Electron Temperature Structures Associated With Magnetic Tearing Modes in the Madison Symmetric Torus

By

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Abstract

Tearing mode induced magnetic islands have a significant impact on the thermal characteristics of magnetically confined plasmas such as those in the reversed-field pinch. Using a state-of-the-art Thomson scattering (TS) diagnostic, electron temperature fluctuations correlated with magnetic tearing modes have been observed on the Madison Symmetric Torus reversed-field-pinch. The TS diagnostic consists of two independently triggerable Nd:YAG lasers that can each pulse up to 15 times each plasma discharge and 21 General Atomics polychromators equipped with avalanche photodiode modules. Detailed calibrations focusing on accuracy, ease of use and repeatability and in-situ measurements have been performed on the system. Electron temperature ($T_e$) profiles are acquired at 25 kHz with 2 cm or less resolution along the minor radius, sufficient to measure the effect of an island on the profile as the island rotates by the measurement point. Bayesian data analysis techniques are developed and used to detect fluctuations over an ensemble of shots. Four cases are studied; standard plasmas in quiescent periods, through sawteeth, through core reconnection events and in plasmas where the tearing mode activity is decreased. With a spectrum of unstable tearing modes, remnant islands that tend to flatten the temperature profile are present in the core between sawtooth-like reconnection events. This flattening is characteristic of rapid parallel heat conduction along helical magnetic field lines. The spatial structure of the temperature fluctuations show that the location of the rational surface of the $m=1$, $n=-6$ tearing mode is significantly further in than equilibrium suggestions predict. The fluctuations also provide a measurement of the remnant island width which is significantly smaller.
than the predicted full island width. These correlated fluctuations disappear during both
global and core reconnection events. In striking contrast to temperature flattening, a
temperature gradient within an $m=1$, $n=-5$ island is observed just after a global recon-
nection event. This suggests local heating and relatively good confinement within the
island. Local power balance calculations suggest reduced thermal transport within this
island. During improved confinement plasmas with reduced stochasticity, brought about
by a reduction in tearing instability temperature fluctuations correlated with magnetic
modes are small with characteristic fluctuation amplitudes of $\tilde{T_e}/T_e \sim 2\%$. 
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Chapter 1

Introduction

The magnetic configuration of a reversed field pinch (RFP) is a result of magnetic self-organization. The resulting reversed field configuration is preferred because it minimizes energy while conserving global helicity\(^1\). The complex plasma interactions which create the RFP are what sets this "alternative" fusion reactor concept apart from the standard tokamak where plasma feedback is minimized. They are also what gives the RFP a plethora of rich, interesting physics.

Understanding how heat is transported in the RFP is key to understanding and improving confinement. Temperature quickly equilibrates along magnetic field lines and is much slower to do so across field lines. Magnetic field lines are isothermal and thus detailed magnetic topology can have a great effect on heat transport.

In conventional RFP operation many tearing modes\(^2\) are resonant and often unstable forming magnetic islands. Magnetic islands connect field lines at smaller radii to field lines at larger radii tending to locally flatten the electron temperature profile across the island width\(^3\textit{--}^8\). Tearing modes in the RFP can be large enough that the magnetic islands overlap forming large scale stochastic fields. Therefore the degree to which tearing modes overlap can greatly affect heat transport. To fully understand how the magnetic topology affects heat transport, detailed information about both internal magnetic fields and internal electron temperature must be known.
Magnetic measurements in the core of the Madison Symmetric Torus (MST) fusion experiment are limited although not wholly lacking. The majority of information about the core magnetic field is reconstructed using magnetic measurements from the plasma edge. Localized, spatially detailed measurements of electron temperature are possible with the multi-pulse, multi-point Thomson scattering diagnostic, although they are limited temporally.

This thesis correlates edge magnetic measurements with localized, core, electron temperature measurements to discern information about core magnetic fluctuations and associated heat transport. This first chapter gives an overview of the resistive magnetic instabilities or tearing modes present in MST (Section 1.1), the cyclical relaxation of the RFP magnetic configuration known as the sawtooth cycle (Section 1.2) and the somewhat rare tearing mode events involving core tearing modes only (Section 1.3). This is followed by a description of an improved confinement mode of operation, where tearing mode activity is decreased, in Section 1.4. The final section (1.5) of this chapter outlines the rest of the topics in this thesis.

1.1 Tearing Modes

The equilibrium magnetic field configuration of the RFP is characterized by a toroidal field that reverses direction in the edge relative to the toroidal field on axis. At the center the field is purely toroidal. At $r > 0$ the field twists helically around the torus. The ratio of toroidal to poloidal field changes as a function of radius, going through zero before reaching the edge. The point at which the field is purely poloidal is called the reversal surface. The amount of field line twist is represented by the safety factor:
Here $r$ and $R$ are the minor and major radii respectively and $B_t$ and $B_p$ are the equilibrium toroidal and poloidal magnetic fields. Fluctuations on the equilibrium magnetic field are resonant when the fluctuation wave vector is perpendicular to the magnetic field.

$$q(r) = \frac{rB_t}{RB_p}$$  \hspace{1cm} (1.1)

or,

$$\vec{k} \cdot \vec{B} = 0$$  \hspace{1cm} (1.2)

Here $m$ and $n$ are poloidal and toroidal wave numbers respectively. Rearranging equation 1.3 and substituting in $q$ from equation 1.1, this resonance condition can be written in terms of $q$:

$$q(r) = -\frac{m}{n}$$  \hspace{1cm} (1.4)

This states that modes are resonant for rational values of $q$. Although this condition allows for an infinite number of resonant modes, the largest modes tend to be the ones that have the smallest wavenumbers such as $m = 0$ modes and $m = 1$, $n = -6, -7, -8$, etc. All $m = 0$ modes are resonant at the same place - the reversal surface- and the largest $m=1$ modes are resonant in the core (see Figure 1.1).

Although these modes are resonant at a specific radial location they are global modes. Magnetic coil arrays (toroidal and poloidal) at the plasma edge are used to detect these
Figure 1.1: The safety factor, $q$, peaks on axis at a value of $\sim 0.2$ and goes through zero near the edge at the reversal surface. Magnetic tearing modes are resonant at rational values of $q$.

modes. At each point in the array there are three coils; one to measure each toroidal, poloidal and radial fields. A spatial Fourier mode decomposition is automatically done after each plasma shot. The magnetic data used in this thesis uses measurements from thirty-two poloidal coils in the toroidal array. With thirty-two coils a spatial analysis can be done to resolve mode numbers $|n|=1-16$. The poloidal mode number cannot strictly be resolved by the toroidal array alone. The distinction between $m = 0$ and $m = 1$ modes can be made by the fact that for an $m = 0$ mode the $B_p$ component of the fluctuation is very small for any $n$ number. Therefore the $B_p$ signal is mainly dominated by $m = 1$ modes. So, the poloidal mode number, $m$, is generally a function of $n$ with $m = 0$ for $|n|=1-4$ and $m = 1$ for $|n| > 6$. The $n = 5$ mode can either be $m = 1$ if it is resonant in the core ($q(0) > 0.2$) or $m=0$ resonant at the reversal surface. It is also important to note that $n$ can either be positive or negative. The standard directions in MST use
a left handed coordinate system (see Figure 1.2). According to these directions, in the core $B_t$ is parallel to $\vec{z}$ whereas $B_p$ is anti-parallel to $\vec{\theta}$. To avoid confusion, $B_p$ is defined as being positive in this thesis, another way of saying this is $B_p = -B_\theta$. Therefore in the core, $m$ and $n$ have opposite signs with $m$ defined as positive and $n$ defined as negative.

There are also $m = 1$ modes which are resonant beyond the reversal surface. These have a positive designation of $n$. In order to resolve positive and negative $n$ values an analysis must also be done in the time domain to separate negative and positive velocities and frequencies. The edge modes are typically quite small with large toroidal wave numbers ($n > 9$) and they are generally not confused with the core modes studied in this thesis. However during plasmas with deep reversal, such as improved confinement plasmas, the $m = 1, n = 6$ mode becomes resonant in the edge and steps must be taken to separate it from the $m = 1, n = -6$ mode in the core.

From the mode analysis done at the end of each shot several mode characteristics can be determined: the amplitude of the modes at the edge, velocity of each mode, and phase (or position) of each mode.

A peaked parallel current profile provides a free energy source for resonant modes allowing them to grow. As the modes grow field lines can ”tear” and reconnect forming magnetic islands. Figure 1.3 shows a reconnected or tearing mode. Reconnection of the mode short-circuits field lines inside of the resonant surface to field lines outside the resonant surface. Providing a direct path for particles and heat to move outward. As tearing modes grow they begin to overlap with neighboring modes. This is especially true for high toroidal wave number modes and $m = 0$ modes where radial separation is small or zero. This results in a magnetic field which is stochastic in the mid to outer radii. The core most mode is often far enough from its neighbor that a partial island
Figure 1.2: Directions of magnetic fields in MST. A top view is shown on the left and a poloidal cross section is on the right.

Figure 1.3: Tearing modes, formed at rational surfaces, connect field lines that would otherwise be radially separated. This provides a mechanism for radial transport.
structure remains. Simulations\textsuperscript{12} have been performed at realistic Lundquist numbers (S \(\sim 3.8 \times 10^6\)) using the 3D resistive MHD DEBS code\textsuperscript{13}. The magnetic fields produced by the DEBS code, scaled such that the toroidal flux matches the toroidal flux measured in MST, are mapped by the field line tracing code MAL\textsuperscript{14}. Figure 1.4 shows that a residual magnetic island structure is often present for the \(m = 1, n = -6\) mode. The \(m = 1, n = -6\) mode is usually the core-most resonant mode and the largest amplitude mode. However if \(q(0)\) reaches 0.2 the \(m = 1, n = -5\) mode becomes resonant closer to the axis than the \(n = -6\) and can often be comparable or larger in magnitude.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.4.png}
\caption{Magnetic field line tracing performed by the MAL code\textsuperscript{14} with results of DEBS\textsuperscript{13} simulations. A remnant \(m = 1, n = -6\) island is visible at \(R \sim 1.56\ m\). Plot courtesy of Josh Reusch.}
\end{figure}
1.2 Sawtooth Cycle

MST’s reversed-field configuration is the result of a self-organization towards a relaxed state. A toroidal electric field is inductively applied to the plasma. This results in a peaked parallel current profile that provides a free energy source for tearing modes. With a free energy source readily available, the fluctuations grow, changing the magnetic topology and moving it away from the relaxed state. In general these fluctuations grow slowly. However, during quasi-periodic discrete sawtooth events\textsuperscript{15,16} mode amplitudes spike up. Nonlinear interactions between fluctuations drive a dynamo which works to flatten the parallel current profile and bring the plasma back towards the relaxed state. The event is called a sawtooth because several diagnostic signals appear to crash down (or spike up) giving a sawtooth-like appearance. It is a global event; the total magnetic energy decreases, the total toroidal flux goes up and there is a loss of confinement. Then once again the applied toroidal electric field provides a peaked parallel current starting the cycle over again. The cycle is several ms long with the discrete sawtooth crash happening on much shorter timescales $\sim 0.2$ ms. Figure 1.5 shows the magnetic fluctuation amplitude through a sawtooth. The $n = -5$ mode is only large during and briefly after the sawtooth crash. It is of particular interest because of its proximity to the axis. When present, it dominates the mode activity in the core.

1.3 Core Tearing Mode Events

Another type of tearing mode event takes place in standard MST plasmas that, in contrast to sawteeth (which are global events), only involves core modes. Figure 1.6 highlights the difference between a sawtooth and a core event. Core events are characterized
Figure 1.5: An ensemble of $m = 1, |n| = 5-10$ magnetic fluctuation data through a sawtooth crash. The $n = -6$ fluctuation is the largest mode before the crash. The $n = -5$ fluctuation is small before the sawtooth crash and is briefly the largest mode after the crash.
by the $m = 1, n = -6$ mode slowly ramping up over a period of several milliseconds, reaching a magnitude several times the magnitude of the next largest mode and then quickly (over $\sim 200 \mu s$) dropping down. The length of time that the core mode is ramping up and the maximum amplitude can vary. Just prior to the event other $|n| > 7$ core modes begin to rise. The mode activity is limited to the core modes and the $m = 0$ modes do not change through the event. The magnetic field structure does not appear to change on global scales. The $n = -5$ mode which spikes up at a sawtooth crash does not appear during a core event.

The ramp-up phase of these events is similar to the ramp-up phase of a plasma state of particular interest called Quasi-Single Helicity (QSH)\textsuperscript{17}. During QSH the amplitude of the core most mode can grow to be several times the amplitude of the sum of all other modes. In certain cases QSH seen on the RFX RFP will reorganize the plasma into a helical equilibrium state\textsuperscript{18,19} greatly impacting confinement in the core. During the ramp-up phase of a core event in MST, QSH status is usually not achieved. In MST QSH is often achieved in non-reversed plasmas. In these conditions the plasma rotation usually halts soon after the initial formation of QSH. The correlation analysis done in this thesis requires a rotating plasma and because of this QSH plasmas are not part of this study, however the similar ramp-up stage of a core event is investigated.

1.4 Improved Confinement Plasmas

Flattening of the current density profile is necessary for RFP sustainment. While this is normally done through the dynamo activity during a sawtooth crash this is not ideal, because it results in a loss of plasma confinement. The necessary flattening can also
Figure 1.6: The difference between core and global (sawtooth) tearing mode events is shown (MST shot 1090629117). a) The core most mode \((m = 1, n = -6, \text{ black})\) and a mode resonant at the reversal surface\((m = 0, n = 1, \text{ red})\) both peak during a sawtooth, but only the core mode shows activity for a core event. b) Core tearing modes with \(n = -7, -8\) increase at the core event, but the \(n = -5\) mode does not.
be achieved by inductively controlling the parallel current so that the resulting parallel current profile is near the relaxed state. The result is a sawtooth-free plasma with tearing modes suppressed\textsuperscript{20}. Tearing modes no longer have a strong overlap in the core and the magnetic field is significantly less stochastic. Electron thermal diffusion can be as low as 5-10 \(m^2/s\).\textsuperscript{21} Confinement is significantly improved and maintained in the absence of sawteeth\textsuperscript{22,23}. In practice plasma current is easily controlled, but the level of confinement may vary from shot to shot when applying this technique. The overall reproducibility of shots is not great making it difficult to ensemble data.

1.5 Thesis Overview

The remainder of this thesis is dedicated to measuring electron temperature fluctuations associated with magnetic tearing modes in the MST plasmas described above. Chapter 2 provides details of the hardware, calibration and data analysis for the state-of-the-art Thomson scattering diagnostic used to make electron temperature measurements. Chapter 3 describes how electron temperature is mapped to magnetic tearing modes. It also describes the innovative method of ensembling data using Bayesian analysis to reveal correlated temperature fluctuations and proposes two possible scenarios which may be measured: a temperature flattening and a temperature peaking within island separatrices. Chapter 4 describes the electron temperature fluctuation results and physical implications for four cases. The first case is between sawteeth, where temperature is flattened across remnant islands. In the second case, fluctuations are studied through sawteeth. Correlated fluctuations are found to disappear at the crash with the destruction of remnant island structures. After the crash a peaked helical temperature structure
appears coincident with the $m=1$, $n=-5$ tearing mode. The third case looks at electron temperature through core events where increased magnetic stochasticity (limited to the core) also destroys the correlation between magnetic and electron temperature fluctuations. In the final case electron temperature fluctuations are studied in plasmas with low magnetic fluctuations and are found to be similarly small. Chapter 5 provides a summary and suggestions for future work. The appendices contain an overview of Bayesian probability theory and the full sets of sawtooth fluctuation data discussed in Chapter 4.
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Chapter 2

Thomson Scattering Diagnostic on MST

Local electron temperature measurements on MST are made using a multi-pulse, multi-point Thomson scattering diagnostic. Although this diagnostic has operated in some form on MST since 2004 it has only had short periods of continuous operation. The system is constantly evolving and has been through several upgrades since it’s initial operation. This chapter describes the diagnostic and how it was used to gather data used in this thesis. The basics of Thomson scattering (TS) are reviewed in section 2.1. This is followed by a description of the TS diagnostic hardware in 2.2, a discussion of calibration procedures in 2.3 and the electron temperature fitting procedure in 2.4.

2.1 Basics of Thomson scattering

Thomson scattering is the absorption of a photon and subsequent emission of a different photon by a free, charged particle\textsuperscript{1,2}. In general the emitted particle differs from the incident particle in both trajectory and wavelength. The diagnostic described in this chapter analyzes the scattered photon spectrum of light produced by high power lasers and scattered off plasma electrons. The characteristics of the emitted or “scattered”
photon depend on the incident photon’s polarization and energy as well as the energy of the electron. The electron absorbs the incoming photon and experiences a Lorentz force from the oscillating electric and magnetic fields. Since the magnetic force is dependent on the velocity of the particle it is much less than the force due to the electric field and only becomes important at relativistic speeds. The oscillating electric field of the incident photon accelerates the electron and the accelerated electron emits a photon, consistent with general scattering theory. The probabilistic direction of this scattered photon follows a dipole radiation pattern. If many photons are incident on a volume of plasma electrons and all incident photons have the same polarization the intensity of scattered light is proportional to $\sin^2(\chi)$, where $\chi$ is the angle between the incident electric field and the direction of the scattered photon. Figure 2.1 shows the geometry of this process. Momentum and energy are conserved in this process so that:

$$\vec{k}_s = \vec{k}_i + \vec{k}$$  \hspace{1cm} (2.1)

Where $\vec{k}_s$, $\vec{k}_i$ and $\vec{k}$ are the wave vectors for the scattered photon, incident photon and charged particle respectively. This is depicted graphically in Figure 2.2. Since the energy of the scattered photon depends on the energy of the charged particle, Thomson scattering can be used to measure the energy of the particle if the wavelength of the incoming photon is known.
Figure 2.1: A schematic of non-relativistic Thomson scattering from polarized incoming radiation. For the MST Thomson scattering diagnostic the incoming radiation is polarized in the toroidal direction and the observer is a collection lens located at $\chi=90^\circ$. 
Figure 2.2: A diagram of the relationship between the scattered and incident photons and the electron. The component of electron velocity sampled depends on the direction of the scattered photon. Note that the direction of $\vec{k}$ is different in a) and b).
2.2 Hardware

The hardware for the Thomson scattering diagnostic\textsuperscript{3} on MST falls into two main categories; incident photon production and scattered light detection. Incident photon production is done with two independently triggerable Nd:YAG lasers and a 15 m remote-controlled beam line. Upgrades to this system between 2007 and 2009 have allowed for several different operational modes. The details of this hardware and the various operating modes are given in Sections 2.2.1 and 2.2.2 respectively. Light detection hardware includes collection optics, twenty-one General Atomics polychromators equipped with avalanche photodiode modules and a set of 1 GS/s Acqiris digitizers described in Sections 2.2.3 through 2.2.6.

2.2.1 Lasers

The production of photons incident on the plasma electrons is done with two Nd:YAG Spectron SL858g lasers. The lasers are physically separated from MST, located in a room on the second floor of Chamberlin Hall. Laser light is directed down to MST with a set of mirrors on remotely moveable mounts. The mirrors are imaged by a set of CCD cameras. Optimum alignment positions are imaged by the cameras. A LABVIEW program is used to automatically or manually align the beam to the optimal images by comparing images taken after each plasma shot. Figure 2.3 shows a layout of the laser path. Each laser can produce a maximum pulse amplitude of $\sim 2.8$ J in 9 ns at a wavelength of 1064 nm. The original design of the laser system has been modified so that the lasers are each able to fire up to 15 times during one MST discharge of $\sim$50-80 ms\textsuperscript{4}. The various modes of operation used to take the data in this thesis are described
below.

Figure 2.3: The laser light follows a 15 m remote controlled beamline from the second floor down to MST. Diagram courtesy of Josh Reusch.

2.2.2 Operational Modes

Through the course of the research presented here the laser system has gone through several upgrades which have enabled various operational modes. Three different modes have been used to take electron temperature fluctuation data; original Spectron operation, "multiple q-switch Spectron" operation, and "fast Spectron" operation. These modes are summarized in Figure 2.4.
Figure 2.4: The laser is operated in three different modes: a) original Spectron operation with just 2 pulses per discharge, b) multiple q-switch Spectron operation with 4 pulses per discharge and c) fast Spectron operation, in this mode a burst of 6 pulses is repeated 5 times for a total of 30 pulses per discharge. Red and blue pulses are from separate lasers. The minimum time separation between the two lasers is variable. Here they are shown interleaved but they can also be sequential. The time between pulses from a given laser is fixed.
The original design of the laser system allowed for one pulse at maximum energy (~2.8 J) from each laser during a plasma discharge. The time separation between the two pulses can be adjusted to as little as 1 µs, limited by the Jorway 221 timing module used for triggering. The frequency of tearing modes in MST is ~ 20 kHz, or a period of ~ 50 µs. This original operation mode has sufficient time resolution to resolve fluctuations associated with tearing modes although with only two time points per MST discharge this requires a large ensemble.

In order to overcome some of the limitations of the original design, external Pockel cell drivers made by Bergmann Meßgeräte Entwicklung KG (Part No. ds11d/KD*P) were installed. This separated the control of the q-switch from the control of the flash lamps and enabled pulsing the q-switch twice during each flash lamp pulse. Hence this is called "multiple q-switch Spectron" operation. For a flash lamp pulse ~ 160 µs long two laser pulses ~ 100 µs apart with an energy of ~ 2 J can be obtained. In this operational setup each flash lamp is still limited to fire once per plasma giving a total of four laser pulses per MST discharge. If the pulses for the lasers are interleaved this gives a 50 µs time resolution (although the minimum time separation is still variable down to 1 µs). In addition to opening up a new operating mode and allowing more information to be gathered in one shot, the new Pockel cell drivers also reduced high frequency electromagnetic noise created previously when the q-switch fired and thus improving data quality.

For "fast Spectron" operation the flash lamps in the Spectron lasers are driven by QXF54 power supplies (built by Physical Science Laboratory in Stoughton, WI) originally designed for a fast Thomson scattering system now being commissioned. In this operational mode both the duration of a flash lamp pulse and the number of times a
flash lamp is pulsed can be controlled. This makes for a very versatile system. For fluctuation data the "fast Spectron" system is operated in such a way that the flash lamps are pushed closer to their limit by both increasing the length of time for which they are on and the number of times which they are pulsed during a discharge. By increasing the duration of each flash lamp pulse (from 150 $\mu$s to 310 $\mu$s) the number of times the q-switch can be triggered increases from 2, in multiple q-switch operation, to 3. The flash lamps can be pulsed in this manner 5 times during the plasma discharge giving a total of 15 laser pulses (5 sets of 3) for each laser or a total of 30 for the two. Once again the lasers can be interleaved giving an effective data rate of 25 kHz during each burst of pulses. The versatility of this system allows for other modes of operation (such as 2 kHz operation for 15 ms) but they were not used for data collection in this thesis.

2.2.3 Collection Optics

Scattered photons are collected by a lens (see Figure 2.5 ) that sits at 222 ° toroidal and 20 ° poloidal on the MST. The lens is comprised of seven elements with a transmission of over 80% between 750 and 1064 nm. During MST operation without the Thomson scattering diagnostic the lens is outside of the vacuum vessel and closed off behind a gate valve. During operation with the diagnostic, the gate valve is opened and the lens is inserted into the vessel. Figure 2.6 shows the lens apparatus and bellows system outside of MST. Although Thomson scattering is, in general, a non-invasive procedure, the relatively large porthole required (114 mm) for the lens insertion is the maximum allowable size on MST because of field error production. The field error produced is
small enough that it probably has little effect on the plasma away from the port but it is still large enough at the porthole to cause significant plasma-lens interaction. The first element of this lens needed to be replaced every few weeks during continuous operation due to pitting on the surface and darkening of the anti-reflective coating. To limit this interaction a boron nitride limiter was placed on the inside wall of MST around the porthole and lenses without anti-reflective coating on the plasma facing side have been used. Since these changes were implemented the amount of lens damage from the plasma has been less and lenses need to be replaced less frequently.

A collection of 21 incoherent fiber bundles\(^3\) sits on a curved mount at the image plane of the lens outside of MST. Sampling volumes are larger in the edge (\(\sim 2\) cm radial extent) and smaller in the core (\(\sim 1.3\) cm radial extent). The fiber bundles occupy 21 of the 34 possible positions in the fiber optic mount. Each fiber is 18 m long and runs parallel to the laser beam path from the collection lens image plane back to the laser room where they are coupled to a set of polychromators.

The position of the fiber optic mount is adjusted in order to place it on the image plane. A 100 mW 1064 nm cw laser is coupled to an insertable integrating sphere (ISIS) that traverses the vertical laser beam path through MST (see Figure 2.7). The light is then emitted from the ISIS through a pinhole and the fiber optic mount is adjusted until the signal level in the 0th channel of the polychromators is maximized along the beam path.
Figure 2.5: A schematic of the collection lens apparatus as it sits on the side of MST is shown. Fiber optic cables sit on the image plane gathering scattered light at 21 locations between the core and edge of MST. Diagram courtesy of Rob O’Connell.
Figure 2.6: A picture of the bellows system used to move the lens in and out of MST. Fiber optic cables sit behind the lens on the image plane. Photo courtesy of Josh Reusch.

### 2.2.4 Polychromators

The light detection system was designed to handle the substantial range of MST’s operating space as well as unique applications such as fast fluctuation measurements and investigations into non-Maxwellian temperatures. The chosen system is a set of polychromators with spectral channels equipped with avalanche photodiodes (APDs). The output of each APD is digitized by a set of 1 GS/s Acqiris digitizers\(^7\). This system not only covers MST’s operating space but has proven to be easily expanded and upgraded.

Thomson scattered light is coupled to 21 polychromators (see Figure 2.8) each used to measure a separate radial location in the plasma. The polychromators are the same as those used on DIII-D\(^8\), General Atomics polychromator model No. GAPB-1064-4-1K. Fifteen are equipped with four spectral channels over a range of 868-1065 nm and six
Figure 2.7: A stepper motor drives the insertable integrating sphere (ISIS) along the TS beam path inside of MST. The sphere has small pinhole on top and is coupled to a fiber optic cable accessed by a port on the outside of MST. During regular MST operation the ISIS is retracted into the pumping duct. Courtesy of Mike Borchardt.
are equipped with eight spectral channels over a range of 715-1065 nm. Each spectral channel is equipped with a General Atomics APD module. The first channel in each polychromator measures scattered light at the laser wavelength (1064 ± 1.5 nm) and is currently overwhelmed by laser light reflected off the inside of the vacuum vessel\(^9\). Therefore there are three and seven spectral channels available for electron temperature measurements on the four and eight channel polychromators respectively. Transmission curves have been measured over the appropriate ranges for each polychromator (see Section 2.3.4). The four channel polychromators are designed to be able to measure temperatures between 10 eV and 2 keV, whereas the eight channel polychromators can measure electron temperatures up to at least 10 keV. In addition to the increased temperature range the eight-channel polychromators also have the capability of distinguishing non-Maxwellian temperature distributions that have been observed in other machines\(^\text{10}\) and may be present in certain MST plasma conditions.

### 2.2.5 Detectors

Each polychromator spectral channel is equipped with a Perkin-Elmer C30956E APD for light detection. Two different amplifier modules constructed by General Atomics are used\(^\text{11}\). The first amplifier module (1999 construction) is used with all four channel polychromators and three of the six eight channel polychromators. The remaining three polychromators use the second amplifier module (2006 construction).

The two types of amplifier modules are similar in design but have key differences in construction and operation. Both amplifiers require ± 8 V input power, both have a pulsed and DC output and a four step DC gain selection (currently only the DC coupled
Figure 2.8: A four channel polychromator is shown; a) schematic, b) photo. Light is collimated as it enters the polychromator from the upper left through a fiber optic cable. As light hits each station in turn a section of the infrared spectrum is passed through an interference filter to an APD.
output is used on the maximum gain setting). The first difference is the operational amplifier used in the input stage; the 2006 construction amplifiers use LMH6624 (low-noise voltage-feedback surface mount), whereas the 1999 construction amplifiers use CLC425 (low-noise voltage-feedback dual in-line package (DIP), with adjustable supply current/bandwidth). As originally constructed, the 2006 modules had a higher bandwidth and a gain about 2.2 times the 1999 modules, though noise pickup and oscillation in the 2006 modules made operation difficult. Modifications were made to the 2006 modules to lower the gain, shield noise pickup, and stabilize oscillations.

The gain of the APDs is highly dependent on temperature and applied bias voltage. For example, at 22 °C a temperature change of just 0.5 °C can change the gain by 1%. Similarly a 5 V deviation in the applied bias voltage from the recommended voltage will change an APD’s gain by approximately 6%\(^\text{12}\). It is therefore important to both monitor and keep constant the temperature and applied bias voltage during calibration and during regular operation.

The detectors are located in an air-conditioned clean room to keep the ambient air temperature and humidity constant. During regular operation the APD modules are mounted on polychromators that are water cooled with an external chiller. The APD modules are then cooled by conduction with the polychromators. The temperature of each polychromator is monitored with a thermocouple attached to the top of the polychromator box. Individual polychromator temperature measurements are recorded with a LABVIEW program and can be checked throughout any day the system is in operation for consistency. These measurements show that the temperature variation of any given polychromator is \(< 0.12 \, ^\circ \text{C}\) with many below \(0.06 \, ^\circ \text{C}\).

Bias voltage for the APDs is supplied with a serial combination of power supplies
located externally to the clean room. The bias voltages are fed thru to the detector modules via panels in the ceiling. Each panel has voltages between 275 V and 425 V in 5 V increments with the bias voltage for each APD fixed inside the panel. A change in any 5 V increment will be represented as a change in the maximum bias voltage.

The APD amplifier modules require +8 V and -8 V inputs. If these drop below a certain threshold the amplifiers do not work. These inputs are also provided by a power supply external to the clean room and fed through the ceiling via the same panels and monitored at each panel.

2.2.6 Digitizers

The APDs not only detect Thomson scattered light but also background light coming from the plasma. The amount of background light present in any given shot depends on the specific plasma conditions. Background light is also not constant throughout a plasma and sometimes varies quite rapidly. For example, the background might vary wildly in just 10 µs as the confinement drops during a sawtooth event (See figure 2.9). Therefore the digitizers used to record the scattered light signal must also be able to accurately capture the background light. The digitizers used are 1 GS/s Acqiris digitizers (Acqiris DC270) with a bandwidth of 300 MHz and 8 bit vertical voltage resolution with variable gain. For most plasma conditions this is sufficient to capture both the background and scattered light. With very high background light the vertical bit resolution of the scattered pulse can be compromised. A second set of digitizers is being added to enable separate recordings of the scattered pulse and the background light to ensure the bit resolution of the pulse is always sufficient.
Figure 2.9: Background light from the plasma changes rapidly. A linear estimation of the background over a 5 µs window would severely misrepresent the background during the laser pulse.
The digitizers can run in a burst mode which breaks up the memory into discrete short segments where the signal is being recorded separated by longer segments where the signal is not stored. Breaking up the memory like this is a way to record several scattered light pulses in one plasma discharge while conserving memory.

2.3 Calibration

This thesis applies the Thomson scattering diagnostic to measure electron temperature fluctuations. In order to quantify fluctuations, accurate calibrations must be done and frequently checked to enable correct temperature and uncertainty derivation. With the calibration procedures now in place it is possible to quickly and easily check system stability and functionality over time. Calibrations performed include in-situ spatial calibration, APD gain and noise determination, and spectral instrument function measurement. To improve the ease and accuracy of these calibrations an absolutely calibrated APD (for which the gain and relative quantum efficiency has been measured) is used as a reference detector. Procedures are in place that make it possible to easily and quickly check system stability and functionality over time. This includes in-situ calibrations that can be done without disturbing the system. In-situ measurements include a daily calibration that checks the functionality of electro-optical system components and a pulsed spectral calibration (implemented with an insertable integrating sphere and a wavelength-tunable laser) that checks all optical system components. Section 2.3.1 describes the in-situ spatial calibration done using an insertable integrating sphere. Section 2.3.2 describes the gain and uncertainty calibration of the APD detectors. This includes the implementation of using one absolutely calibrated APD to calibrate the remaining
APDs, as it is easier to compare two similar detectors. Section 2.3.3 describes a daily calibration system in place to check the functionality of the APDs over time. Section 2.3.4 includes a description of necessary spectral calibration of the polychromators in order fit temperature to Thomson scattering data. This also includes a measurement of a relative quantum efficiency curve for the APDs and a pulsed in-situ spectral calibration that can be compared to lab calibrations to check the entire system.

2.3.1 Radial Calibration

Each of the 21 Thomson scattering polychromators\(^8\) views a different radial location between the core and edge of the MST. In order to determine the exact viewing location and resolution of each polychromator an in-situ radial calibration is performed. A Nd:YVO4 laser diode (Lasermate GMF-1064-100FBC2) operating at 1064 nm is coupled via fiber optic to the ISIS (see Section 2.2.3 for a description of the ISIS). The traverse of the ISIS is scanned inwards at a constant rate and the signal for each laser line polychromator channel is digitized using 10 ms sampling time and 42,000 samples. The ISIS scans from the edge to the core over the 420 seconds. When the ISIS is in the viewing location of a given polychromator the laser line channel picks up the emitted laser light. As the ISIS moves inward each polychromator lights up in turn. The light gate which is normally used to ’home’ the ISIS is also digitized, giving the precise location during the scan. Figure 2.10 shows the radial calibration. The radial resolution of the diagnostic is 2.0 cm in the edge and 1.3 cm in the core, with a precision of approximately 1 mm.
2.3.2 APD Absolute Calibration

The APD calibration process is two-step. First the gain (a combination of detector response and amplification) near the laser wavelength of one APD is obtained by comparison to an InGaAs detector absolutely calibrated by Optronic Laboratories. Then this APD that has been absolutely calibrated is used as a reference to obtain the gain near the laser wavelength and noise enhancement factors of the rest of the APDs. The unbiased InGaAs detector setup is noisy, relative to the APD module, and prone to oscillations making it unstable. A substantial amount of work has been put into fixing this but it can not be made reliable in all operating conditions. In addition to this the InGaAs detector, and corresponding current to voltage amplifier, has a much lower overall gain-bandwidth product than a typical APD and amplifier module. In a side by side calibration setup drastically different light levels are needed to get reasonable signal levels for the InGaAs detector and an APD. This difference in light levels results
in a calibration setup that does not take advantage of the full scale bit resolution of the digitizers. The variance in light pulses detected by the APD becomes determined by digitizer bit resolution washing out the detector photon statistics. It is therefore advantageous to use an APD as an absolutely calibrated reference detector over the InGaAs in the second part of the calibration mainly to get accurate noise levels but also for ease of use. With the absolutely calibrated APD detector the calibration setup becomes simpler and more accurate. Similar light levels can be used for the absolute detector and APD to be calibrated which allows for a physically simple setup and allows the full scale vertical resolution of the digitizers to be used. This is essential because it is important to have the bit transition voltage resolution much less than the statistical variation of the pulses.

**Characterization of light source**

The light source used in the calibration process is an infrared InGaAs LED (PerkinElmer, Model C30116/F). The light is pulsed to mimic a laser pulse with a width of $\sim 30$ ns at 1 kHz. The normalized spectral output of the LED pulsed in this manner, $P_{\text{norm}}$, is characterized using the absolutely calibrated InGaAs detector. To do this the light from the LED is sent through a 0.5 m Czerny-Turner monochromator with a 1200 1\text{mm}^{-1} grating. The output of the monochromator is coupled to the InGaAs detector and the monochromator is scanned through the output wavelengths of the LED. It is assumed the throughput of the monochromator in the region of interest is relatively flat.

The InGaAs detector, which has been absolutely calibrated by Optronic Laboratories, outputs a current proportional to the power hitting the detector. The gain as a function of wavelength, $C(\lambda)$, is shown in Figure 2.11. The current output is sent to a current to
voltage amplifier with a gain, $G_{\text{amp}}$, of $3.3 \times 10^7 \text{ V/A}$. Assuming the current to power ratio is the same as the charge to energy ratio\textsuperscript{13} the integral under the digitized output pulse, $S_{\text{InGaAs}}$, gives the total Coulombs per pulse.

\[ S_{\text{InGaAs}}[V_s] = G_{\text{amp}}[V/A] = Q[C] \]  \hspace{1cm} (2.2)

The number of photons detected for a pulse at a given wavelength, $j$, by the InGaAs detector can be calculated using the energy per photon = $hc/\lambda$.

\[ N_j = \frac{Q_j}{C(\lambda_j)\frac{hc}{\lambda_j}} \]  \hspace{1cm} (2.3)

Figure 2.12 shows the spectral output in photons for the pulsed LED. The measured peak output is $\sim 1020$ nm even though it is nominally specified to be at 1060 nm by the manufacturer. The normalized output is then
Figure 2.12: Photons output by the pulsed LED and measured by the InGaAs absolutely calibrated detector at the monochromator exit. Measurements show a peak wavelength of 1020 nm with a FWHM of 70 nm.

\[
P_{\text{norm}} = \sum_j \frac{N_j}{S_{\text{InGaAs}_j}} = \sum_j \frac{Q_j \lambda_j}{S_{\text{InGaAs}_j} C(\lambda_j)hc}
\]  
(2.4)
defined to be a convenient conversion so that the number of photons detected in a single LED pulse is simply: \[N_{\text{InGaAs}} = S_{\text{InGaAs}} P_{\text{norm}}\]

**Measure the gain for one APD**

For the first step in the calibration process light from the LED is split into two fiber optic cables. One is sent to under fill the InGaAs detector and the other under fills the APD to be absolutely calibrated. Both detectors sit on a water cooled block to ensure that their temperatures remain constant. The system is on and warmed up for ten minutes before the calibration to allow everything to reach an equilibrium temperature and
the temperature is monitored continuously during the calibration. If the temperature changes by more than 0.2°C the calibration is redone. The pulse to pulse variation of the LED is assumed to be small and \( N_{\text{InGaAs}} \) is averaged over several thousand pulses to give \( N_{\text{InGaAs}_{\text{avg}}} \). In addition to the data taken while the LED is pulsing data are taken when the LED is off to get a dark signal necessary to measure the background electronic noise and APD noise enhancement factor (See figure 2.13). The signal level of the APD is simply the dark signal subtracted from the light signal. The APD signal level is averaged over the several thousand pulses to give, \( S_{\text{APD}_{\text{avg}}} \). The fiber optic cables are then switched to find the light ratio, \( L \), between the two detectors. Finally the gain of the absolutely calibrated APD, \( G_{\text{absAPD}} \), is calculated.

![Figure 2.13](image.png)

Figure 2.13: a) A pulse of the LED as seen by an APD. The first half of the raw signal is integrated to get the light signal (orange). The second half of the light signal is integrated to get a baseline dark signal level (blue). Histograms of the 100,000 pulses for the b) light and c) dark signals are shown.
\[ G_{absAPD} = \frac{S_{APD_{avg}} L}{N_{InGaAs_{avg}}} \quad (2.5) \]

In actuality, \( G_{absAPD} \) is a function of wavelength, but the variation is assumed to be small and it is taken to be a constant of wavelength and used in the second calibration step.

**Finding the gain and noise enhancement of all APDs**

For the second step of calibration the light from the LED is input into an integrating sphere with four outputs. The outputs are sent to 3 APDs to be simultaneously calibrated and the one absolutely calibrated APD. All four APDs in this setup are mounted on a water cooled block for temperature control. The light ratios \( L_i \) for each position, \( i \), on the cooling block are calculated relative to the absolutely calibrated APD. 100,000 pulses of the LED are recorded (see Figure 2.13 for typical LED pulse and histogram of light and dark signals) and the gain can be calculated.

\[ G_{APD_i} = \frac{S_{APD_{avg_i}} G_{absAPD}}{S_{absAPD_{avg}}} \quad (2.6) \]

From the combination of light and dark pulses \( F/QE \) values for each APD are calculated where \( QE \) is the quantum efficiency and \( F \) is a factor describing the noise enhancement beyond Poisson statistics\(^{14} \) for each APD so that

\[ SNR = \frac{N \ast QE}{\sqrt{N \ast QE \ast F}} = \sqrt{\frac{N \ast QE}{F}} \quad (2.7) \]

Calculating both the variance of the pulsed LED signal, \( \sigma^2_{pulsed} \), and dark signal, \( \sigma^2_{dark} \), the value of \( F/QE \) for each APD is then calculated to be
Quantum efficiency is a function of wavelength but for uncertainty analysis we assume this value of $F/QE$ is constant. However, Section 2.3.4 describes how a relative $QE$ curve is obtained which can transform this constant $F/QE$ measurement into one that is a function of wavelength. Future refinements will also include repeating this calibration with pulsed LED’s at other wavelengths.

Since this calibration step can take up to 30 minutes the temperature of the APDs during this process must be monitored closely. Both gain and dark current are highly dependent on APD temperature. Temperature differences of just $0.5^\circ C$ can make this measurement unreliable if the temperature difference is not taken into account. With the environmental control and monitoring system currently in use the temperature can be kept constant to within $0.2^\circ C$ and this is not a concern.

The linearity of the detectors has also been tested by comparing the integrated APD signal with the integrated signal on an absolute detector known to have a linear gain in this wavelength region (see Figure 2.14).

### 2.3.3 Daily Calibration

A system is in place to monitor APD functionality on an as needed basis while the APDs are attached to the polychromators in the normal operation setup. This provides a quick and convenient way of measuring the relative gain and functionality of the APDs on a daily basis. The cooling and power setup for these calibrations is the same as the setup during normal operation (Section 2.2.5) so that checks can be made at the
Figure 2.14: The integrated signal of four APDs compared to the integrated signal of an (noisy) absolute detector shows a linear relationship. The slope represents the APD and amplifier module gain, the horizontal width at zero represents the noise on the absolute detector and the vertical width at zero represents the noise of the APD and amplifier module.
beginning of any day without much set up. Two large integrating spheres sit on top of each polychromator rack. The light input to each integrating sphere is an infrared LED pulsed at 1 kHz. Fiber optic outputs from the spheres are fed into bypass holes on the top of the polychromator boxes so the light hits each APD directly. After several pulses of the LED the recorded light level of each APD can be determined. Comparing the light level ratios of each APD to previous light level ratios can reveal changes in electro-optical component functionality of the detection system.

### 2.3.4 Spectral Calibration

In general the spectrum of Thomson scattered light is a relativistic Maxwellian and depends on two parameters: the temperature and the density\(^\text{15}\). The width of the Maxwellian is determined by the temperature and the integral underneath the spectrum determined by the density. In order to correctly fit Thomson scattering data to a Maxwellian the instrument function, a combination of transmission and gain of the spectral channels of each polychromator, must be known.

Two different methods of measuring the polychromator instrument function are described below. The first, a more traditional method\(^\text{16}\), uses a DC light source and is done in the lab whereas the second uses a pulsed light source and is done in-situ. The DC calibration can be done quickly and does not require access to the MST machine area. The ease of this calibration allows it to be performed more frequently to check changes in the polychromators and its components over time. It is an advantage to use a pulsed light source in the second calibration to reveal any slight dependencies on
pulsed light of the system performance. It is also beneficial because this is done in-situ so that it includes the entire path of Thomson scattered light. It is used to check for any systematic changes in the spectral response of the non-polychromator optical components. This includes plasma facing components that are difficult to access and which may be damaged during operation\textsuperscript{17,18}. However, the pulsed light source used for this calibration does not have a consistent pulse to pulse power output nor is the maximum power output of the spectral region of interest consistent. This makes the needed measurements very difficult. In addition to this to get the same resolution as the DC measurement the procedure is tedious and time consuming. A different light source such as a super continuum light source used for spectral calibrations on RFX\textsuperscript{19} could solve these problems but is currently not an option for this system. Because of this the pulsed in-situ calibration is only done before and after long periods of Thomson Scattering operation when the first element of the collection lens has been exposed to plasma and at much lower spectral resolution.

As with the gain and noise enhancement calibrations discussed above the absolutely calibrated APD used as a reference detector is also used in these calibrations. This allows for a simpler setup and an easy comparison between the DC and pulsed in-situ measurements. The DC calibration is also used to find a relative quantum efficiency curve for the APD which allows for the separation of the quantum efficiency and transmission of the instrument function.

**DC measurement of polychromator instrument function**

The first step in the DC measurement serves two purposes: to characterize the DC light source used in the calibration and obtain a relative spectral calibration between the
APD detector and the absolutely-calibrated detectors.

The light source used is a quartz-halogen DC light source. The output of the light source is coupled to the monochromator. The monochromator is scanned at a constant rate from 700 nm to 1100 nm. The light source has a relatively constant output in this range. The output of the monochromator is coupled to a fiber optic that is coupled directly to each of three detectors: The absolutely calibrated APD in Section 2.3.2 biased and amplified in the normal way and InGaAs and Si absolutely calibrated detectors, not biased, amplified by a 3.3e7 transimpedance voltage amplifier. Each detector was illuminated in turn, then the process was repeated. This was done to check two things: that the output from the light source did not change over time and that re-insertion of the fiber into the SMA mount did not change the amount of collected light. The result is shown in Figure 2.15.

To a very high degree, the output did not change (only 3 curves can be seen). At the end of the scan, typically > 1080 nm, the light source was blocked allowing the offset to be measured for each detector. After this a new fiber was connected to each of the polychromators in turn and the signal from the four (or eight) channels was recorded.

The spectral response of the InGaAs and Si detectors is known (shown in figure 2.11). This is used to determine the number of photons as a function of wavelength shown in Figure 2.16.

There are some important differences between the two detectors, even in the regions where the response for both detectors overlap and should be trustworthy. Perhaps this is due to differences in the absolute calibration setup for the two detectors, differences in the absolute calibration process and use (collimated vs diverging light sources) or changes that have occurred since the detectors were calibrated. This disagreement is
Figure 2.15: Raw signal of the three absolute detectors in response to the halogen-quartz light. Two curves for each detector are shown although only 3 curves can be seen which shows the reproducibility of the measurement.

Figure 2.16: Number of photons collected as a function of wavelength in 5 ms period for Si and InGaAs detectors.
currently unresolved. The answer given by the Si detector is the one used in the following calculations for two reasons: 1) It has a good wavelength response over the entire region of interest for the eight channel polychromators, 2) Its response closely resembles that of the APDs.

The instrument function for a given wavelength, $i$, spectral channel, $j$, and polychromator, $k$, is given by:

$$R_{i,j,k} = \frac{N_{\text{abs},i}}{N_{\text{APD},i,j,k}}$$

where $N_{\text{abs}}$ is the number of photons collected by the absolute detector and $N_{\text{APD},i,j,k} = \frac{S_{\text{APD},i,j,k}}{G_{\text{APD},j,k}}$. Figure 2.17 compares instrument functions for a polychromator using the Si and InGaAs detectors. Again, for the reasons above the instrument function using the Si detector is chosen.

Figure 2.17: Instrument function, which is the product of transmission and quantum efficiency for a four channel polychromator.
The instrument function is a product of the optical transmission of the polychromator system, \( T_{i,j,k} \), and the quantum efficiency of the APDs, \( QE_i \).

\[
R_{i,j,k} = T_{i,j,k} QE_i
\]  

(2.10)

The gain of the APD detector which is a combination of quantum efficiency (a function of wavelength) and APD amplifier gain (a constant of wavelength) is only measured at 1020 nm (see Section 2.3.2). As it has been applied thus far the dependence on wavelength has been ignored. However, using the absolutely calibrated APD these two components can be separated. This is an important measurement because the statistical photon noise analysis is tied to the ratio of \( F/QE \) (see above discussion). \( F \), a wavelength independent number, and \( QE \) can not be measured independently and currently their ratio is only measured at one wavelength. The measurement of a relative \( QE \) curve of the APD can be applied to get \( F/QE \) as a function of wavelength and therefore improve uncertainty analysis.

A relative spectral calibration between the absolute APD detector and the absolutely calibrated detectors is made.

\[
QE_i^* = \frac{S_{absAPD_i}}{G_{absAPD}N_{absi}}
\]  

(2.11)

This is not absolute because the quantum efficiency at 1020 nm is coupled to \( G_{absAPD} \).

Figure 2.18 shows \( S_{absAPD_i}/G_{absAPD} \) and the \( QE^* \) curve.

A purely optical transmission function can now be found:

\[
T_{i,j,k} = \frac{R_{i,j,k}}{QE_i^*}
\]  

(2.12)
Figure 2.18: a) Number of photons collected as a function of wavelength in 5 ms period for the absolute APD detector. b) Relative quantum efficiency of the APD.
The next section describes how this optical transmission is found in-situ.

2.3.5 In-situ spectral calibrations

These calibrations are made possible by two key pieces of equipment: the ISIS and a tunable-wavelength laser or optical parametric oscillator (OPO). Similar calibrations have been done before using an OPO\textsuperscript{20} but with this coupled to the ISIS this is a unique in-situ calibration. The OPO which is located in a separate lab from MST is coupled to an integrating sphere with two fiber optic outputs: one to the calibrated reference APD on the cooling block and one to the ISIS inserted to a depth corresponding to the first polychromator to be calibrated. The OPO is pulsed at 8 key wavelengths (740 nm, 790 nm, 830 nm, 880 nm, 950 nm, 990 nm, 1025 nm and 1056 nm). At each of these wavelengths 500 OPO pulses of varying height are recorded by the calibrated reference APD and the corresponding polychromator APDs. These pulses are filtered for good pulses (i.e. light level above noise, not saturated, etc). If the number of good pulses exceeds a threshold value then a line is fit through $N_{APD,abs}$ vs $N_{APD}$ data taking the slope to be the transmission. This is then compared to the DC transmission measurement. This measurement is made initially as a baseline measurement and then again after the first element of the collection lens has been exposed to plasma for long periods of time. If a significant change in spectral transmission is seen the first element of the collection lens can be replaced. Figure 2.19 shows a comparison between these measurements. Even though a damaged lens exhibits a decrease in overall transmission this is not a function of wavelength and does not affect temperature measurements. However, the signal to noise ratio is increased and future density measurements must take this drop
in to account.

Figure 2.19: Transmission for an 8 channel polychromator measured in the lab with a broad DC light source (solid line), measured in-situ with a lens that has not been exposed to plasma using the OPO light source (diamonds), and measured in-situ after the lens has been exposed to the plasma for several weeks of operation (triangle). Note that the drop in transmission does not have a strong wavelength dependence. The output of the OPO decreases above 980 nm and the measurements become unreliable.

### 2.4 Electron Temperature Fitting

This section discusses how data collected using the above described hardware during MST operation are fit using the calibration information to give electron temperature. Analysis of the raw Thomson scattering data to get electron temperature requires two main steps: determining the number of background and scattered photons from the raw signal and using the number of background and scattered photons in each polychromator channel to find the most likely temperature and associated uncertainty. In addition to
this, the precise timing of each laser pulse is recorded by a diode located in a box above MST through which the laser passes (see Figure 2.20). The diode detects stray light from the laser and is digitized on a TR1612 at 6 MHz.

![Figure 2.20: A laser diode near the beam path records stray laser light and recording precise timing information of the laser pulses relative to the master MST clock. Red dots identify the laser fire time of quickly spaced pulses.](image)

2.4.1 Fitting the raw signal

The detector output is digitized for 2 $\mu$s around the laser pulse. The first step in fitting the raw signal is to locate the peak in the scattered light pulse, $t_0$, within this 2 $\mu$s. Wildly varying background light levels (see Figure 2.9), and a combination of high noise and low signal can make this more than a trivial task. Since each channel on the polychromator is digitized for the same time window and the same scattered light pulse is recorded in each channel it follows that for a given laser pulse the scattered light signal
is at the same time in the digitized output of each channel in a given polychromator. The peak (lowest point for a negative going signal) is found by adding the signal from all channels used for fitting in a single polychromator together. A rough polynomial fit is done to this added signal to subtract off the background light. The lowest point of the added-background-subtracted signal is taken to be \( t_0 \) (see Figure 2.21).

Given a short (\( \sim 10\text{-}40 \text{ ns} \)) input light pulse to an APD with corresponding amplifier the digitized output pulse has a known shape. The area under the pulse has been shown to scale linearly with number of photons (see Figure 2.14) up until the APD saturates. This characteristic signal, \( C(t) \), shown in Figure 2.22, can be used to extract the scattered light signal from background noise. For each channel, \( j \), a functional fit is performed:

\[
F_j(t) = a_j C(t - t_0) + b_0 + b_1 t + b_2^2
\]

(2.13)

Here \( a_j \) is the height of the characteristic pulse and \( b_0, b_1 \) and \( b_2 \) are polynomial coefficients that describe the background light. \( C(t) \) is normalized so that:

\[
\int C(t)dt = 1 \quad (2.14)
\]

The scattered photons detected, \( N_{j,\text{scat}} \) is then simply:

\[
N_{j,\text{scat}} = \int a_j C(t)dt \quad (2.15)
\]

The uncertainty in this measurement scales with \( \sqrt{N} \), where \( N \) is the total number of photons. So the number of background photons detected is also important. The number of background photons is more complicated because the APD response is not a top-hat
Figure 2.21: a) Raw voltage signals from the three polychromator channels used in the temperature fitting. The signal from APD 3 is low and comparable to background noise levels and is difficult to pick out without additional information. b) Adding the raw voltage signals for all three fit channels unambiguously determines the timing of the laser pulse within the 2 µs window.
Figure 2.22: a) Characteristic output pulse of an APD and amplifier module when a short pulse of light is incident onto the APD. b) Raw voltage signals (blue), initial fit guess (yellow), polynomial background fit (red-dash), and fit of characteristic pulse shape plus background (red). Using the characteristic pulse helps to find the signal out of the q-switch noise in channel 1 and provides and effective higher bit resolution for channel 3.
shape, but instead rises quickly and falls off slowly. A Monte Carlo method was used to
determine how the background photons contribute to the uncertainty over the pulse.\textsuperscript{7}

2.4.2 Fitting the most-likely temperature

The collected scattered light is broken up by the polychromators into either 4 or 8 chan-
nels depending on the polychromator. Currently the 0th channel on each polychromator,
which detects light at the laser wavelength of 1064 nm, is flooded by light reflected off of
MST at the laser exit.\textsuperscript{9} Because of this only channels 1-3 or 1-7 are used to fit electron
temperature. It is assumed that the photon spectral density is a relativistic Maxwellian
given by the Selden equation.\textsuperscript{15} The Selden equation, \( S \), depends on the two parameters
electron temperature, \( T_e \), and electron density, \( n_e \) and is a function of wavelength, \( \lambda \).
The electron temperature determines the shape and the electron density determines the
integral of the spectral photon output. Since the 0th channels of all the polychromators
are currently saturated absolute electron density measurements are impossible. So while
a density fit is made it is only a relative measurement. For an APD channel, \( j \), with
and instrument function \( R_j(\lambda) \), the number of expected photons is

\[
N_{\text{model}_j} = \int S_{T_e,n}(\lambda) R_j(\lambda) d\lambda
\]

(2.16)

The number of photons actually measured is \( N_{\text{APD}_j} \) which differs in practice from
\( N_{\text{model}_j} \). The uncertainty associated with this measurement is given by \( \sqrt{N \times QE \times F} \),
where \( N \) is the total number of photons hitting the detector (see section 2.3.2). However,
converting the number of photons in each channel to temperature is not a trivial task
since equation 2.16 is not solvable for temperature. The framework for which this is
Bayesian analysis (see Appendix A for an introduction to Bayesian probability theory) which has proven to be a useful TS analysis technique\textsuperscript{21}.

Bayes theorem tells us that:

\[
P(T_e, n_e|d, \sigma, I) \propto P(d|T_e, n_e, \sigma, I)P(T_e, n_e|I) \tag{2.17}
\]

It is expected that the \(T_e\) and \(n_e\) measured will be within the ranges of the instruments. So the prior is taken to be a constant probability across the range of the instrument. The posterior is then calculated as such:

\[
P(d|T_e, n_e, \sigma, I) = \frac{1}{\prod_{j=1}^{N_d} \sqrt{2\pi} \sigma_j} \exp \left( -\frac{1}{2} \chi^2 \right) \tag{2.18}
\]

\[
\chi^2 = \sum_{j=1}^{N_d} \frac{[N_{APD_j} - N_{model_j}(T_e, n_e)]^2}{\sigma_j^2} \tag{2.19}
\]

Here, \(N_d\) is the number of polychromator spectral channels used in the fit. At the present time, calibrations have not been done to make an absolute measurement of density so it is marginalized out.

\[
P(T_e|d, \sigma, I) = \int P(T_e, n_e|d, \sigma, I)dn_e \tag{2.20}
\]

Here, the integral is over all possible values for \(n_e\). The value for \(T_e\) is the value with the highest probability with an error that describes the width of the probability distribution function.
Figure 2.23: a) The probability distribution as a function of $T_e$ and $n_e$. b) The marginalized probability distribution function for temperature. The red bar shows a characteristic $1/e$ error approximation.
REFERENCES


[12] PerkinElmer, “Model c30956e.”


Chapter 3

Detecting Electron Temperature Fluctuations

Typically fluctuation measurements are made by analyzing a continuously time (or space) resolved signal by decomposition via a Fourier transform. Generally the Nyquist theorem puts a limit on the highest frequency (wavenumber) resolvable to be less than twice the sampling rate (spacing)\(^{1-3}\). The number of resolvable frequencies is also limited by the number of samples to be half that number. Since the nature of the Thomson scattering (TS) diagnostic does not allow for a continuous sampling of data - the total sample number is limited - a single shot analysis by Fourier decomposition is not useful. Instead a pseudo-spectral analysis is performed with the assumption that the electron temperature is correlated with magnetic tearing modes. Pseudospectral fluctuation analysis is often done on MST however it is usually done with signals that have a continuous sample rate greater than the Nyquist limit. Bayesian analysis techniques are employed to properly map \(T_e\) to magnetic fluctuations over an ensemble of plasmas and resolve a pseudo-spectrum.

The following describes how Bayesian analysis techniques are used to perform a pseudo-spectral analysis of electron temperature correlated to magnetic fluctuations (measured by the toroidal array). Section 3.1 describes how electron temperature is
mapped on to magnetic islands in the plasma. Section 3.2 describes how Bayesian probability theory is used to ensemble the data from many different plasma discharges. Finally two hypothesized outcomes of this analysis are presented in Section 3.3.

3.1 Definition of a phase

The sampling rate of the TS diagnostic can easily be adjusted to twice the frequency of tearing modes (∼20 kHz). The problem lies with the number of samples taken during a single plasma discharge which can be as few as two or up to six in a short burst (the burst of pulses can be repeated a millisecond later, however the equilibrium and/or fluctuation may have changed and each burst must be treated as a separate set of data). The information from a single burst of pulses is insufficient to unambiguously resolve the fluctuation spectrum (See Figure 3.1). Bursts of laser pulses are separated by periods of time much longer than the fluctuation timescales of interest and the fluctuation spectrum may change enough over that time that a simple temporal analysis is fruitless. Instead phase information from magnetic fluctuations is used to combine data from different sets of laser bursts. This pairing of information gives a clearer picture of both the magnetic field and electron temperature that either diagnostic could give alone. The weaknesses of one diagnostic is complemented by the strengths of the other; the local, radially-resolved electron temperature measurements reveal intricacies of the magnetic field structure in the core of the MST and the detailed time resolution of the edge magnetic field measurements provide a clear picture of the time (and spacial) dependence of the electron temperature.

Since plasmas rotate in the MST in both the toroidal and poloidal direction, spatially
Figure 3.1: a) In an ideal world a fluctuating signal could be sampled continuously in time with no uncertainty. This shows an ideal single frequency signal measurement in time. b) In reality experimental measurements have uncertainty associated with them. This plot shows a continuously sampled signal with uncertainty equal to the fluctuation level. c) Two red points here represent the information gained in a single plasma discharge with TS. d) With only two data from a single plasma discharge the fluctuating signal can not be resolved. Several possible signals are shown (dashed lines).
resolved and temporally resolved signals are related. Measurements made at a single
location at different times samples different spatial locations within the plasma as it
rotates past the measurement point.

The TS diagnostic sits at a fixed toroidal location on MST ($\phi_{TS} = 222^\circ$) and measures
electron temperature at a fixed poloidal location ($\theta_{TS} = -90^\circ$) along the minor radius.
The phases of the magnetic fluctuations stored in the MST database are phases measured
by the toroidal array at the poloidal gap ($\phi_{TA} = 0^\circ$, $\theta_{TA} = 241^\circ$). At any given time
the poloidal component for a given mode (with poloidal wavenumber, $m$ and toroidal
wavenumber $n$) at the toroidal array is:

$$b_{pTA} = b_{pm,n} \cos \Delta_{m,n}$$  \hspace{1cm} (3.1)

Here, $b_{pm,n}$ is the poloidal amplitude and $\Delta_{m,n}$ is the phase stored in the MST database.
The poloidal component of the fluctuation at the TS location is

$$b_{pTS} = b_{pTA} \exp \left(i[m(\theta_{TS} - \theta_{TA}) + n(\phi_{TS} - \phi_{TA})]\right)$$  \hspace{1cm} (3.2)

or

$$b_{pTS} = b_{pm,n} \cos(\Delta_{m,n} + m(\theta_{TS} - \theta_{TA}) + n(\phi_{TS} - \phi_{TA}))$$  \hspace{1cm} (3.3)

The phase of the magnetic fluctuation at the TS location is:

$$\Delta_{TS_{m,n}} = \Delta_{m,n} + m(\theta_{TS} - \theta_{TA}) + n(\phi_{TS} - \phi_{TA})$$  \hspace{1cm} (3.4)

In MST $m$ and $n$ are related with $m = 1, n = -6, -7, -8...$ and $m = 0$ for
$n = -1, -2, -3, -4^4$. In standard plasmas the only ambiguity is with $n = -5$ which
can either be \( m = 1 \) or \( m = 0 \). For an \( m = 0 \) mode the poloidal component of the fluctuation is zero, so from here on out the \( m \) subscript will be dropped. For plasmas with deep reversal, such as those discussed in Section 4.4 the higher \( n \) modes of interest may become resonant in the plasma edge past the reversal. In this case the positive and negative modes are counter-propagating and can be separated. The phase of the fluctuation is defined so that when the O-point of the mode is being viewed by the TS diagnostic \( \Delta_{TS_n} = 0 \) see Figure 3.2.

\[ \Delta_{TS,n} = 0 \quad \Delta_{TS,n} = \pi \]

Figure 3.2: As a tearing mode rotates around MST different phases (positions) of the mode are sampled by the diagnostic. The black line is the TS viewing location and the grey object is the tearing mode. The phase of the mode is zero when the TS diagnostic is viewing the O-point of a tearing mode (a) and \( \pi \) when viewing the X-point (b).

The electron temperature at a given radial location is modeled as a simple fluctuations on top of the mean temperature,

\[
T_e(r) = T_{e0}(r) + \tilde{T}_{e,n}(r) \cos \Delta_{TS_n}
\]  

(3.5)

where \( T_e \) is the measured electron temperature, \( T_{e0} \) is the mean electron temperature
and $\tilde{T}_{e,n}$ characterizes the fluctuating component in phase with the given magnetic fluctuation.

### 3.2 Using Bayesian Analysis to ensemble data

In a single plasma shot $T_e$ is measured between two and six times in rapid succession (5-100 µs apart). An analysis method is needed which properly combines data from several different discharges and bursts of laser pulses while keeping in mind that shot to shot (or burst to burst) equilibrium changes may be on the order of the fluctuation level. Bayesian analysis (See Appendix A) provides a framework in which the data can be ensembled taking full advantage of all of the available information. For the case of two time points in a given shot, only one model representation can be drawn directly through the two data points. The data, however has some uncertainty associated with it which Bayesian probability theory allows us to keep in our analysis. Figure 3.3 shows that by keeping the uncertainty we can now draw an infinite number of model realizations through the data. The two model realizations shown in the figure have a finite and unequal probability of being true.

The probability that a specific model realization is true can be found with Bayes Theorem. For the case of two temperature measurements:

$$P(\tilde{T}_{e,n}, T_{e0}|T_{eA}, T_{eB}, I) \propto P(T_{eA}|\tilde{T}_{e,n}, T_{e0}, I)P(T_{eB}|\tilde{T}_{e,n}, T_{e0}, I)P(\tilde{T}_{e,n}, T_{e0}|I)$$

(3.6)

Here, $T_{eA}$ and $T_{eB}$ (or generally $T_{ei}$) are electron temperature measurements taken in quick succession. Equation 3.6 is explicitly for two temperature measurements but this could be expanded to include as many as the TS diagnostic can provide. A gaussian
Figure 3.3: An example of two separate model realizations that have a finite probability of being true.

The probability distribution for the electron temperature measurement is assumed to calculate the likelihood PDF.

\[ P(T_e|\tilde{T}_e, T_{e0}, I) = \frac{\exp(-\frac{1}{2}\chi_i^2)}{\sigma_i\sqrt{2\pi}} \]  

(3.7)

\[ \chi_i^2 = \left(\frac{T_{e,\text{measured},i} - T_{e,\text{expected}}(\tilde{T}_e, T_{e0}, \Delta T_{S_n})}{\sigma_i}\right)^2 \]  

(3.8)

The prior PDF, \( P(\tilde{T}_e, T_{e0}|I) \), is taken to be flat over physically reasonable values. With this, the right hand side of Equation 3.6 can be evaluated.

Equation 3.6 does not find the true values of \( \tilde{T}_e \) and \( T_{e0} \) rather it finds the probability that specific combinations of \( \tilde{T}_e \) and \( T_{e0} \) are true. The difference between Bayesian probability theory and orthodox statistics is highlighted here, although it may be subtle. Typically orthodox statistics deals with the distribution of measurements - if you flip a coin a thousand times how many times it lands on heads and how many times it
lands on tails. Bayesian probability theory answers a different question: what is the true fairness value of the coin? Each flip of the coin is evidence gathered to judge the fairness of the coin, but fairness itself is not measured with each flip. Both methods will eventually come to the same conclusions about the coin, but will get there by different means. Bayesian probability theory is chosen here because it is impossible to measure with no uncertainty the values of $\tilde{T}_e$ and $T_{e0}$ each shot. In essence the question is not what is the distribution of $\tilde{T}_e$ and $T_{e0}$, but rather what is the true value of $\tilde{T}_e$ and $T_{e0}$.

In order to do this the assumption must be made that there is a true value. At any given time this is certainly the case, however the plasma system is dynamic and there may be shot-to-shot variation. Because of this data chosen to ensemble together have tight restraints to ensure that the true value of $\tilde{T}_e$ and $T_{e0}$ are very close for all each data points in the ensemble.

Shot-to-shot variation\(^1\) in $T_{e0}$ is on order of the expected value of $\tilde{T}_e$. In order to find $\tilde{T}_e$ it is assumed that $\tilde{T}_e$ is constant shot to shot, but $T_{e0}$ is not necessarily constant. Thus $\tilde{T}_{e,n}$ becomes the interesting quantity and $T_{e0}$ a nuisance parameter which is marginalized out for each shot. The probability for a single burst of pulses is marginalized with respect to $T_{e0}$ and the PDF becomes a function of $\tilde{T}_e$ alone.

\[
P(\tilde{T}_e|T_{eA}, T_{eB}, I) = \int P(\tilde{T}_e, T_{e0}|T_{eA}, T_{eB}, I) dT_{e0}
\] (3.9)

Once the PDF for each burst has been found in this form the product rule is used to combine data from all shots.

\(^1\)From here on out "shot" means a temporal grouping of data. Bursts of data separated by 1 ms are considered separate "shots" as well as data taken in different plasma discharges.
\begin{equation}
    P(\tilde{T}_e|T_{\text{all}}, I) = \prod_{i}^{\text{shots}} P(\tilde{T}_e|T_{eA_i}, T_{eB_i}, I)
\end{equation}

Figure 3.4 shows how the PDF of Equation 3.10 collapses down to a narrow range of possible values for $\tilde{T}_{e, 6}$ as information from additional MST shots is added to the ensemble.

Figure 3.4: The probability evolves with the data from each shot as more information is added.

In order to find $T_{e0}$ the assumption must be made that shot-to-shot variation in the mean temperature is small. This was an assumption explicitly not made above because the mean variation is probably greater than the variation in the fluctuation. The PDF
for $T_{e0}$ is found similarly with the marginalization step done with respect to $\bar{T}_e$, treating it as a nuisance parameter and keeping $T_{e0}$ as an interesting quantity. For data sets which are tightly constrained this may be a good assumption. However, if the shot-to-shot variation is too large it is an ill posed problem and all values of the PDF drop to zero over a large ensemble.

### 3.3 Two possible temperature structures

Two different scenarios for electron temperature fluctuations associated with an isolated magnetic island are presented in Figures 3.5 and 3.6. The first scenario is an isothermal island in a background plasma with a temperature gradient, with hotter temperatures inside the resonant surface and cooler temperatures outside the resonant surface. Fluctuations associated with an isothermal island structure will exhibit a phase flip (fluctuation amplitude passes through zero and reverses sign) across the resonant surface for the mode. This is because the fluctuations on the inside of the resonant surface tend to lower the temperature and fluctuations on the outside of the resonant surface tend to raise the temperature to produce a flat temperature in the island region. In this case heating sources for the electron temperature are located far from the island region, rapid equilibration is assumed along the helical magnetic field lines and slow transport processes are assumed across magnetic surfaces.

On the other hand if a heating source is present within the island separatrix and intact magnetic surfaces exist, a peaked temperature profile inside the island separatrix is present with a hot spot located at the O-point of the island. For this case, no phase flip is observed. Fluctuations across all radii within the island tend to raise the temperature
Figure 3.5: Electron temperature model for an isothermal island. Electron temperature is measured along the black vertical line from the core to the edge of MST. With $T_e$ constant inside the island (light blue): Inner circle (red) - $T_e$ is lower than circle average at O-point, Middle circle (orange) - $T_e$ is constant, Outer circle (green) - $T_e$ is higher than circle average at O-point.
at the O-point, with the largest fluctuation amplitude present at the location of the hot spot.

Figure 3.6: Electron temperature model for an island with a temperature peak. Electron temperature is measured along the black vertical line from the core to the edge of MST. With a $T_e$ peak (dark blue) inside island structure: $T_e$ fluctuations are highest for the middle circle (orange) sampling the hot spot. The Thomson scattering diagnostic samples a hotter temperature for the red, orange and green circles when viewing an O-point rather than the X-point.

These are two possible scenarios, but not an exhaustive list, nor is it necessary that fluctuations which exhibit a phase flip represent a flattening temperature or fluctuations of all the same sign exhaustive evidence for a peaked temperature. Mean electron temperature and the fluctuating component must be combined to give the bigger picture.
REFERENCES


Chapter 4

Electron temperature in and around magnetic islands

Electron temperature fluctuations associated with tearing modes are studied in four cases and several relationships between electron temperature and magnetic structure are observed. In the first case, standard MST discharges are studied in the relatively quiescent period between sawtooth crashes and the electron temperature is observed to flatten across core remnant islands (Section 4.1). Section 4.2 deals with electron temperature associated with core tearing modes through a sawtooth crash, where in addition to temperature flattening, the electron temperature is found to peak in relation to a mode near the magnetic axis just after the crash. Fluctuations through core tearing events are presented in Section 4.3. Finally, electron temperature fluctuations are found to be small during plasmas with reduced magnetic tearing fluctuations in (Section 4.4).

4.1 Quiescent plasmas

Measurements have been made of electron temperature fluctuations correlated with magnetic modes present in MST between sawteeth. The ensemble of data was collected far
away (> 3 ms) from the closest sawtooth crash, in so-called quiescent periods when tearing mode activity is low. Plasma current is 400 kA ±15%, plasma density is \( \sim 10^{13}/cm^3 \) and the reversal parameter, \( F \), is \( \sim -0.2 \). All data was taken during plasmas where the mode of interest is present and rotating. Furthermore the data was filtered for Thomson scattering data quality.

Three modes are studied: \( m = 1, n = -6, -7 \) and \(-8\). All three modes are resonant in the plasma according to the reconstructed q-profile (see Figure 4.1) and the core most resonant mode, \( n = -6 \), has the largest signal. The \( n = -6 \) mode is presented in Section 4.1.1 and the higher \( n \) modes are presented in Section 4.1.2.

![Figure 4.1: Safety factor profile between sawteeth.](image-url)
4.1.1  \( m=1, n=-6 \) Isothermal Island

Electron temperature fluctuations associated with the \( m = 1, n = -6 \) tearing mode are observed to be consistent with the isothermal island scenario presented in Section 3.3. Fluctuation amplitudes are around 15 eV in a background of \( \sim 350 \) eV with a clearly defined point where the fluctuations pass through zero and flip sign, pinpointing the resonant surface of the mode. These observations (see Figure 4.2) are consistent with the presence of a remnant island structure between sawtooth crashes. By combining \( \tilde{T}_{e,-6} \) and \( T_{e0} \) results it is obvious that the slope across the X-point and O-point are different. A steep gradient exists across the X-point of \( \sim 800 \) eV/m (decreasing with increasing radius) while the temperature across the O-point is relatively flat at \( \sim 154 \) eV/m. As a consistency check the expected gradient across the X-point of the mode should be:

\[
\left[ \frac{\partial T_{e,n}}{\partial r} \right]_{\text{X,expected}} = \frac{2(\tilde{T}_{e,n}\text{max} - \tilde{T}_{e,n}\text{min})}{w_{n,\text{measured}}}
\]

(4.1)

Here, \( w_{n,\text{measured}} \) is the width of the island measured from the minimum to the maximum in the electron temperature fluctuations. For the \( n = -6 \) mode \( w_{-6,\text{measured}} \sim 0.08 \) m and the difference in the minimum and maximum temperature fluctuations is 30 eV making the expected gradient across the X-point to be 750 eV/m. This is very close to the measured gradient of 800 eV/m showing that the fluctuation results and equilibrium results are consistent with one another. This is not trivial. The variation in temperature measured by one polychromator and the polychromators adjacent to it may be larger than the calculated uncertainty and still may not be a significant difference because of systematic errors. The fluctuation analysis eliminates any systematic calibration errors.
which may be present when comparing data from different polychromators. Fluctuation results are therefore more robust than measurements in the absolute temperature from any one polychromator. However, the consistency between the expected gradient derived from fluctuation measurements and the gradient measured from combined equilibrium and fluctuation results show that the equilibrium results are also fairly accurate.

The expected island width is given by:

$$w_n = 4 \sqrt{\frac{r_{sn} b_{rn}(r_s)}{n|q'_n|B_p}}$$

Here, $r_{sn}$ is the radial value of the rational surface, $b_{rn}(r_s)$ is the radial magnetic fluctuation amplitude at the resonant surface, $B_p$ is the equilibrium poloidal magnetic field and $q'_n$ is the gradient in the $q$-profile at the rational surface for mode $n$. The temperature fluctuation data shows that $r_{sn} = 0.13 \text{ m}$. This is closer in than the reconstructed $q$-profile shown in Figure 4.1. Since the fluctuation data represents a direct measurement and not a reconstruction it is assumed to be the more accurate of the two. $b_{rn}(r)$ cannot be measured directly although it can be inferred from the poloidal magnetic fluctuation measured at the wall, $b_{p,n}(a)^1$. The necessary conversion factor, found in Dave Ennis’ Ph.D. thesis, gives the relationship to be $b_{rn}(r) = 4.2b_{p,n}(a)$. The gradient in the $q$-profile is estimated from the reconstructed $q$-profile to be $\sim 0.29 \text{ m}^{-1}$. It is important to note that small changes to the gradient in $q$ can result in changes in the island with of several centimeters. Since the $q$-profile doesn’t match the fluctuation results exactly this is a known source of great uncertainty. Finally, $b_{p,n}(a) \sim 8 \times 10^{-4} \text{ T}$ and $B_p = 0.18 \text{ T}$ giving an estimated island width of $\sim 15 \text{ cm}$. This is almost double the measured island width (from min to max in the fluctuation amplitude) of $\sim 8 \text{ cm}$. The measured island width is much smaller than the expected island width because the $m = 1$, $n = -6$
Figure 4.2: a) Electron temperature fluctuation amplitude associated with the $m = 1$, $n = -6$ tearing mode between sawteeth. Green vertical line estimates the resonant surface. b) Combining mean temperature and fluctuation amplitude results. The electron temperature is plotted across an X-point (left side) and across an O-point (right side). Red dashed lines are a linear best fit across island (highlighted blue) region.
fluctuation is not the only fluctuation in the field. The observed structure is only a remnant island structure in a background field with some level of stochasticity\(^2\). The field is not completely stochastic since there are fluctuations correlated with the magnetic fluctuations but there is enough background fluctuation in the magnetic field to affect the island structure produced by the \(m = 1, n = -6\) fluctuation.

**Estimating the safety factor profile**

Electron temperature fluctuation measurements, such as those presented above, provide an estimate of the position of rational surfaces within the plasma to within 2 cm. Currently \(q\)-profiles (such as those shown in Figure 4.1) are reconstructed using MSTFit. The value of \(q\) on axis can be constrained with Motional Stark Effect measurements, but further constraints of \(q\) in the core have not been applied in MSTFit. The ability to locate rational surfaces in the core is a valuable and unique ability of the TS diagnostic. Figure 4.3 shows how the \(q\)-profile may be estimated using fluctuation data from the TS diagnostic. According to the symmetric island picture (i.e. Figure 1.3) the zero-crossing of the electron temperature fluctuations should pinpoint the resonant surface for a given mode. However, if the island is asymmetrical (perhaps because of interactions with neighboring modes) and the O-point and X-point are not at the same radial location the zero-crossing in the fluctuations may be misleading. Temperature fluctuation data is sensitive to the O-point of the island and the X-point, which should always lie on the resonant surface.
In addition to fluctuations associated with the large $n = -6$ tearing mode, fluctuations associated with the $n = -7$ and $n = -8$ modes have been measured. The amplitudes of the magnetic fluctuations associated with $n = -7$ and $n = -8$ are smaller than the $n = -6$ mode and their proximity to each other and higher $n$ modes results in a significant overlap of modes and a stochastic field. DEBS simulations predict a fully stochastic field in the region of the resonant surfaces for these modes.

Although it is not as clear, the measured $n = -7$ fluctuations between sawteeth (see Figure 4.4) are similar in character to the $n = -6$ fluctuations, indicative of the presence of a remnant island structure. Fluctuations are 13 eV outside the rational surface and 6 eV inside. The temperature gradient across the O-point ($\sim 4$ eV/m), is
flatter than the slope across the X-point (∼233 eV/m) suggesting a remnant island structure. This is close to the expected temperature gradient across the X-point of 290 eV/m from Equation 4.1. The measured resonant surface is at approximately 0.3 r/a (or 16 cm) with an approximate island width of 13 cm. There is therefore significant overlap between the observed $n = -7$ and $n = -6$ structures. The pseudo-spectral analysis method described in Chapter 3 eliminates all modes which are not always in phase with the desired mode. In general the two modes are not locked together and since by definition have a different spatial structure, it is likely that these fluctuations are indeed due to the $n = -7$ mode and not an aliasing effect from the $n = -6$ mode. Overlapping remnant island structures are difficult to picture, but the data suggests that the $n = -7$ mode has some effect within the $n = -6$ structure.

Electron temperature fluctuations associated with the $n = -8$ tearing mode (see Figure 4.5) show little similarity to the $n = -6$ and $n = -7$ modes. The magnitudes of the fluctuations are small (<10 eV). Non-zero fluctuations seem to be localized to a small radial location and do not appear to be centered on the $n = -8$ rational surface. Instead the fluctuations are inside of the remnant $n = -6$ island structure. The amplitude of the temperature fluctuations do appear to flip sign, but the phase relation is opposite of that observed for the $n = -7$ and $n = -6$. The phase relation is such that the fluctuations tend to raise the temperature at inner radii and lower the temperature at the outer radii. For this to be the case there would need to be a hot bump in the temperature profile in the mid radii of the plasma. Indeed this hot bump is observed in the electron temperature profile across the X-point of the mode in Figure 4.5b. The temperature gradient across the X-point is ∼280 eV/m (increasing with increasing radii). The temperature across the O-point is flatter with a gradient
Figure 4.4: a) Electron temperature fluctuation amplitude associated with the $m = 1$, $n = -7$ tearing mode between sawteeth. Green vertical line estimates the resonant surface. b) Combining mean temperature and fluctuation amplitude results. The electron temperature is plotted across an X-point (left side) and across an O-point (right side). Red dashed lines are a linear best fit across island (highlighted blue) region.
of $\sim 150$ eV/m (decreasing with increasing radii). The expected gradient across the X-point of this structure (from Equation 4.1) is 350 eV/m. Both the equilibrium and the fluctuation measurements show this slight bump in the temperature profile in the frame of reference of the $m = 1$, $n = -8$ magnetic fluctuation, suggesting a heat source located around $r/a = 0.4$. As another consistency check, Figure 4.6 shows $T_e(r, \Delta r_s = \pi/2)$ for $n = -6, -7$ and $-8$. All three plots should be the same, although they will not be exactly the same because the assumption that the temperature is only correlated with one mode is not exact. Indeed the mean temperatures for each mode are very close. The small bump in the profile for $T_{e0}$ does persist except for the one data point at $r/a = 0.4/4$. It is possible that the dynamo for the $n = -6$ is flattening the current profile in the core in such a way to cause a local bump in the current density around $r/a = 0.4$. However more work than what is presented here will be needed to confirm this hypothesis.

**Electron temperature in 3D**

Electron temperature is not simply a 1D quantity as it is usually depicted, instead it follows the complicated magnetic structure of the plasma. Figure 4.7 shows a contour plot of temperature between sawteeth for a poloidal cross section of MST. Here, the $n = -6$, $n = -7$ and $n = -8$ modes are 120 degrees out of phase with one another. At another toroidal location where the modes have a different phase relationship the temperature contour would be different.
Figure 4.5: a) Electron temperature fluctuation amplitude associated with the $m = 1$, $n = -8$ tearing mode between sawteeth. b) The mean temperature and fluctuating component show the temperature gradients across the assumed X and O-points of the magnetic fluctuation are relatively flat. Red dashed lines are a linear best fit across the observed fluctuation region (highlighted pink).
Figure 4.6: Electron temperature between sawteeth for modes $n = -6, -7$ and $-8$ away from both the X-point and O-point of the mode, $\Delta T_{S_n} = \pi/2$.

4.2 Correlated temperature fluctuations through a sawtooth

Electron temperature fluctuations, correlated with magnetic modes, exhibit a strong sawtooth dependence. As magnetic fluctuation amplitudes increase and the magnetic field becomes stochastic, temperature fluctuations correlated with the $m = 1, n = -6$ tearing mode decrease until they are completely gone at the sawtooth crash. After the sawtooth crash the $m = 1, n = -6$ electron temperature structure reappears as well as a $m = 1, n = -5$ helical peak in the electron temperature. Results for the $m = 1, n = -6$ mode are presented in Section 4.2.1 and results for the $m = 1, n = -5$ mode are presented in Section 4.2.2. All data was taken with the same plasma parameters described in Section 4.1 and binned relative to the sawtooth crash in 200 $\mu$s bins.
Figure 4.7: Electron temperature across a poloidal cross section of MST with the $n = -6, -7$ and $-8$ modes 120 degrees out of phase with one another. Electron temperature fluctuations have a significant impact. A simple 1-dimensional profile is not sufficient to describe the temperature.
4.2.1 \( m=1, n=-6 \), Island Structure Disappears

Electron temperature fluctuations associated with the \( m = 1, n = -6 \) tearing mode have been observed through a sawtooth crash. Temperature fluctuations show a strong sawtooth dependence which is summarized in Figures 4.8, 4.9 and 4.10. For reference, the full set of fluctuation plots and mean plus fluctuation profiles of the temperature are in Appendix B. As the tearing fluctuations peak at the sawtooth crash and the field becomes stochastic, heat is rapidly transported outward, the gradient in the mean temperature is reduced and correlated electron temperature fluctuations disappear. Before the sawtooth crash the temperature profile looks similar to the profile during quiescent plasmas with a slight peak in the temperature on axis, a flattening across the remnant \( n = -6 \) structure and a slight bump in the temperature around \( r/a = 0.4 \). As the magnetic field becomes more stochastic the temperature structure within the core region is washed out; at -0.7 ms there is no recognizable on-axis peak in the temperature profile. After the temperature is flattened in the entire core region the global temperature begins to drop and does not rise again until \( \sim 0.5 \) ms after the crash when the \( n = -5 \) temperature structure appears (see Section 4.2.2). All correlated fluctuations drop (within uncertainties) to zero at the crash. This reinforces the hypothesis that a remnant island structure is present between sawteeth but the field becomes stochastic at the crash. If the field were always stochastic the electron temperature in the core would be flat (without even small localized gradients seen in Section 4.1) and would not be correlated to magnetic modes. The canonical isothermal island temperature fluctuation does not appear again until there is a temperature gradient in the mean temperature and the \( n = -6 \) mode becomes relatively large at \( \sim 1 \) ms after the sawtooth crash.
Figure 4.8: a) Amplitude of the $m = 1, n = -6$ magnetic tearing mode relative to the sawtooth crash is shown. Dashed lines correspond to plots b)-e) of electron temperature fluctuation amplitude over the minor radius for various times. b) Fluctuations are still large 0.9 ms before the crash. c) - d) Fluctuations correlated with $m = 1, n = -6$ tearing mode disappear at the crash. e) Canonical isothermal island fluctuations reappear 1.1 ms after the crash.
Figure 4.9 shows the time history of the peak in electron temperature fluctuation both inside and outside the rational surface through the sawtooth crash. The time history of the inside peak and outside peak are different. Inside the rational surface, the amplitude of the temperature fluctuations begin to decrease at -2 ms. At about the same time the temperature fluctuations outside the rational surface begin to rise, until \( \sim 1 \) ms before the crash where they begin to quickly ramp down. The difference in inside and outside time history reveals intricacies of the temperature profile; the gradient inside and outside the rational surface is not constant and the island isn’t symmetric about the rational surface.

The radial position of the remnant magnetic island structure can be mapped relative to the sawtooth crash. Figure 4.10 shows that the position of the temperature structure associated with this mode moves outward after the crash from the broadening of the \( J(r) \) profile and increase in \( q \) on axis\(^3\). The rational surface before the crash is constant at \( r \sim 10 \) cm. When the correlated temperature fluctuations reappear after the crash the rational surface for the \( n = -6 \) mode has moved outward to \( r \sim 18 \) cm. It then moves slowly inward. This trend is consistent with magnetic structures seen in Figures 1.4 and 4.11 from 3D resistive simulations. When the remnant island is present the measured island width is approximately the same (\( \sim 8 \) cm) for all times except just before the sawtooth crash when the width of the island jumps up to over 20 cm. At that point the amplitudes of the fluctuations are small since the gradient in the core is nearly flat.
Figure 4.9: The amplitude of the electron temperature fluctuations show a strong saw-tooth dependence, disappearing at the crash. a) Peak in temperature fluctuation amplitude inside (a) and outside (b) the rational surface through a sawtooth crash. The time histories are different inside and outside the rational surface.
Figure 4.10: Rational surface location(a), island width(b) and temperature gradient(c) for $m = 1$, $n = -6$ through a sawtooth crash. Structure disappears at the crash and reappears $\sim 1$ ms after the crash.
4.2.2 \( m=1, n=-5 \): Helical Temperature Peak Forms

During the sawtooth crash the q-profile changes so that \( q(0) \gtrsim 0.2 \). The \( m = 1, n = -5 \) mode, which has a small amplitude before the crash and no correlated temperature fluctuations, is the largest mode shortly after the crash (see Figure 1.5). 3D resistive MHD simulations\(^4\) performed at experimental values of the Lundquist number with the DEBS code\(^5\) show the presence of a \( m = 1, n = -5 \) magnetic structure with intact magnetic flux surfaces just after the crash (see Figure 4.11). Electron temperature fluctuations associated with this mode appear \( \sim 0.5 \) ms after the sawtooth crash and show a strikingly different structure than the flattening seen with the \( m = 1, n = -6 \) mode. Figure 4.12 summarizes the results and a full set of plots is in Appendix C. The temperature fluctuations are all the same sign; there is no phase flip across a rational surface (see Figure 3.6). This results in a temperature profile that peaks off the geometric axis and rotates with a \( m = 1, n = -5 \) helical characteristic. Just after the sawtooth crash, correlated fluctuations are not present, indicating the field is still stochastic. Approximately \( 0.5 \) ms after the crash the helical temperature structure appears. The peak in temperature ramps up reaching its maximum around \( 1.3 \) ms. The length of time in which the \( n = -5 \) mode has a relatively large signal (as measured by the edge coils) varies from sawtooth to sawtooth, but it usually lasts between \( 2-4 \) ms. The temperature fluctuations associated with the \( n = -5 \) mode appear to decrease as the amplitude of the magnetic mode decreases. However, since there is variability in when this happens the uncertainties in the temperature fluctuations are large at that point. Even though electron temperature fluctuations only appear to be correlated with the \( n=-5 \) just after the crash, electron density fluctuations correlated with the \( n=-5 \) are present throughout
the entire sawtooth cycle\textsuperscript{6}. The reasoning behind this is not yet well understood.

![Figure 4.11: Field line puncture plot from resistive MHD simulations shows intact flux surfaces inside the $m = 1, n = -5$ island.](image)

**Estimating local thermal diffusion**

Power balance calculations can be used to estimate the local thermal diffusion within the helical temperature structure. The power balance equation is:

$$\frac{3}{2} \frac{\partial}{\partial t} (p_e) = S - \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho Q_e)$$  \hspace{1cm} (4.3)

The electron pressure is $p_e = n_e T_e$. The peak in electron temperature defines the flux surface coordinate $\rho = 0$. The coordinate $\rho$ represents an appropriate "radial-like" label
Figure 4.12: a) Electron temperature fluctuation amplitude associated with the $m = 1$, $n = -5$ tearing mode at $t = 1.9$ ms after the sawtooth crash. b) Combining mean temperature and fluctuation amplitude results, the electron temperature is plotted across an X-point (left side) and across an O-point (right side) showing an off-axis temperature peak with a helical $m = 1$, $n = -5$ characteristic. c) The maximum fluctuation amplitude grows after the sawtooth crash and peaks at $t \sim 1.3$ ms.
relative to the helically rotating structure. The quantity $S$ describes sources and sinks. The heat flux, $Q_e$ is modeled in terms of the electron thermal diffusivity:

$$Q_e = -\chi_e n_e \nabla \rho T_e$$

(4.4)

Rearranging Equation 4.4 to solve for $\chi_e$ provides the appropriate relation:

$$\chi_e = \frac{\frac{1}{2} \int \left( \frac{\partial}{\partial \rho} \left( \frac{3}{2} n_e T_e \right) - S \right) \rho d\rho}{n \nabla \rho T_e}$$

(4.5)

Assuming the only source is the Ohmic power, $\eta J^2$, and that $n_e$, $\eta$ and $J$ are constant ($n_e$ has not been measured and it may also fluctuate) in the small region near the helical temperature peak we have the following relation:

$$\chi_e = \frac{3 n_e \int \rho T_e \rho d\rho - \rho^2 \eta J^2}{2 n_e \rho \frac{\partial T_e}{\partial \rho}}$$

(4.6)

The electron thermal diffusivity near the $m = 1, n = -5$ mode is shown in Figure 4.13. Before the associated temperature structure appears $\chi_e$ is high (1000 $m^2/s$). As the temperature peak appears $\chi_e$ drops, reaching 30 $m^2/s$ at its lowest, one to two orders of magnitude lower than a stochastic field and about an order of magnitude larger than that in improved confinement plasmas discussed in Section 4.4.

**Estimate of a helical axis perturbation**

During the brief 2-4 ms period after the sawtooth crash when the $n = -5$ mode is large equilibrium reconstructions show $q(0)$ just above 0.2, placing the resonant surface for the mode near the axis. In this case the $n = -5$ mode unlike the $n = -6$ mode would have a resonant surface very close to the axis, perhaps preventing a proper island structure
Figure 4.13: Electron thermal diffusivity near the $m = 1$, $n = -5$ magnetic mode, $\rho = 5$ cm and $\rho = 10$ cm is shown.

from forming. Such a description might explain the difference in character between the two observed temperature structures. However, axi-symmetric calculations of the equilibrium may not be sufficient to describe a plasma with a large helical temperature structure and it may be the case that the $n = -5$ mode is not resonant. It is possible that the off-axis temperature peaking associated with the $n = -5$ mode is because the mode is an ideal kink mode. If the core of the plasma experiences a kink, toroidal axi-symmetry is no longer a valid assumption. The length of the magnetic axis is longer in the helical case than the axi-symmetric one. Equilibrium reconstructions of $q$ assuming axi-symmetry will tend to over estimate $q(0)$. Assuming a force-free equilibrium ($\nabla \times \vec{B} = \lambda \vec{B}$) the normalized parallel current density ($\lambda = \mu_0 \vec{J} \cdot \vec{B}/B^2$) on axis can be written in terms of $q$ and $R$: 
\[ \lambda(0) \propto \frac{2}{qR} \]  

(4.7)

In the axi-symmetric case \( R = R_0 \) and in the helical case it has a slight displacement, \( \xi \).

Assuming that \( \lambda \) is relatively constant in the core, the displacement of the magnetic axis is estimated by setting \( \lambda(0) \) from axisymmetric case equal to \( \lambda(0) \) in the helical case, providing the relation:

\[ R = \frac{q_0 R_0}{q} \]  

(4.8)

\[ \xi = \frac{q_0 R_0}{q} - R_0 \]  

(4.9)

On the helical axis, \( q = 1/5 \) and in MST \( R_0 = 1.5 \) m. As a consistency check the location of the off axis peak in the temperature profile is compared to the approximate displacement in Figure 4.14. The estimated displacement and the measured displacement in the electron temperature peak are very similar. This supports the idea that the \( n = -5 \) produces an ideal kink like magnetic surface disturbance and not a tearing mode. 3D resistive MHD simulations show magnetic flux surfaces intact inside of an \( m = 1, n = -5 \) magnetic structure (Figure 4.11) after the crash in contrast to flux surfaces which have been washed out inside of an \( m = 1, n = -6 \) island before the crash (see Figure 1.4). Such an \( m = 1, n = -5 \) structure is superficially similar to the Single Helical aXis (SHaX) structure observed in the RFX-mod\(^7\). However, in MST the \( n = -5 \) mode does not dominate the spectrum, and is only slightly larger than the next largest mode \( (n = -6) \) immediately after the sawtooth crash in MST. This is in marked contrast to the SHaX structure in which a single mode dominates the spectrum. There
is some similarity to SHaX operation as confinement in the vicinity of the post-sawtooth crash $m/n = 1/5$ structure is substantially improved.

### 4.3 Fluctuations through a core event

Core events happen much less frequently in standard MST plasmas than sawteeth. They are of interest because of their similarity to both sawteeth and QSH states. The ramp-up phase of the core mode is similar to the initial ramp-up to a QSH state, although the crash of the mode often happens before a QSH state is achieved. As the mode ramps up $q(0)$ is lowered slightly so that the resonant surface for the $n = -6$ mode is near the core\(^8\). In this way the $n = -6$ mode is similar to the $n = -5$ mode just after a sawtooth crash. However results presented in this section show that the temperature fluctuations associated with this mode are more typical of the temperature flattening across a remnant island.

In both sawteeth and core events magnetic fluctuation amplitudes rapidly change. In sawteeth this rapid change is due first to a rapid growth of tearing modes and then a quick decrease as the result of the reorganization of magnetic field lines by magnetic reconnection. During a core event the magnetic fluctuation amplitude for the $n = -6$ mode grows slowly and then rapidly decreases as the $n = -7$ and $n = -8$ modes increase. Other than the rapid change in the $n = -6$ magnetic fluctuation amplitude typical signs of magnetic reconnection are not present; ion heating does not occur, the toroidal flux does not change and only a slight change in magnetic energy is observed\(^8,9\). By studying the time history of electron temperature fluctuations associated with core modes through both sawteeth and core events the similarities and differences between
Figure 4.14: a) Reconstructed $q$ on axis (using MSTFit) through a sawtooth crash using ensembled data. Red line shows $q = 0.2$ the $n = -5$ resonant surface. b) Estimated helical displacement (black) and measured electron temperature profile peak (red) after the sawtooth crash.
these two types of events is investigated.

Section 4.3.1 describes the selection criteria and typical magnetic behavior of core events. Section 4.3.2 presents the equilibrium and temperature fluctuation measurements correlated with the $n = -6$ mode through a core event.

4.3.1 Selection of Core Events

The ensemble of core events corresponding to the results presented in Section 4.3.2 has the same plasma parameters as the rest of the standard plasma discharges presented in this chapter ($I_p \sim 400$ kA, $n_e \sim 10^{13} \text{ cm}^{-1}$, $F \sim -0.2$). The ensemble for each time/radial location is fairly small with only 10-20 events included in each. Special care is taken to ensure the similarity of each event in the ensemble. One key difference between a sawtooth event and a core event is the absence of $m = 0$ mode activity during a core event. Figure 4.15 shows the ensemble averaged magnetic fluctuation amplitudes through a core event. All core events in this ensemble have $m = 0$, $n = 1$ mode amplitudes which are nearly constant and small throughout the drop-off in the $n = -6$ mode. Core events are often followed closely by a sawtooth event and so even though the $m = 0$ modes are flat during the core event they often begin to spike up at a sawtooth soon afterward. The drop-off of in the $n = -6$ mode occurs over no more than 200 $\mu$s and the normalized toroidal magnetic fluctuation for $n = -6$ is at least 0.4 before the drop in amplitude. The normalized toroidal magnetic fluctuation is defined as:

$$b_{t,norm} = \frac{b_{t,-6}}{\sum_{n=5}^{5} b_{t,n} - b_{t,-6}}$$

(4.10)
Figure 4.15: Magnetic fluctuations ensembled through a core event.

Here, $b_{t,n}$ is the toroidal magnetic fluctuation for tearing mode $n$. During the ramp-up of the $n = -6$ mode the amplitude of the $n = -7$ and $-8$ magnetic fluctuations are constant until $\sim 400 \ \mu s$ before the dominant mode drop-off when they slowly rise, peaking $\sim 100 \ \mu s$ after the dominant mode begins to drop-off. The $n = -5$ magnetic fluctuations are small and show no change throughout the entire core event. The next section focuses on the evolution of the electron temperature equilibrium and fluctuations associated with the $n = -6$ mode through a core event.
4.3.2 $m=1, n=-6$ Electron Temperature Evolution Through a Core Event

Electron temperature fluctuation analysis is done in 200 µs bins relative to the core event. The ramp up time can vary from event to event and core events are often followed by sawteeth. To maximize consistency between events in the ensemble the analysis is only performed for 1 ms prior and 0.5 after the event. Results are presented in Figures 4.16 and 4.17.

Before the core event temperature fluctuations associated with the $n = -6$ mode have the same characteristics of $n = -6$ temperature fluctuations before a sawtooth. The only notable difference is that the fluctuations are larger in magnitude by about a factor of two. By combining mean and fluctuating temperature, a slight temperature peak can be seen in the temperature on axis, with a flattening across the $n = -6$ island and a slight bump in temperature around $r/a = 0.4$. In a similar manner as $n = -6$ temperature fluctuations before a sawtooth the temperature fluctuations gradually decrease before the core event and all fluctuations drop to (within uncertainty) zero at the event. The amplitudes of the temperature fluctuations begin to decrease when the secondary $n = -7$ and $n = -8$ modes begin to slowly ramp up before the event ($\sim 400$ µs). At this same time all structure in the core temperature begins to wash out. This implies increased stochasticity in the core. However, unlike the sawtooth crash, global temperatures do not decrease. The decrease in temperature seems to be only localized in the core region. The rapid equilibration of the core temperature is likely due to a change in the magnetic configuration of the core caused by magnetic reconnection. Although the resonant surface for the $n = -6$ mode moves in close to the magnetic axis and the
Figure 4.16: Electron temperature fluctuations for $n = -6$ mode through a core event are shown. Negative $r/a$ values correspond to $\Delta_{TS-6} = \pi$ (X-point) and positive $r/a$ values correspond to $\Delta_{TS-6} = 0$ (O-point)
Figure 4.17: Electron temperature profiles across $n = -6$ structure through a core event. Negative $r/a$ values correspond to $\Delta T_{S-6} = \pi$ (X-point) and positive $r/a$ values correspond to $\Delta T_{S-6} = 0$ (O-point)
mode amplitude is large before the event, temperature structures in this case show little resemblance to the $n = -5$ fluctuation seen after a sawtooth crash or QSH/ShaX events seen in RFX$^7$ as hypothesized.

4.4 Parallel Current Profile Control

Magnetic tearing mode amplitudes in the RFP can be reduced by inductively controlling the parallel current profile and reducing the free energy source for these modes$^{10}$. The $m = 1, n = -6$ fluctuation goes from $\sim 10$ G in standard plasmas to $\lesssim 3$ G in plasmas with parallel current profile control. The sawtooth cycle disappears. Electron thermal diffusion is low ($\sim$5-10 $m^2/s$), the global energy confinement time increases and the core electron temperature increases four-fold$^{11,12,13}$, in this case. The quality of plasma confinement can vary greatly in these plasmas, making it difficult to find many shots that are qualitatively and quantitatively similar enough to ensemble. Selection of plasmas for the ensemble is based on peak core electron temperature, low total magnetic fluctuations and low $n = -6$ fluctuation. Temperature measurements have then been further binned into time from the last burst in magnetic activity. Electron temperature fluctuations have been studied in these plasmas with plasma currents of 200 kA (Figure 4.18) and 400 kA (Figure 4.19).

Electron temperature fluctuations associated with tearing modes are reduced to $\sim 1 - 2\%$ of the mean temperature, compared to $5 - 30\%$ in standard plasmas. No clear electron temperature structure is associated with these modes, although in the 200 kA case all fluctuations appear to be positive indicating that the O-point of the mode is slightly warmer than the X-point. Temperature fluctuations also do not seem to show
a temporal evolution. In general when magnetic fluctuations are small they have small
temperature fluctuations associated with them.
Figure 4.18: Electron fluctuations associated with small \((m = 1, n = -6)\) magnetic fluctuations during 200 kA plasmas where inductive parallel current profile control is applied. Time bins are measured relative to the last burst in magnetic activity. Background temperatures are \(\sim 300\) eV at 1-2 ms up to \(\sim 600\) eV at 3-4 ms.
Figure 4.19: Electron fluctuations associated with small \((m = 1, n = -6)\) magnetic fluctuations during 400 kA plasmas where inductive parallel current profile control is applied. Time bins are measured relative to the last burst in magnetic activity. Background temperatures are \(\sim 800\ \text{eV}\) at 1-2 ms up to \(\sim 1300\ \text{eV}\) at 3-4 ms.
REFERENCES


Chapter 5

Summary

Advanced Thomson scattering diagnostic capabilities and innovative Bayesian analysis techniques enable exploration of fast electron dynamics associated with tearing modes in relatively quiescent periods during standard discharges, through dynamic sawteeth, core reconnection events and plasmas with improved confinement and reduced tearing mode activity. A summary of the work presented in this thesis is presented below in Sections 5.1 thru 5.3 and suggestions for future work are in Section 5.4.

5.1 Advanced Thomson Scattering Diagnostic

A multi-point, multi-pulse Thomson scattering diagnostic has been, built, calibrated and commissioned on MST. The system is based on two commercial Spectron, 2.5 J, Nd:YAG lasers, which have been upgraded by increasing flash lamp pulse lengths and the number of q-switches for each flash lamp pulse to produce up to 15 pulses each per plasma discharge. Thomson-scattered light is detected with 21 General Atomics polychromators equipped with avalanche photodiode modules and digitized with 1 GS/s Acqiris digitizers. Overall the system is capable of measuring changes in the radial electron temperature profile with a temporal resolution of 25 kHz in a six pulse burst every millisecond with a spatial resolution of 2 cm or less.
Detailed calibrations, suitable for electron temperature fluctuation measurements, of the Thomson scattering system have been made. All calibrations have taken place focusing on accuracy, ease of use and repeatability with in-situ measurements wherever possible. APD gain, quantum efficiency and intrinsic noise levels have been measured. Spectral calibrations have been performed on the bench with a DC light source and in-situ with a pulsed light source. Benchmarks for spectral transmission of the entire collection optics and individual APD performance have been made and procedures are in place to regularly check them throughout a regular operation schedule. Novel calibration processes have been developed including using an avalanche photodiode as a reference detector for absolute photon calibration to simplify setup and reduce noise, using an insertable integrating sphere to couple to an optical parametric oscillator to perform in-situ, pulsed spectral calibrations and measuring the quantum efficiency of an APD through a DC spectral calibration. The system is highly dependent on the temperature of the detectors. So the environment of the detectors is controlled in an air conditioned room with water cooling and detector temperatures are individually monitored for stability.

Raw electron temperature data is fit simultaneously in all channels of a polychromator with a characteristic pulse shape. This helps to reduce uncertainty from q-switch noise, low pulse amplitude paired with low bit resolution and pulse timing. The most likely electron temperature is fit with Bayesian analysis techniques.
5.2 Innovative fluctuation analysis

The lack of a continuous electron temperature measurement makes it difficult to measure fluctuating quantities. Electron temperature is mapped onto tearing modes and a simple fluctuation model is used to describe the relationship between a single tearing mode and electron temperature. Bayesian probability theory is used to properly ensemble temperature data from different discharges. Two possible scenarios of the electron temperature and tearing mode relationship are presented: an isothermal island and an island with a temperature peak. Isothermal islands exhibit a change in sign of the amplitude of the electron temperature fluctuation across the rational surface for the mode. In islands with a temperature peak the amplitude of the fluctuations are all positive with the largest amplitude sampling the hot spot.

5.3 Physics results

During quiescent plasmas between sawteeth, the electron temperature has been measured to have a complex 3-D structure which follows the structure of magnetic tearing modes. Correlated fluctuation measurements show that electron temperature is flattened across magnetic tearing modes inside island separatricies. Fluctuations associated with the largest tearing mode \((m = 1, n = -6)\) give a clear picture of a remnant isothermal island, pinpoint the rational surface for the mode and provide an estimate of the island width. Correlated fluctuations have also been measured for the \(n = -7\) and \(n = -8\) modes. Fluctuations associated with \(n = -7\) mode are similar in character to the \(n = -6\) fluctuations. Fluctuations correlated with the \(n = -8\) mode show an opposite phase relation to those of the \(n = -6\) and \(n = -7\) modes indicating an off-axis bump in
the temperature profile. This off-axis bump is also seen in equilibrium temperature and may be caused by a localized heating source such as a local maximum in the parallel current profile.

Correlated electron temperature fluctuations for the $m = 1, n = -6$ mode have also been measured through a sawtooth crash and are observed to have a strong sawtooth dependence. Correlated temperature fluctuations disappear at the sawtooth crash as the magnetic field becomes stochastic and the remnant island structure is destroyed. Shortly after the sawtooth crash the electron temperature exhibits a large helical peaking associated with $m = 1, n = -5$ magnetic perturbation indicating relatively good confinement within the island. Local power balance calculations suggest reduced thermal transport with values of the electron thermal diffusivity near those for improved confinement plasmas. These results also suggest that a helical equilibrium may be better suited than an axi-symmetric toroidal equilibrium to describe the plasma after a sawtooth crash. Estimates of the position of the helical axis have been compared to measurements of the electron temperature peak and are consistent. This requires a lower value of $q$ on axis such that the $m = 1, n = -5$ mode is resonant on axis or non-resonant altogether. Although the plasma is not in a QSH/ShaX state at this point in time, results parallel those for SHaX states seen in the RFX device\(^1\).

Electron temperature fluctuations for the $m = 1, n = -6$ mode through core events are similar in nature to those through a sawtooth crash. The remnant island structure as observed through temperature fluctuations disappears as the magnetic field becomes stochastic. Mean electron temperature is flattened in the core region but global electron temperature does not drop suggesting that the magnetic reconnection taking place is limited to the core region only. Although the gradual ramp-up of the dominant mode
before a core event is similar to the beginning of a QSH state electron temperature does not peak with this mode as expected from a QSH state.

During improved confinement via parallel current profile control, tearing instabilities are reduced. Correlated electron temperature fluctuations are also reduced. Observed fluctuations in 200 kA and 400 kA plasmas are small and do not represent the canonical isothermal island or a hot spot within an island. Instead the fluctuations suggest a slightly hotter temperature (1 – 2%) in the O-point of the island as compared to the X-point.

5.4 Future Work

As with any interesting scientific endeavor many more questions are likely to be brought about by the course of study than answered. Looming diagnostic improvements, suggested improvements to equilibrium reconstructions and some interesting studies for the future are discussed below.

5.4.1 Diagnostic Upgrades

The Thomson scattering diagnostic on the MST is in a constant state of flux. There are currently several upgrades to the system in the works. Soon the current Spectron lasers will be superseded by a pulse-burst laser system\(^2\) currently being commissioned. This fast laser will produce \(\sim 200\), 1 J pulses at rates between 5 and 250 kHz significantly reducing the number of plasma shots needed to do electron temperature fluctuation analysis. Single-shot fluctuation analysis may even be possible which is especially beneficial to studies where it is hard to obtain reproducible plasmas for an ensemble such as for
core events and improved confinement plasmas discussed in this thesis.

All four-channel polychromators are currently being upgraded to six-channel polychromators. The extra channels will extend the collected scattered light spectrum to higher frequencies. This will increase the upper boundary of resolvable temperature for these polychromators from \( \sim 2 \) keV to \( \sim 10 \) keV. The hottest MST plasmas are already near the current 2 keV limit. By upgrading to six channel polychromators it will ensure the usefulness of the diagnostic as MST plasmas get hotter in the future and also enable fluctuation measurements at these hot temperatures as uncertainties will be comparatively lower for \( \sim 2 \) keV plasmas.

A set of 100 MHz, 14 bit digitizers has recently been purchased and is currently being installed so that both the DC and 100 ns delay line subtraction output from the APD modules can be simultaneously digitized. This will improve the quality of data taken by the system by eliminating shots lost to digitizer saturation and reducing bit noise. Fluctuation measurements will benefit from this increase in data quantity and quality.

Although the Thomson scattering diagnostic has the ability to measure electron density in addition to electron temperature the necessary calibrations have not been performed to obtain an absolute density from the data gathered by the diagnostic. Absolute density calibrations via Raman or Rayleigh scattering should be performed and measurement of electron density fluctuations can be made in a similar manner as the temperature fluctuations described in this thesis.
5.4.2 Improvements to equilibrium reconstructions

Results from electron temperature fluctuations correlated with magnetic tearing modes can improve equilibrium reconstructions. Fluctuation measurements can be used to locate the rational surface of tearing modes as with the \( m = 1, n = -6 \) measurements presented in this thesis. Currently this information cannot be used by equilibrium fitting routines such as MSTFit, however this could be a useful addition to the routine with future revisions to help further constrain the q-profile in MST. In light of the realization that the \( m = 1, n = -5 \) mode causes the plasma core to become helical in nature, and to be applicable to SHaX states, MSTFit could be revised or rewritten to use a helical equilibrium instead of being constrained by axi-symmetry. Reconstructions could also be improved with the implementation of integrated data analysis using Bayesian probability theory which is being utilized more and more frequently in the plasma physics community\(^3\)\(^-\)\(^5\).

5.4.3 Further investigation of the characteristics of islands

The electron thermal characteristics of magnetic islands in the RFP are only a small part of the overall picture. For example, a complete analysis of the electron thermal diffusivity requires not only knowledge of correlated electron temperature fluctuations but electron density fluctuations as well. With density calibrations for the Thomson scattering diagnostic this will be possible. Adding in other diagnostics will also help in working towards a more complete picture. TS density measurements compared with the line integrated FIR measurements may shed light on the question as to why FIR measures density fluctuations with the \( n = -5 \) mode before a sawtooth, but TS does not
detect temperature fluctuations. Correlated FIR measurements may also help identify the \( m = 1, n = -5 \) mode after a sawtooth crash to be either a kink or tearing mode by probing the internal magnetic structure.

Studies in other plasma regimes may also be interesting, particularly at higher current and during QSH. QSH is difficult to study for three main reasons. QSH occurs relatively infrequently, so it is hard to catch with the Thomson scattering diagnostic. However, with the new pulse burst laser system described above 10 kHz temperature measurements can be made for 20 ms, making it easier to catch QSH occurrences. The reproducibility of one QSH occurrence and the next QSH occurrence is difficult to qualify, making it difficult to put together a large ensemble of similar shots. Although with the large amount of data that will available in just one shot from the pulse-burst laser system the number of shots required to quantify the electron temperature fluctuations will be significantly lower than it currently is. Finally, QSH is difficult to study because once QSH forms the plasma usually stops rotating and in order to quantify fluctuations, the electron temperature must be measured at different phases of the tearing mode. Still, with the diagnostic improvements it may be possible to ensemble shots locked at different phases relative to the TS diagnostic. Do magnetic islands formed by QSH flatten the temperature profile like the \( m = 1, n = -6 \) mode during the ramp-up phase before a core event or do they exhibit a temperature peaking like the \( m = 1, n = -5 \) mode after a sawtooth crash? Although results from core events presented in this thesis may suggest the former evidence from high current plasmas might support the latter. Figure 5.1 shows the \( n = -5 \) tearing mode signal as measured by the edge coils for a plasma with \( I_p \sim 600 \text{ kA} \). The \( m = 1, n = -5 \) mode instead of decaying away after the sawtooth crash, remains flat for a while and then rapidly increases into a QSH state. Future
studies into the nature of the temperature structure associated with this kind of event should be very interesting.

Figure 5.1: Poloidal magnetic fluctuation for the $m = 1$, $n = -5$ mode as measured by edge coils in a 600 kA plasma.
REFERENCES


Appendix A

A brief introduction to Bayesian Probability Theory

A full and complete tutorial on Bayesian probability theory can be found in D.S. Sivia’s book “Data Analysis: A Bayesian Tutorial”. What follows here is a brief introduction which should be sufficient to understand the data analysis techniques used in this thesis.

The reasons for using Bayesian analysis for finding the most likely electron temperature and for fluctuation measurements discussed in this thesis are twofold. First is that Bayesian analysis provides a useful, straight-forward framework to analyze data given all of the available information. Secondly, and perhaps more importantly, this framework properly deals with propagation of measurement uncertainties in a easy straight-forward manner.

Bayesian probability theory, from which Bayesian analysis is derived, is a different framework than orthodox statistics. The main difference lies within the definition of ”probability”. In orthodox statistics probability represents the long-run relative frequency of an event. A common example is how many times a coin flipped a thousand times turns up heads verses how many times it turns up tails. While this may seem like a reasonable definition it is not very useful since often in experiments we are not able to measure the same thing a thousand times. In contrast to this Bayesian probability
theory defines probability as the plausibility that something is true given the available information. In the coin example Bayesian probability theory quantifies the plausibility of whether or not the coin is fair given the information at hand and has a built in uncertainty.

First a brief note on notation. The statement below is read ”The Probability that a specific value of X is true given Y and any background information, I”

\[ P(X|Y,I) \] (A.1)

Bayesian probability theory is based on two simple rules of basic probability theory. The product rule:

\[ P(X,Y|I) = P(X|Y,I)P(Y|I) \] (A.2)

and the sum rule:

\[ P(X|I) + P(\bar{X}|I) = 1 \] (A.3)

(\( \bar{X} \) means that ”X is not true”.)

The basic idea behind Bayesian analysis is that the probability that a given parameter, X, is true given all the data, d, uncertainty, \( \sigma \), and any relevant background information, I, \( P(X|d,\sigma,I) \), can be found by calculating three other easier to obtain quantities. These are the likelihood probability distribution (PDF), \( P(d|X,\sigma,I) \), the probability of getting the data given X, the prior PDF, \( P(X|I) \), the probability of measuring X given relevant background information (e.g. temperature cannot be a negative number) and the evidence, \( P(d|I) \), a normalization factor.

\[ P(X|d,\sigma,I) = \frac{P(d|X,\sigma,I)P(X|I)}{P(d|I)} \] (A.4)

Equation A.4 is called Bayes Theorem and follows directly from the product rule (Equation A.2). The left hand side of Equation A.4 is called the posterior PDF. Bayes
Theorem sets up a framework in which the unknowns can be calculated in terms of the knowns in a straightforward way. Another useful corollary of probability theory is marginalization, which allows the probability calculation for interesting quantities while getting rid of the dependence on uninteresting quantities, or nuisance parameters. This is advantageous because the calculation can be done without assuming the value of the nuisance parameter is known. The marginalization procedure is defined as:

\[ P(X|I) = \int_{-\infty}^{\infty} P(X,Y|I) dY \tag{A.5} \]

Because Bayesian analysis returns the entire PDF no information has been lost. As more data is gathered more information is added to the posterior PDF and a true representation of "the answer" would be to display the entire PDF. However, since we are used to thinking in terms of a best fit value plus error bars, the value with the maximum probability is taken to be the "best fit" and the points at which the PDF drops to 1/e of its maximum define the error bars.
Appendix B

m=1, n=-6 Temperature Fluctuations Through A Sawtooth

The following pages contain results of correlated temperature fluctuations associated with the m=1, n=-6 tearing mode through a sawtooth. Time relative to the sawtooth is denoted in ms. Plots on the left are the electron temperature profile across an X-point ($\Delta_{TS_{-6}} = \pi$) on the left (negative radius) and across an O-point ($\Delta_{TS_{-6}} = \pi$) on the right (positive radius). Plots on the right are of the temperature fluctuation amplitude versus minor radius.
Appendix C

$m=1, n=-5$ Temperature Fluctuations After A Sawtooth

The following pages contain results of correlated temperature fluctuations correlated with the $m=1, n=-5$ mode after a sawtooth. Time relative to the sawtooth is denoted in ms. Plots on the left are the electron temperature profile across an X-point ($\Delta_{TS_{-5}} = \pi$) on the left (negative radius) and across an O-point ($\Delta_{TS_{-5}} = \pi$) on the right (positive radius). Plots on the right are of the temperature fluctuation amplitude versus minor radius.
0.1 ms

0.3 ms

0.5 ms

0.7 ms
2.5 ms