\text{sat}} & \text{if } \Phi_{probe} > \Phi_p \end{cases}. \quad (2.14)$$
Single tip probes

A Langmuir probe in a single tip configuration is operated by biasing the probe tip relative to the vessel wall and sweeping that voltage up and down. As the voltage moves from one extreme to the other, current is collected and the particle distribution is sampled. The left panel in Figure 2.9 shows the I-V curve as calculated from the theory presented earlier in section 2.2.1. The right panel shows a more typical I-V curve as measured by a single tip probe in a “soup pot” experiment[50, 35] on the engineering campus of the University of Wisconsin-Madison*.

The nonlinear nature of this characteristic curve is caused by the Debye shielding, as mentioned earlier. At negative voltages the current collected is $I_{i,\text{sat}}$, given by equation (2.8).

*The Langmuir probe data shown has an arbitrary offset on the current axis because the oscilloscope used to collect the data had been offset. The plot is to be used for illustrative purposes only.
Decreasing the applied voltage further does not result in significantly more current. $I_{i,\text{sat}}$ depends on the density of the plasma and the square root of electron temperature. Hence, by measuring $I_{i,\text{sat}}$ with a decent guess of $T_e$, one can make a pretty good measurement of $n_e$.

As the applied voltage is increased towards the plasma potential, electrons start overcoming the potential barrier. In this region, the total current to the probe is the sum of the ion and electron currents, $I = I_i + I_e$, where $I_i$ and $I_e$ are given by equations (2.9) and (2.13) respectively. At $\Phi_f$ the ion and electron currents to the probe are equal and the total current is zero.

As the voltage is further increased, more and more electrons make it to the probe. The I-V curve again rolls over at about $\Phi_p$. At voltages greater than $\Phi_p$, the ion contribution is zero and the electron saturation current is collected.

**Triple tip probes**

A triple tip Langmuir probe, TLP, has three electrodes. Tips 1 and 2 are biased relative to each other while the third is left floating. The benefit of this configuration is that a measurement of $\Phi_f$, $T_e$, and $n_e$ is made without sweeping the voltage. The probe used during this work is shown in Figure 2.6.

Since tip 3 is at the floating potential, the ion current, equation (2.9), is equal to the electron current, equation (2.13). Let the factor in front of the exponential in equation (2.13) be denoted $J_{e,\text{sat}}$. From current balance we have that
\[ J_{i,\text{sat}} = -J_{e,\text{sat}} \exp \left[ -\frac{e(\Phi_f - \Phi_p)}{T_e} \right] \]

After rearranging terms we come to

\[ \ln \left[ \frac{J_{i,\text{sat}}}{-J_{e,\text{sat}}} \right] = -\frac{e(\Phi_f - \Phi_p)}{T_e}. \]  \hspace{1cm} (2.15)

Tip 1 is biased negatively (relative to the floating potential) and collects the ion saturation current.

\[ J_1 = J_{i,\text{sat}} \]  \hspace{1cm} (2.16)

Tip 2 collects an equal but opposite current compared to tip 1 from the electrons. The electron current is given by equation (2.13) and \( J_1 \) is given by (2.16), so we can write

\[ J_1 = -J_2 \]
\[ J_{i,\text{sat}} = -\left( J_{i,\text{sat}} + J_{e,\text{sat}} \exp \left[ -\frac{e(V_2 - \Phi_p)}{T_e} \right] \right). \]  \hspace{1cm} (2.17)

After rearranging terms we come to

\[ \ln 2 + \ln \left[ \frac{J_{i,\text{sat}}}{-J_{e,\text{sat}}} \right] = -\frac{e(V_2 - \Phi_p)}{T_e}. \]  \hspace{1cm} (2.18)

Substituting equation (2.15), we get

\[ \ln 2 + \frac{-e(\Phi_f - \Phi_p)}{T_e} = -\frac{e(V_2 - \Phi_p)}{T_e}. \]

After a little algebra we can arrive at an expression for \( T_e \) that depends solely on measured quantities.

\[ \ln 2 = \frac{e}{T_e}(\Phi_f - V_2) \]
\[ T_e = \frac{e}{\ln 2}(\Phi_f - V_2) \]  \hspace{1cm} (2.19)
We can also get an expression for density that depends on quantities measured by a triple tip Langmuir probe. From the ion current given by (2.8), with a correction for finite ion temperature and \( T_e \) from (2.19), we arrive at

\[
n_{\infty} = \frac{I_{i,\text{sat}}}{0.54 A_e} \sqrt{\frac{m_i}{T_e}}.
\]  

(2.20)

2.3 Mach probe

A Mach probe is also an electrostatic probe. Sometimes referred to as a directional Langmuir probe, it consists of two oppositely facing electrodes. The electrodes are insulated from each other and biased to collect the ion saturation current. Each electrode operates like a double probe with a common reference tip serving as the return path for the current. If the plasma is flowing, the upstream electrode will collect more current than the downstream one, see Figure 2.10.

Figure 2.10: Simplified schematic of a Mach probe.
2.3.1 Bidirectional Mach probe design

The Mach probe used in this work, Figure 2.11, consists of four electrodes biased to $-350 \text{ V}$ (sufficient to collect the ion saturation current) and one central reference electrode. The poloidal and toroidal plasma flows are each measured by a pair of oppositely facing electrodes. The electrodes are enclosed in boron nitride with four 2 mm tapered apertures defining the current collection area. The total probe diameter is 2 cm. The Mach probe is also equipped with five magnetic pickup coils. Four coils manufactured by Coilcraft, Inc. redundantly measure $B_\theta$ and $B_\phi$. One hand wound coil measures $B_r$.

Figure 2.11: Mach probe
2.3.2 Mach probe theory

The ratio of the upstream to downstream currents is related to the Mach number $M$, which is the ratio of the ion flow velocity $v_f$ to the (modified) ion sound speed $c_s$. 

$$M = \frac{v_f}{c_s} \quad (2.21)$$

$$c_s \equiv \left( \frac{Z T_e}{m_i} \right)^{1/2} \quad (2.22)$$

The form that relationship takes depends on the theoretical model employed to explain the phenomena. In general, Mach probe models take the form

$$\frac{I_{i,sat}^{up}}{I_{i,sat}^{down}} = \exp(kM), \quad (2.23)$$

where $k$ is a constant that is model dependent. It is $k$ that has become known as the Mach probe calibration problem. In a strongly magnetized plasma, as is the case for many fusion experiments, the ion Larmor radius $r_i$ is much smaller than the diameter of the probe $d$. The $I_{i,sat}$ that is collected depends on cross field diffusion into a presheath that is elongated along the field. In this regime, a value for $k$ of about 1.7 has been established for some time[38, 13].

If $r_i \ll d$, then a magnetized model should be used. To the other extreme, if $r_i \gg d$, then an unmagnetized model should be used. However, in some magnetically confined plasmas, the situation is not clear[37, 33]. For the MST Mach probe, $r_i \sim 0.5d$. Peterson et al. put forward a methodology for determining the degree to which a probe is magnetized[53]. Kuritsyn applied that test to the MST Mach probe and found that it should be treated as unmagnetized[44].
The theory for unmagnetized Mach probes has only recently been rigorously developed. Back in 1970, Hudis and Lidsky put forth a one dimensional model based on energy conservation\cite{37}, which is what Peterson used. That theory fell under strong criticism by Hutchinson\cite{39} who later performed a simulation using the code SCEPTIC\cite{40}. Following the results of Hutchinson’s unmagnetized model, the flow velocity is calculated from Mach probe measurements by

\[
v_f = 0.746 \sqrt{\frac{ZT_e}{m_i}} \ln \left( \frac{J_{i,\text{up}}}{J_{i,\text{down}}} \right) .
\]  

(2.24)

For the analysis in this work, values in (2.24) were fixed at \( Z = 1, m_i = 2 \text{ u}, \) and \( T_e = 20 \text{ eV}. \)

### 2.3.3 Balance factors

The current to each electrode is measured as a voltage across a shunt resistor, as shown in Figure 2.10. That voltage depends on the value \( J_{i,\text{sat}} \), the resistance \( R \), the effective area of current collection \( A \), and any other miscellaneous gains \( G \). If there are two electrodes labeled “1” and “2”, the voltages measured are

\[
V_1 = R_1 G_1 A_1 J_1 \quad \text{and} \quad V_2 = R_2 G_2 A_2 J_2 .
\]  

(2.25)

(2.26)

Due to causes like sheath effects, machining errors, and uneven degradation from continued use; \( R, G, \) and \( A \) are not precisely known and, furthermore, are most likely different. If these differences affecting current collection are not properly taken into account, it could
lead to an inaccurate inference of plasma flow. Therefore, we define a balance factor \[ F_{\text{bal}} \equiv \frac{R_1 G_1 A_1}{R_2 G_2 A_2}. \quad (2.27) \]

The balance factor accounts for any asymmetry in the probe. It is measured by taking data in one orientation, rotating the probe 180°, and taking data in the opposite orientation. In order to see how this is done, multiply the top and bottom of equation (2.27) by \((J_{\text{up}} + J_{\text{dwn}})\). Then by equations (2.25) and (2.26), \(F_{\text{bal}}\) turns out to be a ratio of measured voltages,

\[ F_{\text{bal}} = \frac{V_{\text{up}}^1 + V_{\text{dwn}}^1}{V_{\text{up}}^2 + V_{\text{dwn}}^2}. \quad (2.28) \]

Once this is measured, it is used to calibrate future measurements. For equivalent current densities on both sides of the probe (no plasma flow), \(V_1 = F_{\text{bal}} V_2\). If electrode 1 is “upstream”, then by equation (2.24), the plasma flow velocity is

\[ v_f = 0.746 \sqrt{\frac{Z T_e}{m_i}} \ln \left[ \frac{V_1}{F_{\text{bal}} V_2} \right]. \quad (2.29) \]

Data from three different run days were used to measure \(F_{\text{bal}}\). During the runs, half the data was taken during shots with the Mach probe in one orientation. Then the Mach probe was rotated 180°, and the second half was collected. Assuming the shots made two similar ensembles, we can assume that the average flow was the same for both data sets. A sawtooth ensemble was performed for the raw voltages on each of the four electrodes. The electrodes are numbered sequentially, so opposite pairs of electrodes are labeled (1, 3) and (2, 4) respectively.

Balance factors for the two pairs of electrodes are plotted in Figure 2.12 for three
Figure 2.12: Mach probe balance factors. Dark colored lines used averages between sawteeth. Light colored lines included data during sawteeth.
different run days. Dark colored lines only used data between sawteeth, while light colored lines average the data through the sawtooth crash. The two different procedures did not result in significant differences in $F_{\text{bal}}$. The final value used in subsequent analysis was the average of $F_{\text{bal}}$ over all radial locations with the uncertainty quoted being the standard deviation,

\[
F_{\text{bal}}(1,3) = 1.05 \pm 0.09
\]

\[
F_{\text{bal}}(2,4) = 1.18 \pm 0.17. \quad (2.30)
\]

As can be seen in Figure 2.12, there was a lot of variation in measured $F_{\text{bal}}$. The variation was between 10–15%. This represents a significant source of systematic error. For a typical voltage ratio of 2 V, the uncertainty in $F_{\text{bal}}$ propagates into a systematic error in velocity from electrode pair (1, 3) as

\[
v_f = 14.9 \pm 2.1 - 1.9 \text{ km/s}
\]

and from electrode pair (2, 4) as

\[
v_f = 12.2 \pm 3.6 - 3.1 \text{ km/s}.
\]

Hence, this uncertainty could amount to an error in flow velocity of about 30%.
Chapter 3

Analysis Techniques

This chapter goes into some detail about analysis techniques used throughout this thesis. Sawtooth averages are frequently used on MST to study the fast dynamics during a crash. This chapter will show that a sawtooth ensemble approximates a flux surface average and will define more precisely what is meant by a flux surface average, namely, that it represents an axisymmetric quantity—one that is averaged over $\theta$ and $\phi$. It will also be demonstrated that a flux surface average is synonymous with the $m = 0, n = 0$ component of a quantity. While the Madison “Symmetric” Torus was designed to be symmetric poloidally and toroidally, there are very important non-axisymmetric components of most measurable quantities, which are especially apparent at a sawtooth crash. A synopsis of the pseudospectral method that is utilized to investigate these non-axisymmetric components will be presented; followed by a more in-depth look at the mode adding method.
3.1 Flux surface average

The concept of a flux surface average is prevalent in magnetically confined plasma research. In the case where magnetic field lines form nested magnetic surfaces, one can define the average value of a quantity $X$ over a flux surface of area $A$ by

$$\langle X \rangle = \frac{\oint X \, dA}{\oint dA}.$$  \hspace{1cm} (3.1)

In this thesis, a fluctuation is considered to be any spatial deviation from $\langle X \rangle$,

$$\tilde{X} \equiv X - \langle X \rangle.$$  \hspace{1cm} (3.2)

It will also be important to talk about the product of two quantities averaged over a flux surface, mathematically found by

$$\langle XY \rangle = \frac{\oint \left( \langle X \rangle + \tilde{X} \right) \left( \langle Y \rangle + \tilde{Y} \right) \, dA}{\oint dA} = \langle X \rangle \langle Y \rangle + \langle X \tilde{Y} \rangle.$$  \hspace{1cm} (3.3)

3.2 Sawtooth ensemble average

Much current research is aimed at understanding the dynamics during a sawtooth crash. Sawteeth are very reproducible during standard MST discharges. Not only is the physics of what is happening interesting, but sawteeth also provide a convenient identifiable event to ensemble over. Because the plasma is rotating rapidly, each sawtooth crash occurs at an approximately random phase. An ensemble average of a signal $X$ measured at a single point
in space over many events, sometimes denoted $[X]$, approximates a flux surface average $\langle X \rangle$.

\[
\langle X(\theta, \phi) \rangle \approx [x(\text{point})]
\] (3.4)

An ensemble average is preferable to a time average, because the equilibrium is rapidly changing through a sawtooth event. The tearing mode frequencies of interest are on the order of 10 kHz, or a period of 10 ms, but relevant dynamics and turbulence are happening at higher frequencies as well. To study these fluctuations, one must be able to subtract out the equilibrium without smoothing over frequencies of interest. The crash occurs within 100 $\mu$s, so this is difficult to do in time for a single shot. However, we rely on the reproducibility of the sawtooth crash to our equilibrium average across an ensemble of events. We relate this ensemble to the flux surface average $\langle X \rangle$, or equivalently, the $m = 0, n = 0$ axisymmetric component. After $\langle X \rangle$ is calculated, the fluctuating component of a given signal is found by (3.2), and the ensemble is performed again over products of fluctuations, as in (3.3).

### 3.3 Pseudospectra and mode addition

It is often the case that one would like to Fourier decompose a quantity measured on an experiment. This is especially true in toroidal magnetically confined plasmas where tearing modes produce magnetic islands that are periodic around the torus. MST is equipped with a toroidal array of magnetic pickup coils that sense the perturbations caused by these tearing modes. Even though the toroidal array is located at the inboard edge of the plasma ($241^\circ$ poloidal), it picks up perturbations caused by modes resonant in the core. Using the
toroidal array, the magnetic field measured at the plasma boundary is Fourier decomposed as
\[ B = B_0 + \sum_{n=1}^{\infty} b_n \cos(m\theta + n\phi - \delta_{bn}). \]  
(3.5)

In (3.5), \( m \) is the poloidal mode number, \( \theta \) the poloidal angle; \( n \) is the toroidal mode number, and \( \phi \) the toroidal angle. The toroidal array does not resolve the \( m \) spectrum. For that, one uses the poloidal array. However, magnetic fluctuations have been studied extensively in MST both experimentally and computationally[9, 66]. The fluctuations arise from tearing modes resonant within the plasma. In the core, the dominant modes are \( m = 1 \) and \( n \geq 6 \). In the edge, \( m = 0 \) modes are resonant at the reversal surface and the lowest \( ns \), namely 1–4, dominate. Therefore, it can be assumed that \( m \) is a simple function of \( n \),
\[ m = \begin{cases} 
0 & \text{if } n = 1, 2, 3, 4 \\
1 & \text{if } n \geq 5 
\end{cases}. \]  
(3.6)

Tearing modes have a profound effect on many quantities. If an array of simultaneous measurements could be made of another signal, \( X \), it could be decomposed as
\[ X = X_0 + \sum_{n=1}^{\infty} x_n \cos(m\theta + n\phi - \delta_{xn}). \]  
(3.7)

This is often impractical to do. However, if one assumes that \( X \) is strongly coupled to \( B \), than the pseudospectrum of \( X \) can be calculated from a single point measurement and correlation with the toroidal array. The pseudospectrum of a signal \( X \), \( X' \), yields the components of \( X \) that are correlated with the magnetic modes \( B_n \) measured by the toroidal array. It results in the pseudospectral mode amplitudes, \( \xi_n \), and their phases relative to
the respective magnetic mode, \( \delta_{x,bn} \). The true \( X_n \) is approximated by the pseudospectral \( X_n \).

\[
X_n \approx X_n = \xi_n \cos(m\theta + n\phi - \delta_{x,bn})
\]  

(3.8)

The geometrical shift between the location where \( X \) is measured and the zero point defined during the Fourier decomposition of the magnetic modes must be taken into account. The mode analysis on MST utilizes a left-handed coordinate system and defines the zero point to be at \((\theta = 241^\circ, \phi = 0)\). Therefore, the geometrical shift is given by

\[
\Delta_n = n\phi_{\text{probe}} + m(\theta_{\text{probe}} - 241^\circ).
\]

(3.9)

When correlating with the magnetic array, we do so in a left-handed coordinate system and add \( \Delta_n \) to the phase of the mode, thereby, changing the reference point to \( X \)'s location.

The pseudospectrum is made possible because the modes tend to rapidly rotate around the machine. A fluctuation in time detected by a single point measurement corresponds to a fluctuation in space rotating around the machine. Furthermore, each sawtooth event occurs at a random phase of the mode. A more in depth description of pseudospectral analysis techniques and a derivation of the pseudospectrum is given in T. D. Tharp’s PhD thesis[66].

Once \( X_n \) is computed, we would like to add the modes back up together. Doing so would give a measurement in the rotating reference frame of the plasma. During a sawtooth crash, the mode amplitudes are large and tend to lock together in predictable ways, so it makes sense to add these modes together. The pseudospectral method yields the phase between \( X_n \) and \( B_n \). To properly add the modes together, one needs to know the relative phase between
the magnetic modes. Because the mode analysis code performs the Fourier decomposition as $b_n \cos(n\phi - \delta_m)$, the relative phase between mode $n$ and a reference mode $n'$ is given by

$$\delta_{n,n'}^{\text{rel}} \equiv \delta_{bn} - \frac{n}{n'} \delta_{bn'}.$$  

(3.10)

The sum of all $m = 0$ modes relative to the $n = 1$ for our pseudospectral $X$ is given by

$$X_{m=0} = X_1 + \sum_{n=2}^4 \xi_n \cos\left(n\phi - \delta_{x,bn} - \delta_{n,1}^{\text{rel}}\right).$$  

(3.11)

First, we check to make sure that the $n = 1$ mode is in a random phase for each sawtooth event, see Figure 3.1. A flat histogram indicates that the mode is at a random phase for each event, and that the measurement evenly samples the entirety of the mode. The other modes can also be checked in this way, see Figure 3.2.

![Figure 3.1: The $n = 1$ phase distribution for a test ensemble. A flat histogram indicates that the random phase approximation is valid.](image)

If we construct a histogram of the relative phases, as defined in (3.10), we observe that it is no longer flat but is strongly peaked, see Figure 3.3. The peaks are separated by $2\pi$ and represent the same phase.
Figure 3.2: The $n = 2, 3, 4$ phase distributions for a test ensemble. A flat histogram indicates that the random phase approximation is valid.

Figure 3.3: Relative phase distributions for a test ensemble given by $\delta_{n,1}^{\text{rel}} = \delta_n - n\delta_1$. Peaks in the histogram separated by $2\pi$ represent the same value.
The modes are not necessarily locked together away from sawtooth crashes. In Figure 3.4, the relative phase 1 ms before the crash is much more random. This could mean that the modes are slipping past each other, or it could mean that because the mode amplitudes are so small, the phase is difficult to measure and not well defined. In either case, the mode adding method is inappropriate away from sawtooth crashes.

Figure 3.4: The relative phase of the modes before a sawtooth crash. It is random because modes are not locked together or because the amplitudes are too small to adequately measure the phase.

If one incorrectly defined the relative phase, for instance a simple subtraction of two phases, then one would get the wrong phase relationship, see Figure 3.5. As another test, Figure 3.6 shows histograms of correctly defined relative phases, but the reference phase is subtracted from a random phase. As expected, the histogram shows a random result.

The pseudospectrum can be calculated after running MST’s sawtooth analysis code, ST_CORR.PRO*. First, one writes a signal routine to ensemble the correct signals. The mode signals must come first in the signal routine so that the phase is correct. After that,

*ST_CORR.PRO is an IDL procedure developed by the MST group. Typewriter font will be periodically used throughout this thesis. It should be read as IDL commands and procedures.
Figure 3.5: Relative phases calculated incorrectly as $\delta_{n,1}^{\text{rel}} = \delta_n - \delta_1$. These distributions are not nearly so peaked.

Figure 3.6: Relative phase distribution calculated correctly but subtracting from a random phase. As expected, it is random.
include both your signal $X$ and a "$X_{\text{fluct}}$" signal for mode correlation. $X_{\text{fluct}}$ is attained by subtracting off a smoothed signal. Smooth over about 5 ms so that the shot evolution is captured but not the sawtooth behavior. The reason for correlating with $X_{\text{fluct}}$ is to prevent shot-to-shot variability and shot evolution effects from inadvertently adding to the fluctuation power. To summarize, the signal routine should return the following signals in the order as listed.

\[
\begin{align*}
    b_n \\
    B_n &= b_n \cos(-\delta_{bn} + \Delta_n) \\
    B_n^\dagger &= b_n \sin(-\delta_{bn} + \Delta_n) \\
    \cos(\nu_{n,1}^\text{rel}) \\
    \sin(\nu_{n,1}^\text{rel}) \\
    X \\
    X_{\text{fluct}}
\end{align*}
\]

After the ensemble is performed, the pseudospectrum is calculated as

\[
\begin{align*}
    \text{Re}(\mathcal{X}_n) &= \frac{2\langle \tilde{X}_{\text{fluct}} \tilde{B}_n \rangle}{\langle b_n \rangle}; \quad (3.12) \\
    \text{Im}(\mathcal{X}_n) &= -\frac{2\langle \tilde{X}_{\text{fluct}} \tilde{B}_n^\dagger \rangle}{\langle b_n \rangle}; \quad (3.13) \\
    \xi_n &= (\text{Re}(\mathcal{X}_n)^2 + \text{Im}(\mathcal{X}_n)^2)^{1/2}; \quad \text{and} \quad (3.14) \\
    \delta_{x, bn} &= \arctan \left( \frac{-\text{Im}(\mathcal{X}_n)}{\text{Re}(\mathcal{X}_n)} \right). \quad (3.15)
\end{align*}
\]

The ensemble average of a quantity, namely $\langle X \rangle$, is stored in the \texttt{ST\_CORR\_PRO} output file as
DXAV[*,nxsig], where nxsig is the index specifying signal $X$. The ensemble average of the product of fluctuations, namely $\langle \tilde{X}\tilde{Y} \rangle$, is stored as DXDXAV[*,nxsig,nysig]. Remember, that one is usually interested in $\langle \tilde{X}_{\text{fluct}}\tilde{Y}_{\text{fluct}} \rangle$

Add the $m = 0, n = 1, 2, 3, 4$ modes together with $n = 1$ as the reference by using equation (3.11), the spectral quantities calculated from (3.12)–(3.15), and the relative phases calculated by

$$\delta_{n1}^{\text{rel}} = \arctan(\sin(\delta_{n1}^{\text{rel}}), \cos(\delta_{n1}^{\text{rel}})).$$ (3.16)

In conclusion, the pseudospectral method of analysis provides a means of extracting the spatial Fourier components of a signal from a single point measurement. It relies on plasma rotation, correlation with the toroidal array, and a coupling between the signal being measured and the magnetic structure. The modes are locked together at the crash, so the pseudospectral modes can be added together to reconstruct a signal’s spatial variation in the rotating reference frame of the plasma. This allows for a detailed examination of the dynamics and spatial structure of measured signals during a sawtooth crash.
Chapter 4

Flux Surface Averaged Measurements

Sawtooth dynamics in MST plasmas occur rapidly (within 100 µs). Sawtooth ensembles are performed to better understand these fast time dynamics. This chapter presents sawtooth ensembles of quantities measured by the three probes described earlier in chapter 2. Each sawtooth event is at a random phase, so sawtooth ensembles approximate a flux surface average $\langle X \rangle$. In that regard, the sawtooth average represents the $m = 0$, $n = 0$, or axisymmetric, component. For $m = 0$ modes, this corresponds to an average over $\phi$, while for $m = 1$ modes, it corresponds to an average over $\theta$ and $\phi$.

This chapter also discusses the root mean square (RMS) fluctuations for measured quantities. The RMS fluctuation of a quantity is calculated via an event ensemble as $(\langle X^2 \rangle)^{1/2}$. Therefore, the RMS value is also a “axisymmetric” quantity. In general, fluctuations are caused by many processes. A lot of the fluctuation power shows up in the tearing mode frequency range, which is 1–10 kHz.
This work focuses on fluctuations, but axisymmetric components play an important role as well. Subsequent chapters will refer to these ensemble averages and utilize them for analyses.

4.1 RIDSP measurements

The RIDSP only views the plasma in the $-\hat{r}$ direction, and for that reason, an absolute velocity measurement is unavailable. The IDSII reproducibly centers on a chosen wavelength, but the precise setting slowly drifts over time. An absolute calibration is possible because the IDSII has a second fiber that could look at a stationary plasma glow discharge. However, such a setup was not available for this work. Furthermore, the equilibrium radial flow is expected to be small. An $\mathbf{E} \times \mathbf{B}$ pinch gives rise to an inward radial velocity, whose measurement is described in detail in chapter 7, but it is small. The pinch velocity is about $-20 \text{ m/s}$, and corresponds to a wavelength shift of the 4686 Å He II line of 0.00133 ch—much smaller than can be measured by the IDSII. An equilibrium radial flow could exist unrelated to the $\mathbf{E} \times \mathbf{B}$ velocity. Previous measurements of $v_r$ showed large ($\pm 5 \text{ km/s}$) inward flow that slowed to zero during a discharge[31]. However, plasma-probe interaction was suspected to be the cause.

In any case, the RIDSP is used to measure radial velocity fluctuations $\tilde{v}_r$ and impurity ion temperature $T_i$. The shot evolution is subtracted from the $v_r$ measurement to get $v_{r,\text{fluct}}$, which is then ensemble averaged and shown in Figure 4.1. Radial velocity is observed to increase positively and then negatively during a crash and resembles P. Fontana’s
observations[31]. This behavior could be attributed to a shift in the plasma column. The magnitude and sign of the shift depends on poloidal location and viewing angle. In the plot on the left, the RDISP was at \((105^\circ \text{ P}, 300^\circ \text{ T})\) and pointed at the magnetic axis. In the plot on the right, the RIDSP was at \((90^\circ \text{ P}, 300^\circ \text{ T})\) and pointed at the geometric axis.

Figure 4.1: Sawtooth ensemble of \(v_r\) fluct. Left: RIDSP at \((105^\circ \text{ P}, 300^\circ \text{ T})\) and pointed at the magnetic axis. Right: RIDSP at \((90^\circ \text{ P}, 300^\circ \text{ T})\) and pointed at the geometric axis.

Measured RMS fluctuations of \(v_r\) are large. For all radii, fluctuations are between 2 and 4 km/s, which is larger than the change in the mean \(v_r\) during a crash. At the crash there is a small increase in the fluctuation level, see Figure 4.2. The total level is slightly lower for edge most locations. However, the edge most locations have more fluctuation power at higher frequencies.

The higher resolution spectrometer, IDSII, allows for the measurement of ion temperature. The temperature is measured from the line width of the He\textsc{II} line, Figure 4.3. Measurements of deuterium are not possible because fully stripped deuterium does not produce line radiation. For this ensemble, \(T_i\) is about 25 eV and increases by about 5 eV
(or 20%) within 200 µs at the crash. The radial dependence of $T_i$ initially increases with depth but then decreases. This behavior is likely a probe effect. As a probe is inserted deeper, the bulk plasma temperature is reduced as particles are lost to the probe.

### 4.2 Mach probe measurements

The Mach probe was used to measure poloidal and toroidal ion flow velocity; $v_\theta$ and $v_\phi$ respectively. The mean flow behavior through a sawtooth crash is shown in Figure 4.4. The poloidal flow $v_\theta$ increases in the outboard up direction during the crash. There is a shear in the $v_\phi$ profile. During the sawtooth crash the toroidal flow rapidly goes to zero across all measured radii.

These observations can be put in a broader picture if one considers the flow behavior
Figure 4.3: Sawtooth ensemble of $T_i$ measured from broadening of the 4686 Å He\textsc{ii} line. Average $T_i$ increases by about 20% within 200\,$\mu$s at the crash.

in the core. It has been established (partly due to this work) that the parallel momentum profile flattens during a sawtooth crash. That is to say that the flow decreases in the core where the parallel direction is primarily toroidal, and it speeds up in the edge where the parallel direction is primarily poloidal. A. Kuritsyn discusses this behavior[43] from the viewpoint of a momentum relaxation theory put forth by C. C. Hegna[34].

Measured RMS fluctuations of $v_\theta$ and $v_\phi$ are comparable to each other and several times larger than RMS $v_r$ fluctuations. Fluctuations decrease with depth; leveling out at about 10\,km/s at the deepest measurement locations. Figure 4.5 shows the temporal and frequency dependance of $v_\theta$ fluctuations. Figure 4.6 shows the temporal and frequency dependance of $v_\phi$ fluctuations. At the crash, fluctuation levels increase to 13–15\,km/s for both directions and all radii. In contrast to the $v_r$ frequency dependance, both $v_\theta$ and $v_\phi$ fluctuation power decreases with depth for all frequencies resolved.
Figure 4.4: Left: Sawtooth ensemble of $v_\theta$, which exhibits an increase in speed at the crash. Right: Sawtooth ensemble of $v_\phi$, which exhibits a flattening of the sheared perpendicular flow profile.

Figure 4.5: Left: RMS fluctuations of $v_\theta$ through a sawtooth crash. Right: frequency spectrum of $v_\theta$ fluctuation power.
4.3 TLP measurements

The TLP is used to measure plasma density $n_e$, electron temperature $T_e$, and floating potential $\Phi_f$. Figure 4.7 shows the sawtooth behavior of $n_e$, which exhibits a rapid increase followed by a rapid decrease. During a sawtooth crash, there is a decrease in confinement time, and particles are rapidly lost. Line average density decreases, so the increase in edge density is more than compensated for by a decrease in core density. The momentary increase in the edge density may be the result of less stochastic field lines in the edge. The topic of particle transport will be covered in greater detail in chapter 7.

The ensemble average of $T_e$ and $\Phi_f$ are plotted in Figure 4.8. Electron temperature exhibits a slight increase during the crash; increasing by 3–4 eV, or about 5%. This is opposite to the behavior in the core where the temperature is known to decrease at the crash by Thompson scattering measurements. The momentary increase in the edge temperature
may be the result of hotter core particles streaming along stochastic field lines to the edge region.

The floating potential will not be discussed in much detail. The inclusion of it in Figure 4.8 is to completely show probe measurements in these experiments. The gradient in \( \Phi_f \) implies a strong radial electric field of 500–1000 V/m.

A higher resolution radial scan was made with the helicity probe to measure radial profiles of \( n_e \) and \( T_e \). The scan consists of 3–4 shots per location and spans a depth of 1.0 to 11.0 cm, Figure 4.9. Quantities are averaged over the flattop, and the errors are the standard deviation of the average, which includes variation due to sawteeth. The three points clumped close together were actually taken at the same depth of 8 cm. Over the course of those 3 shots, \( T_e \) fell noticeably, but the variation remained within the error bars. The probe was scanned forward and backward to verify the profile shape. The decrease in \( T_e \) may be a probe effect, altering the bulk plasma temperature.
Figure 4.8: Left: Sawtooth ensemble of $T_e$, which exhibits a slight increase of 3–4 eV during the crash. Right: Sawtooth ensemble of $\Phi_f$. 
Figure 4.9: Radial profile of $n_e$ and $T_e$ measured with the Helicity probe. The three points clumped close together were actually taken at the same depth of 8 cm. The probe was scanned forward and backward to verify the profile shape. The decrease in $T_e$ may be the probe affecting bulk plasma temperature.
Chapter 5

Reconnection Flow Patterns

Magnetic reconnection is a process whereby magnetic energy is released during a change in magnetic topology and converted into plasma kinetic and thermal energy. Reconnection happens in plasmas all throughout the universe and is responsible for some spectacular phenomena. The aurora is one example. Magnetic field entrained in the solar wind, slams into our own Earth’s dipole magnetic field. The whole field structure gets compressed on the dayside and stretched out on the nightside. Reconnection in the magnetopause can accelerate particles causing beautiful displays of intense aurora.

Magnetic reconnection has also been suggested as a mechanism causing solar flares. Loops of magnetic field can get twisted up by the motion of their footpoints, which anchor each end of the loop to the sun’s surface. The more the field lines get twisted up, the more magnetic energy is stored in them. When a twisted up loop smashes into an oppositely directed flux loop, the field lines reconnect and snap back into a new magnetic configuration
5.1 Sweet-Parker and tearing reconnection

Sweet and Parker put forward the first quantitative description of reconnection in an attempt to explain solar phenomena\cite{52}. In the Sweet-Parker conception of reconnection, oppositely directed field lines get pushed together by flows and reconnect. It is a steady state two-dimensional description.

A similar picture has been established for $m = 1$ tearing reconnection in tokamaks. These modes have been studied extensively because they are thought to play a role in tokamak disruptions\cite{61}. In Figure 5.1, Bateman shows the standard picture of a $m = 1$ tearing mode. The magnetic field perturbation is drawn in Hamada coordinates, where the equilibrium field lines are straight.

This type of reconnection occurs due to the shear in the magnetic field. A typical tokamak $q$ profile increases with radius. Magnetic energy is released when previously nested flux surfaces reconnect and form a new magnetic topology—with magnetic islands. Field lines reconnect at the $X$-point and form magnetic islands at the $O$-point. The energy released both heats the plasma and drives flows. The flows associated with this reconnection are also depicted in Figure 5.1. They appear as counter rotating vortices that are periodic in $\phi$. Plasma flows radially into the $X$-point from both sides and out towards the $O$-point in the $\phi$ direction. Core resonant $m = 1$ tearing reconnection also occurs in the RFP. Previous work has investigated $m = 1$ flows using a CHERS diagnostic and found the
poloidal tearing mode flows to be localized to the resonant surface[25, 26].

Figure 5.1: Standard picture of tearing reconnection taken from Bateman[5], pg. 199. Depicted are the magnetic field, current, and flow perturbations after the mean fields have been subtracted out.

In this work, I have studied $m = 0$ reconnection in the edge of the MST RFP. The picture of this reconnection is neither the same as Figure 5.1, nor can it be thought of in terms of a Sweet-Parker steady state two-dimensional slab picture. Reconnection in MST takes place in a three-dimensional toroidal plasma, and, therefore, important geometrical effects must be considered. It also occurs impulsively and in bursts; not steady state. In MST, there is a highly conducting wall nearby on one side but not the other, making the reconnection geometry asymmetric. There is a gradient in the equilibrium current profile, which also makes the geometry asymmetric.

An excellent illustration of what one such mode may look like was presented in [51] and is depicted in Figure 5.2. The illustration is of a cylindrical mode, but one can imagine what the toroidal version would look like by bending Figure 5.2 around to connect the ends.
One can see that the structure (in the cylindrical approximation) is poloidally symmetric, and the $X$-point should be pictured as a $X$-line that traverses the poloidal circumference. Because there are so many differences between Sweet-Parker reconnection and $m = 0$ reconnection in MST, comparisons are better made with computer simulations.

Figure 5.2: Picture of a cylindrical $n = 1$ tearing mode resonant at the reversal surface in a RFP, courtesy of [51].

5.2 Simulations of $m = 0$ reconnection in MST

Two codes were used to simulate MST plasmas. NIMROD[58] with single-fluid resistive MHD equations was used to predict the linear mode structure in a toroidal geometry, and DEBS[57] was used to get the nonlinear resistive MHD evolution in a cylindrical geometry. Nonlinear NIMROD simulations are currently underway for a two-fluid physics model[private communication with J. King]. This is ultimately what it needed because both this work and T. D. Tharp’s work have shown that two-fluid effects play an important role.
Toroidal NIMROD simulations are described in [51]. The authors started by considering a force free cylindrical equilibrium, or screw pinch. With the widely used $\alpha$-model for the equilibrium current profile, the edge modes are stable. To make the equilibrium configuration unstable, the current profile was slightly modified. In particular, while maintaining a force free current profile, the current gradient was steepened a little, making $n = 1, 2, 3$ modes unstable. This current profile was then given to NIMEQ, which solves the Grad-Safranov equation. A linear version of the NIMROD MHD code was run in a toroidal geometry to evolve the modes. A growing $n = 1$ mode was picked out at a time slice chosen after the mode had grown sufficiently above the noise level, and the amplitude was normalized such that the magnetic toroidal component was a few percent of the equilibrium value, which was inspired by measurements in MST.

The NIMROD results for the magnetic field and flow are plotted in Figure 5.3. The magnetic profiles are in good agreement with experimental measurement[66]. It is interesting to note that the $X$-point is shifted inward slightly, and the $O$-point is shifted outward slightly. The flow structure is different than that of Figure 5.1. Instead of counter rotating vortices, one flow vortex is observed. The flow streams through the $X$-point, turns around on the other side of the reversal surface, and returns through the $O$-point.

DEBS simulations were carried out by J. A. Reusch. The DEBS code uses a semi-implicit algorithm that solves the single fluid, nonlinear, three-dimensional, resistive MHD equations in a cylindrical geometry. It is periodic in both $\theta$ and $z$ and was run at high spectral resolution; 9 poloidal and 342 axial modes with a radial resolution of 160. These
modes are allowed to non-linearly interact. The simulation was performed at a high Lundquist number \((S = 3.8 \times 10^6)\) matching 400 kA standard discharges in MST. However, it was also run at zero \(\beta\), and, therefore, pressure driven effects are absent. The boundary condition was that of a perfect conductor, which is an approximation to MST’s highly conducting shell.

Despite the simplifications to the model, the simulation (which ran for 6 months!) reproduces many features observed in the MST experiment. One such feature is the saw-tooth behavior. “Sawtooth-like” phenomena was seen previously in lower resolution DEBS simulations of MST\([24, 23]\), but at higher resolution and higher Lundquist number, the sawteeth appearing out of DEBS sharpen and the period between the bursts lengthen to closely match that seen in MST. Hence, this simulation provides a good comparison for measured velocity fluctuations and tearing mode dynamics.

The \(m = 0\) spectral components were picked out of the DEBS output for a single time
slice at a sawtooth crash. There is some variability to the sawteeth, and it is better to perform an ensemble average of the sawteeth, not unlike what is done for MST experiments. Ensembles were not available for this work. DEBS normalizes the magnetic field to the magnetic field on axis and velocities to $v_A$ on axis. As mentioned, this simulation was run at a Lundquist number for 400 kA discharges. However, for comparisons to this work, the spectral components were scaled to 220 kA parameters. The modes from DEBS are a little larger than observed in experiment, so they were all scaled ($B$ and $v$) by the same factor to make the DEBS $B_\phi$ at the edge match the experimental value. For reference, the experimental $B_\phi$ mode amplitudes are shown in Figure 5.4

![Figure 5.4: Experimental $B_\phi$ mode amplitudes. The $n = 1$ mode was used to scale the DEBS output.](image)

The DEBS results for the magnetic field and flow are plotted in Figure 5.5. Contours of toroidal flux including the $n = 0$ component are shown on the left and flow components are shown on the right. It should be mentioned that T. D. Tharp compared magnetic measurements to a different DEBS simulation[66], but he did not look at the flow structure.
The \( m = 0 \) components produce a coherent structure in the edge. When higher \( n \) modes are included, the \( O \)-point is compressed, which was also seen in Tharp’s measurements. To see this, compare the top plot in Figure 5.5 of just the \( n = 1 \) component with the bottom two plots. Modes \( n = 1, 2, 3, 4 \) is shown on the bottom left, and \( n \) modes up through 200 is shown on the bottom right. The flow structure looks very similar to the NIMROD results. There is one flow vortex which is inward through the \( X \)-point and returns outward through the \( O \)-point. Somewhat differently than NIMROD, no noticeable shift of the \( X \)-point relative to the \( O \)-point is observed. The amplitude and phase profiles for \( \tilde{v}_r \), \( \tilde{v}_\phi \), and \( \tilde{B}_\phi \) calculated by DEBS are plotted versus radius in Figures 5.6, 5.7, and 5.8 respectively.

### 5.3 Measured flow patterns

Flows measured by the RIDSP and Mach probe were correlated with the toroidal array, and pseudopsectral techniques of section 3.3 were applied to extract \( \tilde{v}_r \) and \( \tilde{v}_\phi \). This was done in an effort to measure a coherent flow structure associated with \( m = 0 \) tearing reconnection in the edge of MST. The extracted flow field is shown in Figure 5.9 for the \( n = 1 \) and \( n = 1, 2, 3, 4 \).

In some ways, Figure 5.9 is similar to the prediction of NIMROD and DEBS. The radial flow is inward at the \( X \)-point and returns outward at the \( O \)-point. However, there are some dissimilarities as well. Even though the experimentally measured \( \tilde{v}_\phi \) is about twice as large as \( \tilde{v}_r \), it is hardly noticeable due to the aspect ratio of the plot. The \( y \)-axis is the toroidal direction spanning 10 m, while the \( x \)-axis is the radial direction spanning only
Figure 5.5: Nonlinear DEBS simulation in a doubly periodic cylindrical geometry. Contours of toroidal flux and plasma flow fields are shown. Top: $n = 1$ mode only. Bottom left: modes $1 < n \leq 4$ added together. Bottom right: inclusion of high mode numbers, $n \leq 200$. 
Figure 5.6: Amplitude and phase profiles of $\tilde{v}_r$ at a sawtooth crash predicted from DEBS simulation. Left: Profiles of the $m = 0, n = 1, 2, 3, 4$ $\tilde{v}_r$ amplitudes. Right: Profiles of the $m = 0, n = 1, 2, 3, 4$ $\tilde{v}_r$ phases.

Figure 5.7: Amplitude and phase profiles of $\tilde{v}_\phi$ at a sawtooth crash predicted from DEBS simulation. Left: Profiles of the $m = 0, n = 1, 2, 3, 4$ $\tilde{v}_\phi$ amplitudes. Right: Profiles of the $m = 0, n = 1, 2, 3, 4$ $\tilde{v}_\phi$ phases.
Figure 5.8: Amplitude and phase profiles of $\tilde{B}_\phi$ at a sawtooth crash predicted from DEBS simulation. Left: Profiles of the $m = 0, n = 1, 2, 3, 4 \tilde{B}_\phi$ amplitudes. Right: Profiles of the $m = 0, n = 1, 2, 3, 4 \tilde{B}_\phi$ phases.

Figure 5.9: Measurements of the reconnection flow pattern correlated to the magnetic modes. The $X$-point occurs at about 0 m on the $y$-axis. The reversal surface is at a radial location of 0.43 m. Left: the $n = 1$ component. Right: $n = 1–4$ components.
10 cm. Simulations predict about a 10 times higher ratio $\tilde{v}_\phi/\tilde{v}_r$ for $n = 1$. The difference between theory and experimental analyses warrants a closer examination.

DEBS predicts the ratio of the $n = 1 \tilde{v}_\phi$ to $\tilde{v}_r$ to be about 17. This can be seen by comparing Figures 5.6 and 5.7. A ratio of about 17 also follows from an incompressibility argument. Tearing modes are generally considered to be incompressible[32]. The equation describing an incompressible flow is

$$\nabla \cdot \mathbf{v} = 0.$$  \hspace{1cm} (5.1)

For $m = 0$ modes in a periodic cylinder, we assume perturbations of the form

$$\tilde{v}_r = v_r_n(r) \exp[i(k_r r + k_z z)],$$

$$\tilde{v}_\theta = v_\theta_n(r) \exp[i(k_r r + k_z z)], \text{ and}$$

$$\tilde{v}_z = v_z_n(r) \exp[i(k_r r + k_z z)].$$  \hspace{1cm} (5.2)

Substituting (5.2) into (5.1) leads to

$$ik_z \tilde{v}_z + \frac{\partial \tilde{v}_r}{\partial r} + \frac{\tilde{v}_r}{r} = 0.$$  \hspace{1cm} (5.3)

At the edge of MST, the last term on the left hand side can be taken as small compared to the previous term, which we approximate as

$$\frac{\partial \tilde{v}_r}{\partial r} \approx \frac{\tilde{v}_r}{d_{rev}},$$  \hspace{1cm} (5.4)

were $d_{rev}$ is the depth to the reversal surface. When the approximation (5.4) is substituted into (5.3) and we take $n = 1$, we find that

$$v_{z1}/v_{r1} \sim R_0/nd_{rev} \approx 17.$$  \hspace{1cm} (5.5)
When the ratio $\tilde{v}_r/\tilde{v}_\phi$ is modified to reflect this, Figure 5.10 is attained. Here, the flow vortices are more clearly seen. A compression of the flow structure is also observed when more modes are added together, as seen on the right.

![Diagram showing flow patterns](image)

Figure 5.10: Measured reconnection flow pattern of Figure 5.9 with the ratio $\tilde{v}_\phi/\tilde{v}_r$ amplified by 10. The X-point occurs at about 0 m on the y-axis. The reversal surface is at a radial location of 0.43 m. Left: the $n = 1$ component. Right: $n = 1$–4 components.

Considerations concerning the relative flow fluctuation amplitudes begs the question; which component if either, $\tilde{v}_r$ or $\tilde{v}_\phi$, is most accurate? DEBS simulations are ill suited to directly compare flow fluctuation amplitudes because the numerical viscosity is a couple orders of magnitude higher than it is estimated to be in experiment. The numerical viscosity is set to stabilize the code, but it probably has the effect of damping flows at all scales. In comparison to experimental measurements, the velocity components from DEBS are smaller by at least a factor of 10 or more. However, one expects the relative amplitudes and the flow structure to be about right.

It is suspected that the measured $\tilde{v}_{r1}$ is too big. Radial components are notoriously hard
to measure because they are both small and depend on the alignment of the diagnostic with a flux surface. The magnetic axis shifts outward in $R$ during a crash. This means that diagnostics are not always looking at the magnetic axis, and that flux surfaces are not concentric circles. Hence, there is often a small amount of poloidal leakage into a radial measurement. When the poloidal component is much larger than the radial (especially true for magnetic field), then the poloidal pickup in a radial measurement can be significant. In Tharp’s work, they ended up assuming $\nabla \cdot \mathbf{B} = 0$ (a solid assumption) in order to get $\tilde{B}_r$[66].

The $\tilde{v}_r$ mode amplitude and phase profiles for the $n = 1$ are plotted in Figure 5.11. The amplitude profile is qualitatively similar in shape to the DEBS profile in Figure 5.6. It reaches a peak amplitude of 2.8 km/s, or about 0.3% $v_A$. The $\tilde{v}_\phi$ mode amplitude and phase profiles for the $n = 1$ are plotted in Figure 5.12. The $\tilde{v}_{\phi 1}$ amplitude reaches a maximum of 8 km/s, or about 1% $v_A$.

![Figure 5.11](image-url)

Figure 5.11: Left: profile of the $n = 1$ pseuduspectral $\tilde{v}_r$ amplitude. Right: profile of the $n = 1$ pseuduspectral $\tilde{v}_r$ phase.
DEBS predicts that the phase difference between $\tilde{v}_r$ and $\tilde{v}_\phi$ to be $\pi/2$ in the edge, Figures 5.6 and 5.7. The reversal surface is at roughly 0.42 m, and a few centimeters deeper, at about 0.38 m, $\tilde{v}_{\phi_1}$ changes sign. This is evident by the $\tilde{v}_{\phi_1}$ amplitude going to zero where the phase flips by about $\pi$. The phase difference between $\tilde{v}_{r_1}$ and $\tilde{v}_{\phi_1}$ is then $-1.15$ rad, close to $-\pi/2$.

The measured phase of $\tilde{v}_r$ is fairly flat across the reversal surface, which is at a depth of about 9 cm. This is in fairly good agreement with DEBS. The phase of $\tilde{v}_\phi$ does change with depth by about 2 rad; less than in DEBS. Clearly, the profiles do not exactly match the simulation. The differences in the amplitudes and phases of the modes give rise to the more skewed flow structure most clearly seen in Figure 5.10.
5.4 Discussion

The flow structure associated with $m = 0$ tearing modes resonant at the edge has been measured and is qualitatively similar to predictions from simulations and analytical results. A robust result is that the flow pattern is unlike the classical Sweet-Parker picture of Figure 5.1. Whereas the Sweet-Parker picture is symmetric on either side of the reconnection layer, the tearing modes in a cylinder are asymmetric in several ways. First, they are characterized by a non-zero current gradient at the resonant surface. In particular, the logarithmic derivative $d \ln \lambda / dr$ of the equilibrium current profile, $\lambda = j_{||} / B$, gives rise to an asymmetry in the radial flow, making $\tilde{v}_r$ non-zero at the resonant surface. This effect is strongly emphasized for the edge-resonant $m = 0$ modes. Another important asymmetry is that there is a highly conducting boundary condition on the outside of the reversal surface and the cylinder axis boundary condition on the inside. These asymmetries lead to the flow structure that is measured in MST and consistent with the analytical and computational predictions. Namely, one flow vortex that is radially inward through the $X$-point and outward through the $O$-point.

There are also differences between predicted and measured flows. In particular, the ratio $\tilde{v}_\phi / \tilde{v}_r$ is larger by about a factor of 10 compared to simulations. Measuring these flow structures is difficult to do. In particular, it is difficult to get a good measurement of the phases. Because these correlations are done over ensembles of event, variability between sawteeth acts to smooth the phase relationships. It remains to be seen if these differences hold with repeated experiments. A non-perturbative method for measuring the flows in
the edge (e.g. toroidal CHERS) would provide the best confirmation of this work. Large ensembles with very narrow selection criteria should be performed. Additional simulations using nonlinear NIMROD in toroidal geometry including two fluid effects are currently underway. A similar analysis with that data will provide further insight.
Chapter 6

Plasma Stresses

In MST standard discharges, the plasma spontaneously rotates without the application of external torques. During a crash, the core rotation comes to an abrupt halt and the momentum is rapidly transported. This can be seen, for instance, in the core mode rotation velocity measured by the toroidal array and shown in Figure 1.5. The poloidal flow at the edge, on the other hand, speeds up, Figure 4.4. Hence, the parallel momentum profile flattens at the crash[43]. That is to say, the parallel flow decreases in the core and increases in the edge. This flattening indicates a relaxation of the parallel momentum profile.

This work has contributed to our understanding of momentum transport processes by measuring the Reynolds stress associated with turbulent flow and the inertia in the edge of MST and comparing that to measurements of the Maxwell stress. It is observed that the parallel momentum balance is dominated by turbulent stresses, Maxwell and Reynolds, both of which become an order of magnitude larger than the rate of change in
inertia. However, they are oppositely directed such that their difference is comparable to
the magnitude of the inertial term. A detailed balance of all the terms in the momentum
equation awaits more precise measurements.

This chapter begins by showing how turbulent fluctuations can give rise to mean field
effects. With some simplifying assumptions, the stresses are made measurable by probes.
Then the measured stresses are presented and compared to the rate of change in inertia.
Unlike chapter 5, which presented pseudospectral components of the flow fluctuations, this
chapter uses the total locally measured flow fluctuations to calculate the turbulent stresses.

6.1 The momentum equation

The magnetohydrodynamic (MHD) force balance equation is given by

\[ \frac{\rho}{D} \frac{Dv}{Dt} = \rho \left( \frac{\partial v}{\partial t} + (v \cdot \nabla) v \right) = j \times B - \nabla p. \]  

(6.1)

The convective derivative \( D/Dt \) is a derivative taken with respect to a coordinate system
moving with velocity \( v \). It is the derivative of a quantity as it follows the reference frame
of the moving fluid. The gravitational force term \( \rho g \) and the viscosity term \( \rho \nu \nabla^2 v \) have
been left out because they are estimated to be small.

One assumes that \( v, j, \) and \( B \) can be decomposed into a flux surface average part and
a non-axisymmetric fluctuating part:

\[ \mathbf{v} = \langle \mathbf{v} \rangle + \tilde{\mathbf{v}} , \quad \langle \tilde{\mathbf{v}} \rangle = 0 , \]

\[ \mathbf{j} = \langle \mathbf{j} \rangle + \tilde{\mathbf{j}} , \quad \langle \tilde{\mathbf{j}} \rangle = 0 , \]

\[ \mathbf{B} = \langle \mathbf{B} \rangle + \tilde{\mathbf{B}} , \quad \langle \tilde{\mathbf{B}} \rangle = 0 . \]

(6.2)

After substituting (6.2) into (6.1), taking the parallel component, and performing a flux surface average, we arrive at

\[ \rho \frac{\partial \langle \mathbf{v} \rangle \|}{\partial t} = -\rho \langle (\tilde{\mathbf{v}} \cdot \nabla) \tilde{\mathbf{v}} \rangle \| + \langle \tilde{\mathbf{j}} \times \tilde{\mathbf{B}} \rangle \| . \]

(6.3)

Products of fluctuating quantities can give rise to a mean field effect. These so called turbulent stresses are referred to as the Reynolds stress and the Maxwells stress respectively.

The equilibrium pressure gradient is expected to be small in the poloidal and toroidal directions because it is in surface, and so that term has has been left out of (6.3). It should be emphasized, however, that the pressure term can lead to a radial transport of momentum through correlation of pressure fluctuations and magnetic fluctuations[6], but its evaluation is beyond this work, where we measure the importance of the turbulent Reynolds and Maxwell stresses by comparing them to the rate of change in inertia.

### 6.1.1 Simplifying assumptions

The terms on the right hand side of (6.3) are not experimentally tractable. They require simultaneous measurements of all three components of \( \mathbf{v} \) and \( \mathbf{B} \) at several locations so as to get their fluctuations and the spatial derivatives of the fluctuations. However, these
stresses can be written in a more manageable form by utilizing $\nabla \cdot B = 0$ and assuming incompressibility, $\nabla \cdot v = 0$. The incompressibility assumption is often used with tearing modes. The poloidal and toroidal components of the stresses may now be written as

$$-\rho \langle (\tilde{v} \cdot \nabla) \tilde{v} \rangle_\theta = -\rho \left( \frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{v}_\theta \rangle + \frac{2}{r} \langle \tilde{v}_r \tilde{v}_\theta \rangle \right) ; \quad (6.4)$$

$$-\rho \langle (\tilde{v} \cdot \nabla) \tilde{v} \rangle_\phi = -\rho \left( \frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{v}_\phi \rangle + \frac{1}{r} \langle \tilde{v}_r \tilde{v}_\phi \rangle \right) ; \quad (6.5)$$

$$\langle \tilde{j} \times \tilde{B} \rangle_\theta = \frac{1}{\mu_0} \left( \frac{\partial}{\partial r} \langle \tilde{B}_r \tilde{B}_\theta \rangle + \frac{2}{r} \langle \tilde{B}_r \tilde{B}_\theta \rangle \right) ; \quad \text{and} \quad (6.6)$$

$$\langle \tilde{j} \times \tilde{B} \rangle_\phi = \frac{1}{\mu_0} \left( \frac{\partial}{\partial r} \langle \tilde{B}_r \tilde{B}_\phi \rangle + \frac{1}{r} \langle \tilde{B}_r \tilde{B}_\phi \rangle \right) . \quad (6.7)$$

With the turbulent stresses written this way, the radial derivatives appear outside the averaging operator. The correlations of flow and magnetic field fluctuations were calculated and averaged over hundreds of sawtooth events at each of five radial locations, and the radial derivatives were then numerically performed after ensemble averaging the fluctuation products.

### 6.2 Turbulent stress measurements and force balance

Insertable probes were used to measure all three components of the flow and magnetic field in the edge region of MST and have provided a means of evaluating the force balance governed by (6.3). The flows were measured with the RIDSP and the Mach probe, and the Reynolds stress was then calculated by correlating measurements of flow fluctuations as described in section 6.1.1. The Maxwell stress was measured by A. Kuritsyn using a magnetic probe equipped with six pickup coil triplets[44].
The left hand side of (6.3) is the rate of change in plasma inertia. The poloidal and toroidal ensemble averaged flows are shown in Figure 4.4. The edge density was not simultaneously measured, so it was assumed to be constant at $0.5 \times 10^{19} \text{ m}^3$. Figure 6.1 shows the rate of change in plasma inertia $\rho \partial \langle v_\theta \rangle / \partial t$ and $\rho \partial \langle v_\phi \rangle / \partial t$. For both components, it is relatively small; getting as large in magnitude as $1 \text{ N/m}^3$.

The Reynolds stress was measured to be much larger than the rate of change of plasma momentum, Figure 6.2. The poloidal Reynolds stress is largest at the innermost radius; reaching about $15 \text{ N/m}^3$ in the outboard down direction. The toroidal Reynolds stress is also large but does not show significant change through the crash. In general, the stress varies from positive to negative with depth.

The Maxwell stress was measured to be equally large and opposite in direction to the Reynolds stress, Figure 6.3. The left plot shows the time evolution of all three terms near
the reversal surface. The rate of change in inertia (green line) is very small by comparison to the Reynolds stress and Maxwell stress, meaning that the turbulent stresses dominate the momentum dynamics. Profiles of the two stresses at the crash are plotted on the right, where the Maxwell stress has been plotted with the opposite sign to show that they are in near balance over the radial extent of the measurements.

### 6.3 Discussion

A strong radial transport of momentum is coincident with the sawtooth crash when tearing mode amplitudes are large. Tearing instabilities have also been shown to drive transport through theoretical work and simulation of RFPs. A single tearing mode can transport momentum faster than predicted from Coulomb collisions, but multiple nonlinearly interacting modes greatly enhance the transport [24, 23]. Furthermore, this enhancement seems
to depend on the presence of edge resonant $m = 0$ modes because when crashes occur without the $m = 0$ edge modes growing to large amplitude, little momentum transport is observed\cite{54, 43}. Hence, the dynamics occurring at the edge are important to understand.

This work advances the understanding of momentum transport mechanisms by measuring the Reynolds stress associated with turbulent flow and the inertia in the edge of MST and comparing that to measurements of the Maxwell stress. The surprising result is that the parallel momentum balance is dominated by turbulent stresses, Maxwell and Reynolds, both of which become an order of magnitude larger than the rate of change in inertia. However, they are oppositely directed such that their difference is comparable to the magnitude of the inertial term.

A new picture for magnetic self-organization in the RFP thus emerges; one in which
two-fluid effects are significant. The momentum dynamics are inextricably coupled with the current dynamics through the relationship between the Maxwell stress and the Hall dynamo, a term that enters into a two-fluid generalization of Ohm’s law. It is known that in the MST, a simple Ohm’s law is not satisfied ($E \neq \eta S J$) over most of the plasma volume[9, 3] and that MHD and Hall dynamos are present and large such that Ohm’s law is everywhere satisfied[17, 19, 20]. The parallel current density profile $J_{||}(r)$ flattens at the crash and has been explained in terms of a relaxation towards a Taylor state[64]. A large Hall Dynamo implies a large Maxwell stress, and indeed, the parallel momentum profile also exhibits a relaxation at the crash; a phenomena predicted by an analytic two-fluid theory[34]. Our growing appreciation for the role that two-fluid physics plays in the RFP is increasing. This work indicates a generalization of the relaxation picture, and that the nonlinear MHD model studied with DEBS needs to be extended to two-fluid. This extension is now underway using NIMROD.
Chapter 7

Particle Transport

The goal of a fusion reactor is to create and confine a hot dense fusing plasma. Magnetically confined plasmas have been plagued by particle and energy transport that is much higher than classical expectations. This anomalous transport is usually attributed to turbulent fluctuations arising from instabilities[7]. Many mechanisms can lead to such fluctuation induced transport and much research has gone into understanding and mitigating it.

In MST, the density profile flattens during a sawtooth crash much faster than classical expectations. This can be seen, for instance, from a far-infrared (FIR) interferometer measurement of the density profile through a sawtooth crash as shown in Figure 7.1 (plot courtesy of W. X. Ding et al.[21]). The density redistributes within $\sim 100–200 \mu s$, which is 100 times faster than a classical collision time of 10 ms. A rough estimate of the particle transport can be made by calculating the total particle loss from the change in density and
assuming that those particles leave through a cylindrical surface at radius $a$.

$$\Gamma_{est} \approx \frac{\Delta n_e r}{\Delta t} \implies 1.3 \times 10^{21} \text{m}^{-2}\text{s}^{-1}$$

(7.1)

Generally speaking, a radial flux of particles is given by

$$\Gamma = n_e v_r.$$ 

(7.2)

The density and radial velocity can be written as a sum of the flux surface average and the non-axisymmetric spatial fluctuations about that average.

$$n_e = \langle n_e \rangle + \tilde{n}_e$$ 

(7.3)

$$v_r = \langle v_r \rangle + \tilde{v}_r$$

(7.4)

Substituting (7.3) and (7.4) into (7.2) yields four terms.

$$\Gamma = \langle n_e \rangle \langle v_r \rangle + \langle n_e \rangle \tilde{v}_r + \tilde{n}_e \langle v_r \rangle + \tilde{n}_e \tilde{v}_r$$

(7.5)
However, not all terms in (7.5) add to a net transport of particles. By definition, the average of a fluctuation is zero. Therefore, when one does a flux surface average of (7.5), only the first and last terms survive.

\[ \langle \Gamma \rangle = \langle n_e \rangle \langle v_r \rangle + \langle \tilde{n}_e \tilde{v}_r \rangle \quad (7.6) \]

Many previous measurements of particle and energy transport have been made on MST both in the core and in the edge. These experiments have utilized different measurement techniques spanning a wide range of plasma conditions and have greatly increased our understanding of transport in the RFP.

In the core, global magnetic fluctuations principally drive particle and energy transport. This has been shown by probe measurements\cite{62, 29} and by FIR interferometer measurements\cite{21}. Magnetic tearing mode fluctuations can be greatly reduced (at least transiently) during a discharge with pulsed poloidal current drive (PPCD). When this is done, transport is also greatly reduced\cite{45, 46, 47}. Electrostatic fluctuations, on the other hand, drive very little transport in the core\cite{49}.

In the edge, it seems the roles are reversed with electrostatic fluctuations driving particle transport\cite{56, 42}. Interestingly, it was also found that those electrostatic fluctuations did not drive energy transport.

Flow shear seems to play an important role. When the plasma is biased in the edge (inducing sheared flows), electrostatic fluctuations decrease and the particle confinement time increases\cite{15}. Flow shears also develop during enhanced confinement and PPCD discharges\cite{8} where confinement times are also improved. The energy confinement time
only increases when magnetic fluctuations are suppressed, which further corroborates the idea that electrostatic fluctuations only drive particle transport.

This work also lends evidence to the importance of flow shear in minimizing transport. It is shown in section 7.2 that particle transport surges at a sawtooth crash and is much lower between crashes. As can be seen from Figure 4.4, high transport coincides with periods of small flow shear, and low transport coincides with periods of large flow shear.

Recall from section 3.3 that tearing mode amplitudes are large at sawtooth crashes and the modes phase lock together forming complex coherent magnetic island structures. This is seen in other RFPs such as RFX[68]. A theoretical explanation for the phase locking of tearing modes in the RFP based on electromagnetic torques was given by Fitzpatrick[30]. In RFX, the phase locked modes also lock to the wall creating a stationary helical perturbation whose outermost flux surface intersects the wall and generates a “hot spot”[69, 67]. A hot spot can lead to an influx of impurities and the eventual termination of the plasma. RFX has mitigated the overheating problem by forcing the perturbation to rotate[10].

There is strong evidence that tearing modes drive transport. Since these modes phase lock together, one might expect that the spatial variation of particle transport to be influenced by the tearing mode structure. This work investigates this spatial variation by using advanced correlation techniques (pseudospectral analysis). This work also presents a new way of measuring transport by correlating radial velocity fluctuations measured spectroscopically by the RIDSP with density fluctuations measured by a TLP method. Because the velocity measurements are spectroscopic, all fluctuations are included regardless of the
mechanism for their origin. They could be electrostatic or magnetic. Furthermore, a local edge measurement resolving the temporal sawtooth behavior is shown for the first time.

7.1 Equilibrium particle transport

The first term in (7.6) is sometimes referred to as the pinch term because it arises from an inward equilibrium radial flow. Unfortunately, the RIDSP was only calibrated for velocity fluctuations and did not measure $v_{\rho0}$. In MST, equilibrium $E$ and $B$ give rise to a pinch velocity via the $E \times B$ drift. Hence, the flux surface averaged $\langle v_r \rangle$ is estimated as $\langle E \rangle \times \langle B \rangle / \langle B \rangle^2$ and will be denoted $\langle v_{\text{pinch}} \rangle$ to distinguish it from RIDSP measurements of $\tilde{v}_r$.

The poloidal and toroidal electric fields at the wall are known by measured poloidal and toroidal loop voltages. The poloidal loop voltage is denoted $V_{tg}$ (tg standing for “toroidal gap”) and the toroidal loop voltage is denoted $V_{pg}$ (pg standing for “poloidal gap”). Plots of $V_{tg}$ and $V_{pg}$ for the ensemble used in this chapter are shown in Figure 7.2. The poloidal electric field at the wall $\langle E_\theta(a) \rangle$ is outboard up and is given by $-V_{tg}/2\pi a$. In general, the measured toroidal loop voltage depends on poloidal angle. However, due to MST’s highly conducting shell, the plasma surface is approximately a flux surface. Therefore, the toroidal loop voltage is nearly independent of poloidal angle. The probes were located at 90° poloidal, so the toroidal electric field at the wall $\langle E_\phi(a) \rangle$ is given by $-V_{pg}/2\pi R_0$. The electric field varies slightly with depth, Figure 7.3, and is calculated using Faraday’s Law
and helicity probe measurements of $B_\theta$ and $B_\phi$ using

$$
\langle E_\theta(r) \rangle = -\frac{V_{tg}}{2\pi r} + \frac{1}{r} \frac{\partial}{\partial t} \int_r^a \langle B_\phi(r') \rangle r' dr', \quad \text{and} \\
\langle E_\phi(r) \rangle = -\frac{V_{pg}}{2\pi R_0} - \frac{\partial}{\partial t} \int_r^a \langle B_\theta(r') \rangle dr'.
$$

(7.7) (7.8)

The pinch velocity $\langle v_{\text{pinch}} \rangle$ (as calculated from $V_{tg}$, $V_{pg}$, and probe measurements of $B$) is about $-20 \text{ m/s}$, Figure 7.4. The $V_{pg}$ component of $\langle v_{\text{pinch}} \rangle$ is dominant and does not change much through a sawtooth crash, Figure 7.5. The $V_{tg}$ component is quite dynamic, changing sign and increasing by over an order of magnitude during a crash, but it remains a factor of about 10 less than the $V_{pg}$ component. The overall trend of $\langle v_{\text{pinch}} \rangle$ through a sawtooth cycle is due the trend of $V_{pg}$ over the course of a typical discharge.

The axisymmetric component of the flux $\langle n_e \rangle \langle v_{\text{pinch}} \rangle$ is plotted in Figure 7.6. It is a relatively small inward flux of particles. This term is of order $10^{18} \text{ m}^{-2}\text{s}^{-1}$, which is only about 1 % of the fluctuation induced particle flux.
Figure 7.3: Left: Poloidal electric field $E_\theta$ calculated by (7.7). Right: Toroidal electric field $E_\phi$ calculated by (7.8).

Figure 7.4: The radial pinch velocity is calculated from signals $V_{pg}$, $V_{tg}$, and $B_\theta$ and $B_\phi$ from the helicity probe.
Figure 7.5: Components of $\langle v_{\text{pinch}} \rangle = \langle E \rangle \times \langle B \rangle / \langle B \rangle^2$ driven by $V_{p_g}$ (left) and $V_{t_g}$ (right).

Figure 7.6: The axisymmetric component of the particle flux $\langle n_e \rangle \langle v_{\text{pinch}} \rangle$. It is small compared to the fluctuation induced flux.
7.2 Fluctuation induced particle transport

The equilibrium transport discussed in section 7.1 is much too small to account for total particle balance in MST. The fluctuation induced transport, on the other hand, is measured to be two orders of magnitude larger. It arises from fluctuations in density and radial velocity, \( \langle \tilde{n}_e \tilde{v}_r \rangle \). Whereas previous measurements relied on a mechanism to infer \( \tilde{v}_r \) (for instance \( E \times B \)), it is now measured spectroscopically and the fluctuations in \( v_r \) can be either electrostatic or magnetic in origin.

In order to calculate \( \langle \tilde{n}_e \tilde{v}_r \rangle \) from measurements, the fluctuations must be extracted, which is done in two steps. First the discharge trend is removed by subtracting a 5 ms smooth from each signal. Secondly, the sawtooth behavior is removed from each signal by subtracting its ensemble average. Then the ensemble average is calculated again, this time averaging the product of the fluctuations to get \( \langle \tilde{n}_e \tilde{v}_r \rangle \). The resulting (flux surface averaged) fluctuation induced particle flux surges to \( 3 \times 10^{21} \text{m}^{-2} \text{s}^{-1} \) at the deepest insertion, see Figure 7.7, and agrees both with the rough estimate from (7.1) and previous measurements of transport in the edge[56, 55, 62, 63, 47, 45, 46, 48].

The dramatic increase of the particle flux at the sawtooth crash suggests that tearing mode instabilities, whose amplitudes also increase dramatically at the crash, are responsible for the flux increase. To investigate this dependence, pseudospectral techniques were employed to look at the spatial structure of the transport. When the fluctuations are written
Figure 7.7: Sawtooth ensemble fluctuation induced particle transport $\Gamma$. An ensemble is performed over density fluctuations multiplied by radial velocity fluctuations.

in spectral form, the radial particle flux becomes

$$n_e v_r = n_{e0} v_{r0} +$$

$$n_{e0} \sum_{n=1}^{\infty} v_{rn} \cos(m \theta + n \phi - \delta_{v,n}) +$$

$$v_{r0} \sum_{n=1}^{\infty} \frac{n_{e0}}{n} \cos(m \theta + n \phi - \delta_{n,e,n}) +$$

$$\sum_{n=1}^{\infty} \frac{n_{e0}}{n} \cos(m \theta + n \phi - \delta_{n,e,n}) \sum_{n=1}^{\infty} v_{rn} \cos(m \theta + n \phi - \delta_{v,n}).$$

(7.9)

The pseudospectrum was calculated for each signal, and the mode adding method of section 3.3 was used to sum modes $n = 1, 2, 3, 4$. The first term of (7.9), the axisymmetric term, was previously shown in Figure 7.6. Figure 7.8 shows plots of $n_{e0} \bar{v}_r$ 1.5 ms before and right at the sawtooth crash. It is plotted versus toroidal angle relative to the phase of the $n = 1$ mode. This is the largest term of the four. Between sawteeth, it is much smaller and shows little spatial structure. At the crash, the amplitude reaches $2 \times 10^{22}$ m$^{-2}$s$^{-1}$. The
flux is inward at the X-point and outward at the O-point as defined by the $n = 1$ mode. However, the net flux arising from this term is zero. Another way to understand this term is that constant density is convected with a fluctuating radial velocity which is inward at the X-point and outward at the O-point.

Figure 7.8: The $\tilde{n}_e \tilde{v}_r$ particle transport term 1.5 ms before the sawtooth crash (Left) and at the sawtooth crash (Right). Modes $n = 1, 2, 3, 4$ are included. It does not lead to net transport in the flux surface averaged sense.

Figure 7.9 shows plots of $\tilde{n}_e v_{r0}$ 1.5 ms before and right at the sawtooth crash. This term is the smallest of the four—being 3 orders of magnitude smaller than $n_{e0} \tilde{v}_r$. The spatial structure of $\tilde{n}_e v_{r0}$ is more pronounced during a crash. It is outward at the X-point and inward at the O-point as defined by the $n = 1$ mode (opposite that of $n_{e0} \tilde{v}_r$). In other words, the plasma density is lower at the X-point and higher at the O-point, and $\langle \nu_{\text{pinch}} \rangle$ is inward.

The last term in (7.9) is the second largest. This term was previously shown in Figure 7.7 plotted versus time from an ensemble average $\langle \tilde{n}_e \tilde{v}_r \rangle$. Figure 7.7 includes fluctuation power
from frequencies up to 100 kHz and should be thought of as the “total content” averaged over a flux surface. The sum of pseudospectral modes \( n = 1, 2, 3, 4 \) are plotted versus \( \phi \) (relative to the \( n = 1 \)) in Figure 7.10. The particle flux resulting from \( \tilde{n}_e \tilde{v}_r \) is everywhere radially outward but is large at the \( X \)-point and \( O \)-point of the \( n = 1 \) island. The spatial structure is complicated and generally increases with depth. It should be noted that the magnetic island structure is also complicated when one includes modes \( n = 1, 2, 3, 4 \)[66].

Finally, we can add together all terms in (7.9) to yield the \( n = 1, 2, 3, 4 \) mode resolved spatial structure of the particle transport, Figure 7.11. Again, between sawteeth, the transport is small and shows small spatial variation. At the sawtooth crash when the tearing mode amplitudes are large, the transport is large and shows a spatial structure that is inward at the \( X \)-point of the \( n = 1 \) and outward at the \( O \)-point.

Most of the total transport is correlated with \( n = 1, 2, 3, 4 \), see Figure 7.12. The
Figure 7.10: The \( \tilde{n}_e \tilde{v}_r \) particle transport term 1.5 ms before the sawtooth crash (Left) and at the sawtooth crash (Right). Modes \( n = 1, 2, 3, 4 \) are included. This is the dominate term that leads to a net transport.

Figure 7.11: Particle transport \( \Gamma \) 1.5 ms before the sawtooth crash (Left) and at the sawtooth crash (Right). All terms in (7.9) with modes \( n = 0, 1, 2, 3, 4 \) are included.
colored lines are the usual ensemble average as shown in Figure 7.7. The black lines are the corresponding integral of Figure 7.11 over $\phi$. At the crash, the transport from tearing accounts for most of the total fluctuation-induced transport. Between crashes, the transport from tearing is small, which agrees with previous measurements that identified electrostatic transport as dominant at that time.

![Graph showing particle flux](image)

Figure 7.12: Total fluctuation induced particle flux $\langle \Gamma \rangle$ calculated by usual ensemble methods (colored lines) and particle flux calculated from the pseudospectrum of $n_e$ and $v_r$ including $n$ modes 1–4 and integrated over $\phi$ (black lines).

### 7.3 The continuity equation

Particle transport and density are related to each other through the continuity equation, which is an expression of mass conservation.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \Gamma = S$$  \hspace{1cm} (7.10)
This seemingly simple equation belies its true complexity. The rate of change in density $dn_e/dt$ versus $t$ can be readily calculated from measurements of $n_e$ and is shown in Figure 7.13. The divergence of the fluctuation-induced particle flux is shown in Figure 7.14. But there is a lot of edge physics tied up in $S$, which includes sources and sinks. A measure of the electron source can be attained by measuring $D_\alpha$ light emission.

![Graph](image)

Figure 7.13: $dn_e/dt$ versus $t$ changes rapidly through a sawtooth crash.

Strictly speaking, a $D_\alpha$ photon is emitted when a bound electron is excited, not ionized. The emission $\gamma_{D_\alpha}$ depends on temperature (through $\langle \sigma v \rangle_{ex}$), electron density $n_e$, and neutral density $n_0$. That emission is given by

$$\gamma_{D_\alpha} = n_e n_0 \langle \sigma v \rangle_{ex}.$$  \hspace{1cm} (7.11)

However, in high temperature plasmas (in particular MST), the ratio of excitations to ionizations $\langle \sigma v \rangle_{ex}/\langle \sigma v \rangle_{ion}$ is nearly constant over a wide range of plasma parameters[48, 2, 41]. That ratio is 0.08–0.09 over the range of typical MST discharges[48]. The source due
to electron impact is proportional to the ionization rate,

\[ S_{ei} = n_e n_0 \langle \sigma v \rangle_{\text{ion}}, \]  

(7.12)

and can then be written as

\[ S_{ei} = \gamma_{D\alpha} \frac{\langle \sigma v \rangle_{\text{ion}}}{\langle \sigma v \rangle_{\text{ex}}}. \]  

(7.13)

MST is equipped with an array of detectors to measure \( D_\alpha \) emission. MSTFIT was used to reconstruct the source term at the probe location using an Abel inversion algorithm. However, the source measured in this way does not take into account helium sourcing because the measurement comes from \( D_\alpha \) emission. Since half of the plasma density is due to helium puffing, we multiply the source term from \( D_\alpha \) emission by two as a rough estimate. \( 2S_{ie} \rightarrow S_{ie} \)

Previous evaluations of (7.10) inferred one or more quantities from measurements of the others. N. E. Lanier used the FIR interferometer to measure the density profile \( D_\alpha \) detectors co-linear with the FIR chords. From this the particle flux was calculated[47, 46, 48]. W. X. Ding expanded on this by using the FIR system to measure both \( \tilde{n}_e \) and \( \tilde{B}_r \). The magnetic fluctuation-induced particle flux was found to balance \( dn_e/dt \) in the core where the source term is negligible[21]. An attempt was made in this work to independently measure all three terms and compare them.

The estimate of the source is plotted alongside \( dn_e/dt \) and \( \nabla \cdot \Gamma \) in Figure 7.14. One can immediately see that (7.10) does not seem to hold true. The change in density is small in comparison to the other terms. The strong negative gradient in the radial flux is not compensated for by \( dn_e/dt \). The source term is always positive and has a significant
contribution between sawteeth. However, this was a naïve comparison, and many other
effects must be taken into account.

Figure 7.14: Continuity equation terms plotted against each other. The change in density
is small in comparison and balance is not seen.

With regards to the flux term, only the fluctuation-induced particle flux was measured
for frequencies up to 100 kHz. Higher frequency fluctuations maybe playing a role. In the
RFX experiment, frequency resolved particle flux was mostly concentrated in the range
30–250 kHz[4]. One might also consider contributions to a mean flux other than the pinch
term, which was found to be small. Particle diffusion, for instance, would manifest as a
skew in the velocity distribution and may not add to a fluctuation-induced flux.

Possible particle sinks have also not been taken into account. These effects are particu-
larly important in the edge. Recombination is expected to be small compared to ionization,
but temperatures are cooler in the edge and, furthermore, temperatures near the probe may
be cooler than the bulk plasma. Particles can also be lost directly to the plasma limiter
or the wall. Gyro-orbit effects play a role in the boundary of limited plasmas. It was also
mentioned that tearing modes change the magnetic topology. If these distorted field lines intersect the wall, particles streaming along them will be promptly lost. Here, again, RFX has done some studies\cite{69, 67}.

### 7.4 Energy transport

Particles carry thermal energy with them. Hence, particle transport is associated with a convective energy transport, or heat flux $q$. Only the radial component survives a flux surface average, so we will use $q$ to denote $q_r$. The heat flux can be written as the product of density, temperature, and radial velocity,

$$ q = \frac{3}{2} \langle n_e T v_r \rangle. \quad (7.14) $$

A novel way of measuring energy transport was done with a bolometer probe\cite{29, 27}. In a pyrobolometer, the heat of the plasma is deposited on a crystal that generates an electric current proportional to the absorbed power. It, therefore, measures the heat flux directly.

The fluctuation-induced energy flux for the ions be calculated from present measurements of $n_e$, $v_r$, and $T_i$.

$$ q_i = \frac{3}{2} \left[ \langle n_e \rangle \langle \tilde{T}_i \tilde{v}_r \rangle + \langle T_i \rangle \langle \tilde{n}_e \tilde{v}_r \rangle + \langle v_r \rangle \langle \tilde{n}_e \tilde{T}_i \rangle \right] \quad (7.15) $$

These terms are plotted in Figure 7.15. The energy transport due to the particle transport is the largest, but $\langle \tilde{T}_i \tilde{v}_r \rangle$ can drive a significant amount. Since $T_e$ is measured by the helicity probe, we can also plot the electron heat fluxes, Figure 7.16. However, in order to do this, $v_r$ for electrons was assumed to be the same as the ions, which is not necessarily true.
Figure 7.15: Ion heat flux terms $\langle n_e \rangle \langle \tilde{T}_i \tilde{v}_r \rangle$, $\langle T_i \rangle \langle \tilde{n}_e \tilde{v}_r \rangle$, and $\langle \nu_{\text{pinch}} \rangle \langle \tilde{n}_e \tilde{T}_i \rangle$
Figure 7.16: Electron heat flux terms \( \langle n_e \rangle \langle \tilde{T}_e \tilde{v}_r \rangle \), \( \langle T_e \rangle \langle \tilde{n}_e \tilde{v}_r \rangle \), and \( \langle v_{\text{pinch}} \rangle \langle \tilde{n}_e \tilde{T}_e \rangle \)
7.5 Discussion

This work presents a new and direct way of measuring the fluctuation induced particle transport $\langle \tilde{n}_e \tilde{v}_r \rangle$ on MST. The radial velocity fluctuations are measured spectroscopically, so all fluctuations are included regardless of the mechanism for their origin. Furthermore, a local edge measurement resolving the temporal sawtooth behavior is shown for the first time. The fluctuation induced flux is measured to be two orders of magnitude larger than the equilibrium transport and surges at the crash to account for total particle losses.

The spatial structure of the particle transport is found to be non-axisymmetric and associated with edge resonant tearing modes during a sawtooth crash. This was done by performing a pseudospectral analysis on the measured signals and adding the components together by knowing the relative phases between the magnetic modes. The total particle transport due to tearing calculated though the pseudospectral method and including modes $n = 1–4$ almost fully accounts for the total fluctuation induced flux calculated by usual ensemble methods.

Between crashes the transport from tearing is small, which agrees with previous measurements that identified electrostatic transport as dominant at that time. Experiments at RFX have shown that most of the particle transport in the edge is driven by electrostatic turbulence concentrated in the frequency range of 30–250 kHz\cite{4}. Since the time resolution of the IDSII is 10 µs (corresponding to 100 kHz)\cite{16}, the flux due to higher frequency fluctuations are not captured by this method of measurement. This can lead to an underestimate of the fluctuation induced flux, and in particular, this is a more important issue between
crashes. During the crash, the tearing fluctuations are dominant with frequencies less than 10 kHz, which is well within the temporal resolution of the spectrometer.
Chapter 8

Summary and Beyond

MST is particularly well suited to studying plasma dynamics associated with reconnection, momentum transport, and particle transport. This work examines the structure of turbulent plasma flows driven by tearing reconnection and the role those flows play in affecting momentum and particle transport.

8.1 Conclusions

The flow structure associated with $m = 0$ tearing modes resonant at the edge has been measured for the first time on MST using probes. The measured flow pattern is in good agreement with theoretical predictions calculated for the RFP using a nonlinear cylindrical DEBS code and a toroidal NIMROD simulation, which are the most realistic theoretical models for the RFP. The flows are predicted and measured to be different than the classical Sweet-Parker picture and are characterized by a single flow vortex that is radially inward
through the $X$-point and outward through the $O$-point. Asymmetries in the reconnection geometry give rise to an asymmetry in the radial flow, making $\tilde{v}_r$ non-zero at the resonant surface.

Flow fluctuations driven by tearing reconnection have a profound effect on momentum transport. The Reynolds stress was measured to be an order of magnitude larger than the rate of change in inertia. The Maxwell stress is also large but oppositely directed such that the two almost cancel. These measurements have advanced our understanding of momentum transport in the RFP. Tearing instabilities drive transport and nonlinear mode coupling greatly enhances it. Two-fluid effects are important. The Maxwell stress is proportional to the Hall dynamo, which has been studied because of its appearance in the generalized Ohm’s Law and only exists in two-fluid theories. The proportionality implies a coupling between the momentum evolution and the current evolution. Indeed, the parallel momentum profile exhibits a relaxation at the sawtooth crash akin to the parallel current density profile evolution.

The fluctuation-induced particle transport was measured directly as $\langle \tilde{n}_e \tilde{v}_r \rangle$. Since the radial flow was measured with the RIDSP (which also measures $T_i$), various energy transport terms were also measured. This is a new method for measuring the fluctuation-induced transport and confirms earlier measurements. The particle transport is now time resolved through a crash in the edge for the first time. It is measured to increase dramatically during the crash. Pseudospectral techniques are applied and indicate that, at the sawtooth crash, the transport is non-axisymmetric with a spatial variation that corresponds to the tearing
mode structure.

This work has used the reproducibility of the sawtooth crash to probe the dynamics of tearing reconnection in MST. In so doing, it has advanced our understanding of physics relevant to the RFP, and additionally, it has given insight into momentum and particle transport processes that may have relevance elsewhere.

8.2 Future work

The CHERS diagnostic on MST has recently been upgraded to provide a toroidal view. A very important and interesting experiment to carry out is to make \( v_\phi \) measurements with CHERS, apply pseudospectral techniques, and compare the results to Mach probe measurements. This will provide a non-invasive measurement of tearing mode flows. One may invoke \( \nabla \cdot \mathbf{v} = 0 \) to infer \( \tilde{v}_r \) and compare the flow structure.

Our appreciation of the role that two-fluid physics plays in the RFP is increasing. The observation that the turbulent stresses are much larger than the rate of change in inertial was an unexpected result. Nonlinear two-fluid NIMROD simulations are currently underway specifically to investigate the interplay between the Reynolds stress and Maxwell stress and to better understand the connection between momentum dynamics and current dynamics.

It is interesting to consider whether a current driven tearing mode like phenomena could also be at work in accretion disks around black holes and other compact objects. Accretion disks also exhibit a loss of angular momentum that is much faster than viscosity alone.
can account for. It is thought that the magnetorotational instability (MRI) is the main mechanism for momentum transport. But it is also possible that turbulent stresses could play a role as well. Perhaps current driven reconnection happening within the disk could give rise to stresses analogous to those measured in MST during sawtooth crashes.

Experiments should also be carried out in $F = 0$ plasmas. This work only investigated standard RFP discharges. The combination of the Mach probe and the helicity probe would be capable of measuring the equilibrium radial force balance $E_r + v_\theta B_\phi - v_\phi B_\theta = 0$. Then comparisons could be made between $F = 0$ and reversed plasmas.

Future experiments should be carried out to confirm the non-axisymmetric nature of particle transport. If indeed it is verified, one could imagine puffing gas in phase with the mode (to get deeper penetration) as an alternative to pellet injection. An attempt was made to individually measure all terms in the continuity equation, but the balance is still not understood. A simulation should be run to model the complicated edge effects in order to better understand particle losses and make valid comparisons to measurement.

Spectroscopic measurements are often utilized to look at the properties of impurity ions. A long standing question is whether or not impurity ions are representative of the bulk properties. In the case of ion heating, there are already some results suggesting that this is not true—that ion heating depends on the species. There is less known about flows. The original IDSP (measuring impurity line emission) can be used in conjunction with a Mach probe (measuring ion saturation currents) to compare the bulk and impurity flows. Addition upgrades to the spectroscopic probe method include more advance line profile
fitting (one that takes into account the fine structure using ADAS) and an absolute cali-
bration to the velocity measurement (using a helium glow discharge to provide a reference
line).

Runaway fast electrons present in MST can have a deleterious effect on probe mea-
surements. The helicity probe was designed with a particle shield to block fast electrons.
Appendix B discusses some of the effects that shields can have. Measuring the change if
the electron distribution function could prove difficult, but one can look for fast electron
effects by varying the bias on a TLP with without a particle shield. These experiment
would be akin to the electron energy analyzer (EEA) probe experiments carried out by
Stoneking[62, 63]. According to calculations in Appendix B, $T_e$ measurements are more
strongly dependent on bias voltage for probes without a particle shield when there is a
fast electron component to the electron distribution. The presence of fast electrons can
be somewhat controlled by changing the line average $n_e$. In particular, one could more
definitively answer the question of whether a particle shield does what it is designed to do.
Appendix A

On the Kinetic Theory of Gases

A.1 The Maxwellian distribution

The 3-D Maxwellian velocity distribution function is given by

\[ f = n_0 \left( \frac{m}{2\pi k_B T} \right)^{3/2} \exp \left( -\frac{mv^2}{2k_B T} \right). \] (A.1)

Here, \( n_0 \) is particle density, \( m \) is the mass, \( k_B \) is Boltzman’s constant, \( T \) is temperature, and \( v \) is speed. In cartesian coordinates, \( v^2 = v_x^2 + v_y^2 + v_z^2 \). The factor out front was chosen such that the distribution is normalized to a density, \( n_0 \). Generally speaking, this density could be a function of space, but it is constant for an isotropic system.

To calculate the density, one integrates the distribution over all velocities.

\[ n = \iiint f \, dv^3 \]


\[ n = n_0 \left( \frac{m}{2\pi k_B T} \right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2k_B T}} \, dv_x \, dv_y \, dv_z \]

\[ n = n_0 \]

Integrals of this nature can readily be performed by using an identity found in any standard integral table book such as [59].

\[
\int_0^{\infty} x^m e^{-ax^2} \, dx = \frac{G[(m + 1)/2]}{2a(m+1)/2} \tag{A.2}
\]

\(G\) is the Gamma function and evaluates as

\[ G(n+1) = n! \quad n = 0, 1, 2, \ldots \]

\[ G\left(\frac{1}{2}\right) = \sqrt{\pi} \tag{A.3} \]

\[ G(n + \frac{1}{2}) = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n} \sqrt{\pi} \quad n = 1, 2, 3, \ldots \]

### A.2 Calculating average speed

To calculate the average speed of particles in a Maxwellian distribution, one multiplies the distribution by \(v = \sqrt{v_x^2 + v_y^2 + v_z^2}\) and integrates over all velocities.

\[
\bar{v} = \frac{\iiint v f \, dv^3}{\iiint f \, dv^3} = \frac{1}{n_0} \iiint v f \, dv^3 = \frac{1}{n_0} \iint f \, dv^3
\]

\[
\bar{v} = \left( \frac{m}{2\pi k_B T} \right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{v_x^2 + v_y^2 + v_z^2} e^{-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2k_B T}} \, dv_x \, dv_y \, dv_z
\]
It is easier to perform this integral in spherical coordinates. Let \( v^2 = v_x^2 + v_y^2 + v_z^2 \). We then have

\[
\bar{v} = \left( \frac{m}{2\pi k_B T} \right)^{3/2} \int_0^{2\pi} \int_0^\pi \int_0^\infty ve^{-\frac{mv^2}{2k_B T}} v^2 \sin \theta \, dv \, d\theta \, d\phi
\]

Integration over \( \theta \) and \( \phi \) yields \( 4\pi \). Integration over \( v \) follows from equation (A.2).

\[
\bar{v} = 2 \frac{2}{\sqrt{\pi}} \left( \frac{2k_B T}{m} \right)^{1/2}
\] (A.5)

### A.3 Particle flux

The flux of particles through unit area from one side only, assuming a Maxwellian distribution, is a question of interest to many areas of physics. Effusion is the process where a kinetic gas leaks through a small hole[36]. The floating potential in plasma physics is understood by an initial difference between ion and electron flux[41]. The partial neutron current shows up in nuclear reactor theory[22]; for instance, the rate that neutrons leave a reactor depends on this. Hence, it is important in calculating neutron flux throughout the reactor core volume and, ultimately, nuclear reaction rates. Particle flux can be calculated in the following way.

\[
\Gamma = \int \int \int v_x f \, dv^3
\] (A.6)

\[
= n_0 \left( \frac{m}{2\pi k_B T} \right)^{3/2} \int \int \int e^{-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2k_B T}} \, dv_x \, dv_y \, dv_z
\]

\[
\Gamma = \frac{n_0}{2\sqrt{\pi}} \left( \frac{2k_B T}{m} \right)^{1/2} = 1\frac{1}{4} n_0 \bar{v}
\] (A.7)
It turns out that the expression given in equation (A.7) is valid for any isotropic distribution. This can be proven in the following way. Let \( f = f(v) \). By equation (A.4) the average speed is

\[
\bar{v} = \frac{1}{n_0} \int_0^{2\pi} \int_0^{\pi} \int_0^\infty v^3 f(v) \sin \theta \, dv \, d\theta \, d\phi
\]

\[
\bar{v} = 4\pi \int_0^\infty v^3 f(v) \, dv
\]  

(A.8)

The flux through one direction is the component of \( v \), \( v \cos \theta \), and \( \theta \) is integrated [0, \( \pi/2 \)].

\[
\Gamma = \int_0^{2\pi} \int_0^{\pi/2} \int_0^\infty (v \cos \theta) f(v) v^2 \sin \theta \, dv \, d\theta \, d\phi
\]

\[
\Gamma = \pi \int_0^\infty v^3 f(v) \, dv
\]  

(A.9)

Solving equation (A.8) for the integral and substituting into equation (A.9) yields \( \Gamma = \frac{1}{4} n_0 \bar{v} \).
Appendix B

Extended Langmuir Probe Theory

The probe theory of section 2.2.1 was a simple model, but it captured the main features of the I-V characteristic curve. However, it neglected many non-ideal effects present in MST. One such idealization, which the simple model made, was that the electron distribution function was an unshifted Maxwellian. Plasma current density in the edge is about 15 A/cm$^2$. Since $J = qn_e v_D$, this corresponds to a drift speed of $3.75 \times 10^5$ m/s. This drift speed is about a tenth of the electron thermal speed, $v_{th} = (2T_e/m_e)^{1/2}$, of $4.2 \times 10^6$ m/s for a 50 eV plasma. A shifted Maxwellian distribution would not affect a basic TLP setup because electrons are collected from both directions. That being said, TLPs used on MST are typically designed with a particle shield to block fast electrons. The particle shield is necessary because fast electrons can have such a deleterious effect on probe measurements. But they are also blocking off half of the distribution function, which is clearly asymmetric.

Modeling is required to understand the measurements made under more realistic con-
ditions and to justify, for instance, the use of a particle shield. The effects just mentioned manifest as a modification to the particle distribution function. Be it a shift, a non-thermal tail, or a particle shield, the theory of section 2.2.1 can be readily modified to incorporate these effects by numerically integrating the electron distribution function. A simple yet illustrative IDL code was written for just this purpose. It allows the user to prescribe any distribution function for the electrons, and it has a particle shield option, which limits the integration to one direction. It also outputs the inference one would make using the TLP method encapsulated by equations (2.19) and (2.20). As a first test of the code, the numerical method was benchmarked against the analytic solution, Figure B.1.

Figure B.1: Benchmark test of the numerical I-V curve code. Left: Theoretical Langmuir I-V curve calculated analytically for a 35 eV plasma. Right: Theoretical Langmuir I-V curve calculated numerically for a 35 eV plasma.
B.1 Modeling fast electrons

It has been known for a long time that there is a runaway effect active in MST that produces fast electrons. The basic mechanism hinges on the fact that plasma resistivity $\eta_S$ is proportional to $T_e^{-3/2}$[60]. That means that the higher energy tails of the electron distribution are preferentially accelerated by electric fields. The faster they go, the less resistance they encounter, and they “runaway.” They are believed to carry most of the current in the edge of MST.

Fast electrons can be problematic for probe measurements. In the TLP method, we assume that the ion saturation current is being collected on the negative tip. Fast electrons can overcome the repelling bias, and change current collected. Fast electrons also alter the electrode voltages. These effects are exaggerated because fast electrons can cause enhanced secondary emission on the electrodes. Many of the assumptions of the TLP method are invalidated. It is important to understand fast electron effects because we are often interested in measuring $\Phi_p$.

The electron distribution function was modified to include a fast electron component. Stoneking reported that the fast electron component could be thought of as a drifted Maxwellian with $v_{D,\text{fast}} = 2 \times 10^6 \text{ m/s}$ and $T_{e,\text{fast}} = 100 \text{ eV}$[63]. The overall effect on the distribution function is subtle, see Figure B.2. Subsequently, the effect on the I-V curve is also subtle, Figure B.3. The curve is lifted in the exponential region, which drops $\Phi_I$ by about 50V and changes how negatively one must bias an electrode to collect the ion saturation current.
Figure B.2: Solid line: Maxwellian distribution for electrons. Dashed line: Electron distribution having the same total electron density, but 10% belonging to a fast component with $v_{\text{D,fast}} = 2 \times 10^6 \text{ m/s}$ and $T_{e,\text{fast}} = 100 \text{ eV}$.

Figure B.3: Comparison of I-V curves calculated numerically with and without fast electrons. Left: I-V curve for a 35 eV plasma. Right: I-V curve for a 35 eV plasma plus a fast electron component.
B.2 Modeling a particle shield

On MST, particle shields are often utilized to mitigate fast electron effects on TLP measurements. A particle shield is a wall of boron nitride facing the primary direction that fast electrons are coming from. That is outboard up in standard MST discharges. Particle shields are designed to be larger than the electron gyro-radius, smaller than the ion gyro-radius, and large enough so that electrons moving along a field line will not intersect the electrode. The helicity probe was designed with one such particle shield.

It is important to understand how such a shield affects TLP measurements. Figure B.4 compares numerical I-V curves for a 35 eV plasma with and without a particle shield. The electron saturation current for the particle shield case saturates at half the value of the no particle shield case. This makes sense as we are only integrating half of the distribution function. The particle shield also has the effect of dropping $\Phi_f$ from 100 V to 75 V. Fortunately, the effect on TLP measurements of $n_e$ and $T_e$ are not greatly affected. Assuming that tip 1 is at $-200$ V, a simulated TLP probe measures a $T_e$ of 33.2 eV when it does not have a particle shield and actually does a better job with a particle shield; measuring 34.6 eV.

Next, we turn our attention to plasmas with a fast electron component. Figure B.5 compares numerical I-V curves for a 35 eV plasma with a fast electron component with and without a particle shield. A synthetic probe in this plasma seems to measure the same $T_e$ regardless of the presence of a particle shield, specifically 37.5 eV when tip 1 is biased at $-200$ V. However, this is somewhat of a situation of canceling effects. The TLP method
Figure B.4: Comparison of I-V curves calculated numerically with and without a particle shield. Left: I-V curve for a 35 eV plasma. Right: I-V curve for the same plasma but modeling a particle shield.

assumes that one is collecting ion saturation current. This is equivalent to an assumption that tip 1 is biased at $-\infty$. With tip 1 biased at $-\infty$, the presence of a particle shield has an enhanced effect where the synthetic probe measures a $T_e$ of 73.6 eV when it does not have a particle shield and a $T_e$ of 40.4 eV with a particle shield. Therefore, the measurement is dependent upon bias voltage which is what we are trying to avoid by using a TLP method. The effect, however, is less pronounced for probes with particle shields. It is interesting to note that this discrepancy does not exist in a plasma without a fast electron population.

**B.3 Discussion**

This extended Langmuir probe theory has shown that a particle shield helps mitigate the effects of fast electrons and does not strongly affect TLP measurements in the absence of fast electrons. Particle shields also do not have a strong effect in a plasma with a drifted
Maxwellian distribution, provided that the drift is small in comparison with the thermal speed.

It should be mentioned that another important effect that this extended model does not cover is that of secondary electron emission. Mostly, secondary emission is dealt with through the voodoo of “probe conditioning.” Fast electrons are more likely to cause secondary emission, so the use of a particle shield also helps with this.
Bibliography


